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Monotonicity and the typology of front/back harmony

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Abstract: In this paper, we analyse the surface patterns of suffix harmony in front/back harmony systems as the harmonic values front and back being assigned to harmonic contexts consisting of strings of syllables combining front, back and neutral nuclei. We claim that the harmonic contexts can be arranged in a fixed (universal) scale, the frontness/backness scale, with reference to which the possible (i.e. attested) front/back harmony systems can be characterised in a simple way: only those systems are possible where the assignment of values to the harmonic contexts is monotonic, which makes sure that only contiguous (non-interrupted) sequences of harmonic values are assigned to the harmonic contexts. We give formal definitions of monotonicity in terms of ordering and similarity and discuss predictions about possible harmony systems that follow from monotonicity (which we claim are borne out). These predictions are typological: harmony systems can show disharmonic behaviour, but in a principled way: only those systems exist (at least in front/back harmony) that exhibit a monotonic pattern. In the second half of the paper, we discuss variation in harmony and analyse in detail variation in anti-harmony and transparency in Hungarian, which is thus an example of a variable harmony pattern. We argue that variable front/back harmony patterns assign a "variable" value to some harmonic contexts in addition to the front and back values and can be shown to be constrained by monotonicity, whose definition is naturally extendable to patterns with variation. We discuss both the ordering-based and the similarity-based definitions and the predictions about possible variable harmony systems that follow from the definitions. One of the main predictions of the paper is a consequence of monotonicity: the locus of the variation in a pattern occurs only "in between" non-variable subpatterns. We explore a possible way

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of quantification in which we identify the harmonic values with the relative token frequency of the forms where the number associated with a variable value is p such that $0<p<1$. We show that monotonicity can be defined for quantified patterns too both under the ordering interpretation and the similarity-based one. We conclude by discussing the predictions of the quantified model and showing (based on a corpus study we carried out to discover the 'frontness ratio' of variable sites of suffix harmony) that the quantified variable front/back harmony pattern of Hungarian is monotonic and conforms to these predictions.

Keywords: vowel harmony, front/back harmony, disharmony, transparency, harmonic opacity, anti-harmony, variation, monotonicity, similarity

1 Introduction: disharmony in vowel harmony

In vowel harmony systems, where vowels agree in their specifications for some designated feature(s) (the harmonic features) within a prosodically or morphologically circumscribed domain, there is usually also a certain degree of *disharmony* even within the domain in which harmony applies,[1] i.e. vowels may co-occur that disagree in their specifications for the harmonic feature(s). There are basically two subtypes of disharmony: weak and strong. *Weak disharmony* involves *neutral vowels* co-occurring with harmonic ones, *strong disharmony* does not. For instance, the morphemes *sofőr* /ʃoføːr/ 'driver' and *glükóz* /glykoːz/ 'glucose' are strongly disharmonic in Hungarian because the pairs of vowels in these morphemes disagree in their front/back value, and none of the participating vowels counts as neutral in the language (the morphemes are of the [BF] and [FB] type, respectively). In this paper, we will focus on the typology of weak disharmony. There are several distributional subtypes of weak disharmony (which we discuss in detail below), but the main difference between weak and strong disharmony is that the former can occur systematically in a harmony system as opposed to the latter, which does not.[2]

1 Naturally, there may be a lack of harmony across the boundary of a harmonic domain, but we will only use the term *disharmony* to refer to the lack of harmony *within* a harmonic domain. For example, in many languages the elements of compounds and certain affixes form separate harmonic domains – the lack of harmony that may occur between vowels across these domains does not constitute disharmony. See e.g. Archangeli and Pulleyblank (2007), Rose and Walker (2011).

2 There is a general agreement in literature on vowel harmony that strong disharmony is exceptional, which manifests itself in the low-type frequency and the morpheme-specific character of this type of disharmony. Research into exceptionality in vowel harmony has proposed other (putative) characteristics, e.g. locality (Finley (2010), Mahanta (2012), but see also Bowman and Lokshin (2014) and Törkenczy (2013)).

In this study we will concentrate on front/back harmony, although some of the generalisations we propose are meant to hold for vowel harmony involving other features too. Furthermore, we only examine root/stem-controlled systems where harmony triggers alternations in *suffixes*, i.e. it propagates from left to right (but our analysis would also work in the same way if harmony spread right to left, from stem to prefix).

In analyses of vowel harmony, the classification of vowels into harmonic ones vs. neutral ones in a given harmony system is usually assumed to be categorical and not gradual (e.g. van der Hulst and van de Weijer 1995). This is an over-simplification since neutrality/weak disharmony manifests itself in several different phenomena which do not always classify vowels in the same way (Kiparsky and Pajusalu 2003; Rebrus et al. 2012; Törkenczy et al. 2013). Nevertheless, for simplicity's sake and for ease of exposition, in this paper we will assume that in a given system *all* neutral vowels behave in the same way, i.e. all the neutral vowels meet the criteria for neutrality that are relevant in the given harmony system.

There has been a lot of research on and various typological generalisations have been made about the *quality* of neutral vowels.[3] Since our primary concern here is the possible *patterns* of weak disharmony (in front/back harmony), we will not discuss the quality of neutral vowels and only assume that neutral vowels in front/back harmony systems are (phonetically) front (which seems to be the most frequent case typologically, cf. Aoki 1968; Anderson 1980; Kiparsky and Pajusalu 2003; Archangeli and Pulleyblank 2007; Rose and Walker 2011; Gafos and Dye 2011).

2 Distributional properties of neutral vowels

Let us examine the possible ways in which weak disharmony can arise. In a given harmony system neutral vowels may have the ability to co-occur with members of either harmonic class of vowels *within the morpheme*. In a system where this is permitted there are "mixed roots" in which a neutral vowel may appear to the right or to the left of harmonic vowel(s):[4]

3 It is often argued that the phonological status of neutral vowels in a given harmony system and across systems is non-accidental and follows from their representations (neutral vowels are represented in some special way underlyingly), unmarked character (neutral vowels are unmarked), unpairedness (neutral vowels are prevented from alternating because they have no harmonic counterpart in the inventory), etc. (cf. Harris 1994; Cole and Kisseberth 1994; Kiparsky and Pajusalu 2003; Rhodes 2010; Rose and Walker 2011).

4 Notation: F and B are harmonic front and back vowels, respectively, N stands for a neutral vowel (which we assume is phonetically front), and [,] are morpheme boundaries. Consonants are not indicated since we focus on the case when consonants are invisible to vowel harmony.

(1) a. [B/F N] b. [N B/F]

This state of affairs can be seen as *lexical variation*: vowels of both harmonic classes may appear in the same environment ([_N] and [N_], respectively) in *different* lexical items. In a system that permits this type of weak disharmony, the type frequency of weakly disharmonic mixed roots is relatively high compared to the type frequency of roots with strong disharmony (if the latter occur at all). For instance, in Hungarian, short and long /i, iː/ are neutral and roots combining back vowels and /i, iː/ (e.g. *hamis* /hɒmiʃ/ 'false', *ásít* /aːʃiːt/ 'yawn' and *világ* /vilaːg/ 'world', *kínál* 'offer' /kiːnaːl/) vastly outnumber strongly disharmonic mixed roots.[5]

In a harmony system neutral vowels may co-occur with vowels of both harmonic classes *across morphemes* too. In a language where this occurs, this state of affairs can manifest itself in two ways: the neutral vowel can occur (a) in the suffix or (b) in the stem. In the first case, the neutral vowels occur in non-alternating (invariant) affixes that can combine with stems whose harmonic trigger vowels may be of either harmonic class (F or B); this is shown in (2).[6] While the neutral vowels (can) occur in invariant affixes, the harmonic vowels only occur in harmonically alternating affixes.[7]

(2) [[B/F] N]

In the second case, a root that only contains neutral vowels may co-occur with affixal vowels of either harmonic class (contained in a harmonically alternating affix):

(3) [[N] B/F]

(3) may be interpreted as *lexical variation* when some all-neutral stems consistently take the B alternant and others the F alternant of harmonically alternating affixes. This kind of lexical variation is attested in some front/back harmony

5 The type frequencies of [i(:)B] and [Bi(:)] roots are 521 and 322, respectively; compare strongly disharmonic [FB] and [BF] stems whose type frequencies are 6 and 22, respectively (here F stands for the clearly non-neutral front vowels: [y(:)] or [ø(:)] in Hungarian). The data are from the Szószablya Webcorpus (Halácsy et al. 2004; Szószablya 2014).

6 Naturally, the root can contain neutral vowels too; in this case the form will be [[N]N]. This pattern is not relevant here, so it is not considered in (2) and (3).

7 Invariant affixes with harmonic vowels do exist in some systems (e.g. the Turkish progressive suffix/-(i)jor/, the Hungarian temporal suffix/-kor/(e.g. Kabak 2011; Siptár and Törkenczy 2000), which may cause strong disharmony, considered exceptional/irregular here. These are often analysed by positing separate harmonic domains, cf. footnote 1.

systems (see Krämer 2003; Hungarian: Vago 1980; Rebrus 2000; Siptár and Törkenczy 2000; Rebrus et al. 2012; Törkenczy et al. 2013; Uyghur: Cobb 1993; Halle et al. 2000; Vaux 2000; Kiparsky and Pajusalu 2003). For instance, Hungarian long and short *i*-s in all-neutral stems behave in this way: compare the stems *hír* /hiːr/ 'news' and *sír* /ʃiːr/ 'tomb': e.g. *hír-ben* /hiːrbɛn/ vs. *sír-ban* /ʃiːrbɒn/ 'INESSIVE', *hír-ek* /hiːrɛk/ vs. *sír-ok* /ʃiːrok/ 'PLURAL', *hír-től* /hiːrtøːl/ vs. *sír-tól* /ʃiːrtoːl/ 'ABLATIVE', *hír-ünk* /hiːrynk/ vs. *sír-unk* /ʃiːrunk/ 'PL1.POSS'. (3) may also be interpreted as *vacillation* when the same all-neutral stem may take both the B alternant and the F alternant of harmonically alternating affixes. In systems where (3) is permitted, the occurrence of an affixal harmonic vowel whose phonetic value for the harmonic feature disagrees with that of the neutral vowel ([[N]B]) is called *anti-harmony*, and roots of this type are called anti-harmonic roots (e.g. the stem *sír* 'tomb' above).

In languages where neutral vowels can follow a back harmonic vowel, there are two ways in which neutral vowels can affect suffixation: they may be opaque or transparent. An *opaque* neutral vowel, which is contained in a BNF-type vowel sequence (4a), is only locally weakly disharmonic on the left side of the neutral vowel (BN). A *transparent* neutral vowel, which is contained in a BNB-type vowel sequence (4b), is locally weakly disharmonic on both sides of the neutral vowel (BN and NB).[8]

According to the morphological affiliation of the participating vowels, opacity/transparency can arise in three different ways: (i) monomorphemically, where all the vowels are contained in the same root, (ii) after a mixed [BN]-root in a suffix or (iii) after a [B]-stem followed by a neutral suffixal vowel.[9] This is summarised in (4i–iii).

8 In Hungarian, this is completely productive for the neutral vowels /i iː/: [BN]-type stems almost always get a back harmonic suffix, while [FN]-type stems get a front one, cf. *Madrid-ban* /mɒdrid-bɒn/ 'in Madrid' vs. *Berlin-ben* /bɛrlin-bɛn/ 'in Berlin'. The same is also true if the stem is not monomorphemic: [B[N]] and [F[N]], cf. *bonn-i-ak* /bonː-i-ɒk/ 'Bonn-ADJECTIVAL-PLURAL' vs. *brünn-i-ek* /brynː-i-ɛk/ 'Brno-ADJ-PL'. Note that the vowel *e* /ɛ/ behaves as a neutral vowel in some respects (occurrence in mixed roots, limited and variable transparency, an issue we disregard here, cf. Hayes and Londe 2006). In these examples we consider it harmonic front.

9 Since here we want to focus on productive effects of opacity/transparency as manifested in harmonic alternations, we will discuss the polymorphemic environments (4ii–iii) rather than the monomorphemic case (4i). In addition to (4i–iii) there is another possibility, too: the suffix can be polysyllabic, its first vowel being neutral while the second one (or following ones) harmonising: [[B]NF] or [[B]NB]. This case seems to be less frequent. Opacity/transparency can occur within combinations of suffixes too: in triply suffixed forms where the vowel of the first suffix is back harmonic and the second one's vowel is neutral: [[[[...]B]N]F] and [[[[...]B]N]B] and in doubly suffixed forms that contain a bisyllabic BN-type suffix + a monosyllabic suffix: [[[...]BN]F] and [[[...]BN]B].

(4) Transparency/opacity and morphological constituency

	i. monomorphemic	ii. suffix after stem	iii. suffix after suffix
a. opaque N	[BNF]	[[BN]F]	[[[B]N]F]
b. transparent N	[BNB]	[[BN]B]	[[[B]N]B]

The harmonic behaviour of vowels following an FN sequence is typically straightforward. Since both F and N vowels are phonetically front, both the opacity and the transparency of N result in the same sequence: FNF. The disharmonic pattern FNB is also logically possible (and does occur in some harmony systems, cf. Kiparsky and Pajusalu 2003), but can be attributed to neither the transparent nor the opaque character of the neutral vowel N. This is a kind of anti-harmonic behaviour, which may in principle co-occur with both an opaque and a transparent N in the context BN_ (i.e. with the patterns BNF and BNB, respectively). If it combines with opacity, then the two patterns together (FNB and BNF) yield the opposite of transparency – therefore we will refer to this as *anti-transparency*. If FNB combines with transparency, then the two patterns together (FNB and BNB) yield the opposite of opacity – therefore we will refer to this as *anti-opacity*.[10] The theoretical possibilities of disharmony in the environments BN_ and FN_ are summarised in (5) below:

(5) The four logical possibilities for disharmony after BN and FN sequences

terminology	vowel sequences	
a. opacity	BN**F**	FN**F**
b. transparency	BN**B**	FN**F**
c. “anti-transparency”	BN**F**	FN**B**
d. “anti-opacity”	BN**B**	FN**B**

Not all of the patterns in (5) are attested in harmony systems and the gaps are non-accidental. We will discuss the typology and the relevant constraints in Section 3.

The possibilities of weak disharmony between roots and suffixes are summarised in table (6) below.

10 We will see that the latter of these two strange patterns *is* attested in languages in addition to the often discussed patterns of opacity and transparency.

(6) Types of weakly disharmonic sequences (stem + suffix)

types of disharmony	V_1V_2	cf.
i. **invariant affixes** with N	[[B]N]	(2)
ii. **anti-harmony** with all-N stems	[[N]B]	(3)
iii. **transparency** or anti-opacity of N	[[BN]B]	(5b,d)
iv. **anti-opacity** or anti-transparency of N	[[FN]B]	(5c,d)

Some of these patterns, namely, transparency/anti-opacity (6iii) presuppose the existence of weak disharmony in stems: they involve either weakly disharmonic roots (1a) or stems containing invariant suffixes (2 or 6i). Otherwise opacity/transparency-type disharmony could not arise.

As indicated at the beginning of this section, we will study weak disharmony, specifically, *we examine the ways alternating suffixes harmonise after different types of stems* that contain neutral vowels, i.e. we will analyse the four properties in (6i–iv) above. We focus on the patterns of (dis)harmony that can occur as a result of suffixation by harmonically alternating suffixes after stems that are all-neutral [N], back + neutral [BN] or front + neutral [FN] (where we simplify the problem of neutrality by assuming that the neutral vowels are the ones that can systematically occur in harmonically invariant suffixes (6i: [[B] N])). In other words, the central question is whether the vowels that are weakly disharmonic according to (6i) are also disharmonic according to (6ii), (6iii) or (6iv) in a given system.[11]

Weak disharmony is clearly not homogeneous across languages because disharmony according to (6i), for instance, does not necessarily co-occur with

11 Naturally, this question is also relevant to stems that are longer than bisyllabic (or longer than monosyllabic in the case of all-neutral stems) and the patterns of (dis)harmony we find in suffixes attached to them also apply after longer stems if the vowels that precede the vowels relevant to harmony do not influence the harmonic behaviour in the suffix. Thus, e.g. the trisyllabic stems [FFN], [BFN], [NFN] are of the same harmonic type as [FN] since the vowel sequence relevant to harmony is [...FN] (and this extends to stems of any length). The same holds true for the stem type [...BN] (which covers the trisyllabic types [BBN], [FBN] and [NBN] and longer stems ending in these sequences). In some languages, however, this may be complicated by the fact that the number of stem-final neutral vowels counts, e.g. monosyllabic all-neutral stems [N] behave differently from polysyllabic ones [NN^+] with respect to anti-harmony in Hungarian, where [BN] stems and [BNN^+] stems also behave differently with respect to opacity/transparency. We will discuss these problems in Section 4 below (for some approaches to this 'count effect' see Bowman 2013; Hayes and Cziráky Londe 2006; Krämer 2003; Nevins 2010) anti-harmony (6ii), transparency (6iii), or anti-opacity/anti-transparency (6iv).[12] Thus, based on the possible combinations of disharmonic properties, we can describe theoretically possible types of weak disharmony.

In what follows we will examine which of the theoretically possible disharmony subtypes are attested in front/back harmony systems. Since not all of the possible disharmony types are attested, the question is, if any one of the disharmonic properties (6ii, iii, iv) may be present in or absent from a particular harmony system with weak disharmony, then why is it that not all of the theoretically possible combinations occur. We will argue that the system of occurring types of disharmony is due to a very general principle of patterning, namely, monotonicity.

3 The typology of weak disharmony

3.1 The frontness/backness scale

Let us examine the main types of stems according to the harmonic class of vowels (F, B, N) that (co-)occur in the stem. We will refer to these as *harmonic stem types*. If we disregard mixed (disharmonic) stems, we have three basic types: back [B], front [F] and neutral [N] stems.[13] These stem types can be ordered in a hierarchy, with the "most back" stems and the "most front" stems representing the two extreme points of the scale. The harmonic behaviour of all-neutral stems ([N]) suggests that they are located in between the two extremes ([F], [B]): in some harmony systems they may be systematically followed by front harmonic vowels and in others by back harmonic vowels and in yet other systems by both front and back harmonic vowels. Thus, all-neutral stems are not a third type that is entirely different from the other two: their type instantiates the non-extreme/intermediate case. This is shown in (7) below:[14]

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12 Opacity and transparency are often mutually exclusive in the same system, but they *can* co-occur in some harmony systems, e.g. different neutral vowels or neutral vowels in different domains may differ in transparency/opacity (see, e.g. Gafos and Dye 2011). Since, for methodological reasons, we assume here that all neutral vowels in the system behave in the same way and we examine a single domain, i.e. harmony in suffixes (cf. Section 1), we can disregard this complication here and consider transparency and opacity not to co-occur.

13 [B], [F] and [N] subsume longer non-mixed sequences of vowels, too, i.e. stems with a sequence of any number of Bs, Fs, or Ns: [B^+], [F^+] or [N^+], cf. the discussion following (8).

14 The direction of this relationship is arbitrary. Here < means "more front", but we could have stated the same in the other direction to mean "more back".

(7) Non-mixed stems in the frontness/backness scale

	most back	→			most front
hierarchy of harmony types:	[B]	<	[N]	<	[F]
frequent examples	u, o, a		i, e		y, ø, æ

A hierarchy as in (7) can be used to define the *similarity* between harmonic stem types. For example, type [F] is more similar to [N] than to [B], and [B] is more similar to [N] than to [F]. We can now place the stem types containing neutral and non-neutral vowels which appear in (6iii, iv) in the hierarchy on the basis of their similarity to the stem types in (7). Based on the vowels that co-occur in it, [FN] is more similar to [N] than to [B], and analogously, [BN] is more similar to [N] than to [F]. This determines their position in the harmonic scale in (7). The harmonic stem type [BN] is between the types [B] and [N], and [FN] is between the types [N] and [F].[15] Thus we get the modified scale which contains both the totally harmonic stems from (7) and also the stems where neutral vowels follow non-neutral ones. One of these types ([BN]) is weakly disharmonic. This is shown in (8).

(8) The frontness/backness scale of stem types

	most back	→			most front
hierarchy of harmony types:	[B] <	[BN] <	[N] <	[FN] <	[F]
examples	a	ai	i	æi	æ

Other stem types not included in (8) (e.g. [NB], [NBB], [NNB], [BNB] and [NF], [NFF] [NNF], [FNF]) can be positioned in the frontness/backness scale too. If in a given harmony system the harmonic behaviour of such a "new" type is identical with one of the types in (8), then there is no need to create a separate position for it. Therefore, we can regard it as instantiating an

15 Given the ordering [B]<[N]<[F], there are four possible positions for [BN] in the scale: (i) [B]<[N]<[F]<**[BN]**, (ii) [B]<[N]<**[BN]**<[F], (iii) **[BN]**<[B]<[N]<[F] and (iv) [B]<**[BN]**<[N]<[F]. Of these: (i) is excluded because the relevant similarities do not hold: [BN] would be more similar to [F] than to [B]. (ii) is excluded because the similarity of [BN] to [N] is universally not comparable with the similarity of [BN] to [B], and therefore the order in (ii) does not rule out a state of affairs according to which [BN] would be more similar to [N] than to [B]. (iii) is excluded because [BN] cannot be the most back type. Therefore, the only possible position of [BN] is (iv) as in (8). An analogous explanation holds for the position of [FN].

existing type which subsumes it. To take the examples cited above, if [NB], [NBB], [NNB], [BNB] behave harmonically like [B], then [B] subsumes them, and the same is true of [F] if [NF], [NFF] [NNF], [FNF] behave like [F]. This is also true for strongly disharmonic sequences (e.g. [BF], [FB]). The harmonic stem types that do behave differently have a separate position in the scale, a position which is determined in the same way, i.e. on the basis of their similarity to existing types.[16]

To sum up, we have seen that the members of the set of harmonic stem types are not independent of one another, but form a set which is structured: it is a linearly ordered set of stem types where the order is based on their similarity to one another and to the two extremes which are *prototypical contexts* for suffix harmony in the system (in the case of front/back harmony they are [B] and [F]). This linear order of harmonic stem types (the frontness/backness scale, as in (8) in the case of front/back harmony) is (i) *non-arbitrary* since it is based on and derives from similarity and is (ii) *fixed* for all (front/back) harmony systems: it may contain a different number of elements (stem types) in different systems, but their order is “predetermined” by similarity and is invariant across systems.[17] This scale has a central role in the analysis we propose.

3.2 Contiguity

When we described the possible types of weak disharmony in Section 2, we in effect assigned the harmonic vowel classes *N* or *B* as values to the stem types which serve as contexts for suffix harmony. Let us set aside the case of invariant *N*-suffixes (6i) first and focus on weak disharmony involving harmonically alternating suffixes. This can be seen in (6ii, iii, iv), where the *B*-alternant of a suffix combines with the stem types [N], [BN] and [FN], respectively, resulting in weak disharmony. No weak disharmony occurs when the *F*-alternant of the suffix combines with these stem types. We can view the harmonic stem types (harmonic contexts), which we have arranged in the fixed scale in (8), as *attributes* which are assigned the *harmonic values F* or *B* depending on the harmony they induce in a harmonic suffix in a given language. The harmonic values identify the *harmonic class* of a stem type

16 This is the case with types [NN$^+$] and [BNN$^+$] in Hungarian (cf. footnote 11). This problem will be discussed in Sections 4.1. and 4.2.

17 This scale has properties similar to other scales in phonology, e.g. sonority (e.g. Zec 2007), P-map (Steriade 2009).

(the harmonic class is front (F) if the stem type requires the value *F* in a suffix, and back (B) if the stem type requires the value *B* in a suffix).

We show this explicitly in (9) below, which enumerates the theoretically possible types of front/back harmony systems with weak disharmony (cf. 6ii, iii, iv) in terms of attributes (harmonic stem types as contexts for suffix harmony) and harmonic values assigned to them. In the table in (9) and henceforward we abstract away from the case when the value N is assigned to any of these harmonic contexts, i.e. we disregard the possibility when harmonically invariant suffixes occur in these contexts.

(9) Theoretically possible patterns of weak disharmony (stem + suffix)

stem contexts:	[B]_	[BN]_	[N]_	[FN]_	[F]_
attested language types:					
a. opacity, no anti-harmony (E. Khanty)	*B*	*F*	*F*	*F*	*F*
b. transparency, no anti-harmony (Finnish)	*B*	*B*	*F*	*F*	*F*
c. transparency, **anti-harmony** (Uyghur)	*B*	*B*	*B*	*F*	*F*
d. anti-opacity, **anti-harmony** (E. Vepsian)	*B*	*B*	*B*	*B*	*F*
unattested language types:					
e. opacity, anti-harmony	*B*	*F*	*B*	*F*	*F*
f. anti-transparency, no anti-harmony	*B*	*F*	*F*	*B*	*F*
g. anti-transparency, anti-harmony	*B*	*F*	*B*	*B*	*F*
h. anti-opacity, no anti-harmony	*B*	*B*	*F*	*B*	*F*

It can be seen in (9) that there are $2^3 = 8$ theoretically possible language types: the three relevant stem types ([BN], [N] and [FN]) can be assigned two harmonic values (*F* and *B*) independently. Table (9) shows the three stem types as contexts together with the two *prototypical contexts* [B]_ and [F]_. These contexts are ordered from the most back to the most front context (in accordance with the scale in (8)). The harmonic stem types at the extremes of the frontness/backness scale (the stem types [F] and [B]) have fixed harmonic values (*F* and *B*, respectively) since we are only considering languages with vowel harmony.[18]

18 If *B* is not always assigned to [B], and *F* to [F] in a language, then the language does not have front/back harmony. We also assume that the (phonetically implausible) pattern with

Table (9) also makes a distinction between attested (9a–d) and unattested (9e–h) language types (harmony systems). The data the distinction is based on and the languages exemplifying the attested harmony systems are from Kiparsky and Pajusalu (2003), abbreviated as K&P below. Languages that belong to the attested types are (9a): Eastern Khanty (K&P), Northeastern dialect of Estonian (K&P) and Agulis Armenian (Vaux 1998);[19] (9c) Uyghur (K&P, Vaux 2000) and Western dialect of Estonian (K&P); and (9d) East Vepsian (K&P), Mulgi (K&P) and Karchevan Armenian (Vaux 1998). Kiparsky and Pajusalu (2003) also argue that the missing types are non-accidental gaps in front/back harmony systems and provide an OT analysis. We also assume that the gaps are systematic, but pursue a different analysis.[20]

Table (9) shows that of the eight theoretical possibilities only four language types are attested in front/back harmony systems (9a–d), and the remaining four types (9e–h) are unattested. We will call the series of values *F*, *B* in attested language types *well-formed patterns*, and the unattested series *ill-formed patterns* in (9) (cells with value *B* are shaded, and cells with value *F* are left unshaded to highlight the difference between cells with *F* and cells with *B*). It can be seen in table (9) that the well-formed patterns (9a–d) are those where the *F*-values and *B*-values are not "mixed" in their series, i.e. no sequence of *F*s is interrupted by one or more *B*-values, and no sequence of *B*s is interrupted by one or more *F*-values. By contrast, the ill-formed patterns are exactly the ones in which a sequence of *F*-values is interrupted by one or more *B*-values or a sequence of *B*-values is interrupted by one or more *F*-values. If we define the contiguity of a harmonic pattern as (10) then we can state this as the "contiguity assumption" (11).

(10) **Contiguity**
A (sub)sequence of values is contiguous if no pair of identical values in it is interrupted by a different value.

(11) **Contiguity assumption**
Well-formed patterns only contain contiguous (non-interrupted) sequences of harmonic values assigned to the harmonic stem types.

'counter-harmony', where the context [B]_ is always assigned the value *F* and the context [F]_ is always assigned the value *B*, is excluded (see Section 3.3 below).

19 In type (9a) N always requires a front harmonic suffix alternant, i.e. it seems to behave harmonically like an F-vowel. In this type of system the neutrality of these vowels can only manifest itself in their occurrence in invariant suffixes. The same is true of type (9d) where N behaves like a back vowel with respect to suffix alternation.

20 For a critique Kiparsky and Pajusalu (2003) see Gafos and Dye (2011), Rebrus, Szigetvári and Törkenczy (2012), Rebrus and Törkenczy (2012), Törkenczy (2013).

Assuming that the harmonic values are fixed in the prototypical contexts (i.e. they are *B* in the context [B] and *F* in [F]), the contiguity assumption in (11) has two *formal* consequences concerning (i) the entailments between harmonic values and (ii) the harmonic values in sandwiched contexts.

Let us examine the entailments between the harmonic values first. Given that the harmonic values are fixed in the prototypical contexts, (11) is equivalent to the following propositions:[21] It must be pointed out that (11) and (12) are not logically equivalent unless the harmonic values are fixed properly in the prototypical contexts. Otherwise (12) entails (11), i.e. (12) is stricter than (11), see Section 3.3 for a discussion.

(12) **Entailments between harmonic values**

a. If a context is assigned the value of *F*, then all the contexts in the scale which are "more front", i.e. closer/more similar to the prototypical context [F]_, also have the value *F*

b. If a context is assigned the value of *B*, then all the contexts in the scale which are "more back", i.e. closer/more similar to the prototypical context [B]_, also have the value *B*

The entailments in (12) have direct linguistic consequences. They predict that there should be *no anti-transparent language* (cf. 5c). In such a language the two relevant harmonic contexts [BN], [FN] would have the values *F* and *B*, respectively. These contexts are ordered [BN]<[FN] and the values assigned to them realise a pattern with an *F*...*B* subsequence. This violates (12), i.e. the anti-transparent pattern is non-contiguous, and thus is ill-formed according to (11). This prediction is borne out: anti-transparent harmony systems are unattested; see (9f, g).

A further prediction is that if a system has anti-harmony, i.e. the harmonic context [N] is assigned the value *B*, then it must assign *B* to the harmonic context [BN] too, since [BN]<[N], and a value *F* would violate (12) and make the pattern non-contiguous. Therefore *a harmony system with anti-harmony must have neutral vowels which are transparent for backness* (i.e. it must be of the transparent or anti-opaque type). This prediction is also borne out, see (9c, d).

21 The proof of equivalence is as follows: Given the harmonic scale of contexts in (8), (i) if either of the entailments in (12) does not hold, then there exists a (potentially interrupted) subsequence of values *F*...*B*. We know that the prototypical contexts [B] and [F] are assigned the values *B* and *F*, respectively. Thus, the subsequence ***F*...*B*** must occur somewhere in between the two prototypes, and the whole pattern will be *B*...***F*...*B***...*F* (or *B*...***F*...*B***, or ***F*...*B***...*F* if one of the values in the subsequence ***F*...*B*** is assigned to a prototypical context), which cannot be contiguous. (ii) If the contiguity assumption (11) does not hold, then there must exist a *F*...*B*...*F* (or *B*...*F*...*B*) subpattern, therefore the entailments in (11) do not hold. Therefore, the contiguity assumption in (11) and the entailments in (12) are equivalent.

Let us examine the second formal consequence of the contiguity assumption (11) concerning the harmonic values that are assigned to sandwiched contexts. A sandwiched context is a harmonic context in the scale of harmonic environments that is flanked (not necessarily only in its immediate neighbourhood) by other contexts, e.g. given the scale [X]<[Y]<[V]<[Z], [Y] and [V] are sandwiched by [X] and [Z], [Y] is sandwiched by [X] and [V], and [V] is sandwiched by [Y] and [Z]) while [X] and [Z] are not sandwiched by any contexts. Given this definition, the contiguity assumption (11) is equivalent to (13) below:

(13) Harmonic values in **sandwiched contexts**
If a context is sandwiched by contexts that are assigned identical harmonic values, then the sandwiched context must be assigned the same harmonic value.

This formal consequence of the contiguity assumption (11) also makes cross-linguistic predictions about possible harmony systems with weak disharmony. It follows from (13) that *anti-harmony and opacity are mutually exclusive*: they cannot co-occur in a harmony system, i.e. opacity entails the absence of anti-harmony: [[BN]**F**] ⇒ [[N]**F**]. This is because (i) opacity means that the harmonic contexts [BN] and [FN] are both assigned the value *F* (cf. 5a), (ii) the harmonic contexts [BN] and [FN] sandwich the context [N], and (iii) therefore [N] must be assigned the harmonic value *F* too according to (13) (...*FXF*... ⇒ $X = F$), which is the lack of anti-harmony. This prediction is borne out, type (9e) is unattested.

Furthermore, *anti-opacity entails anti-harmony*: a harmony system with anti-opacity must have anti-harmony too, i.e.: [[FN]**B**] ⇒ [[N]**B**]. This is because (i) anti-opacity means that the harmonic contexts [BN] and [FN] are both assigned the value *B* (cf. 5d), (ii) the harmonic contexts [BN] and [FN] sandwich the context [N], and (iii) therefore [N] must be assigned the harmonic value *B* too according to (13) (...*BXB*... ⇒ $X = B$), which *is* anti-harmony. This prediction is borne out: type (9h) is unattested.

In accordance with (13) it is only systems with transparency which may allow or disallow anti-harmony:...*FXB*... ⇒ ($X = F$ or $X = B$) (compare 9b and 9c). Opaque systems cannot have anti-harmony (9a) and anti-opaque systems must have anti-harmony (9d).

3.3 Monotonicity and ordering

All the well-formed harmony patterns are contiguous (in accordance with 11), but it is not true that all contiguous harmony patterns are well-formed. Constant patterns, which assign the same harmonic value to all the contexts *are* contiguous according to (11), but they are not well-formed. The two constant patterns are shown in (14) below.

(14) Constant patterns

stem contexts:	[B]_	[BN]_	[N]_	[FN]_	[F]_
constant (unattested) language type 1	*F*	*F*	*F*	*F*	*F*
constant (unattested) language type 2	*B*	*B*	*B*	*B*	*B*

Furthermore, "counter-harmonic" patterns, which assign harmonic values to prototypical contexts in the opposite way, i.e. value *F* to context [B]_ and *B* to [F]_, are contiguous by (11) but are also not attested. The four patterns of this kind are shown below in (15).

(15) Counter-harmonic contiguous patterns

stem contexts:	[B]_	[BN]_	[N]_	[FN]_	[F]_
counter-harmonic (unattested) type 1	*F*	*F*	*F*	*F*	*B*
counter-harmonic (unattested) type 2	*F*	*F*	*F*	*B*	*B*
counter-harmonic (unattested) type 3	*F*	*F*	*B*	*B*	*B*
counter-harmonic (unattested) type 4	*F*	*B*	*B*	*B*	*B*

We therefore need a constraint (or a combination of constraints) on harmonic patterns that ensure contiguity and exclude the contiguous *and* ill-formed patterns at the same time. We propose that this general constraint is the principle of monotonicity. In this section we explore two formal models of monotonicity, a stricter one (corresponding to the entailments (12a, b)) and a looser one (corresponding to the contiguity assumption in (11)).

In our discussion of the possible harmony systems with weak disharmony, we have used two linearly ordered sets: (i) the backness/frontness scale of stem types (contexts for suffix harmony) ordered from the prototypical back harmonic context [B] to the prototypical front harmonic context [F] for suffix harmony (see (9)), and (ii) the set of harmonic values of *F* and *B* ordered in the same way as the harmonic stem types: from the most back value to the most front value. These two ordering relations are different, therefore the symbols used to denote the relations have to be distinguished in the notation: we will use $<$ to denote the former and $<'$ to denote the latter relation. The mapping between the two sets is an assignment function f from the set of harmonic stem types (contexts) to the set of harmonic values. This is shown in (16) below.

(16) The mapping between harmonic stem types (contexts) and harmonic values

hierarchy of harmonic stem types: [B] < [BN] < [N] < [FN] < [F]

assignment function f: {harmonic stem types} → {harmonic values}

hierarchy of harmonic values: B <′ F

Each of the patterns of values as in (9a–h) is defined by its assignment function. For instance, the well-formed pattern ⟨*BBFFF*⟩[22] in (9b) is defined by the following assignment function:

(17) An example for the assignment function: the well-formed pattern ⟨*BBFFF*⟩

hierarchy of harmonic stem types: [B] < [BN] < [N] < [FN] < [F]

hierarchy of harmonic values: B <′ F

A function is by definition *monotonic* if it is order-preserving, i.e. the assignment function f is monotonic if f assigns greater harmonic values to greater harmonic contexts, i.e.

(18) Monotonic function

f is *monotonic* iff $X \leq Y \;\Rightarrow\; f(X) \leq' f(Y)$ for all harmonic contexts X, Y.

The patterns with a monotonic function in accordance with (18) are exactly the same that satisfy the entailments in (12). However, only a subset of the contiguous patterns has a monotonic function as defined in (18) above. If we also require that the two prototypical contexts [B] and [F] have the fixed harmonic values B and F, respectively, then these contiguous patterns have a monotonic assignment function. Well-formed harmonic patterns in (9a–d) have monotonic assignment functions, because for any pair of harmonic contexts either the assigned values are identical, or if the assigned values are different, then their ordering follows the ordering of harmonic contexts, see for instance (17). By contrast, the ill-formed patterns (9e–h) are defined by non-monotonic

22 This notation only specifies the order of the harmonic values to identify a pattern and assumes the fixed harmonic scale of the five contexts in (8).

assignment functions. For instance, the ill-formed pattern $\langle B\boldsymbol{FB}FF\rangle$ in (9e) has pairs of contexts that violate monotonicity: the second and third stem types in the harmonic context hierarchy are [BN]<[N], but the harmonic values assigned to them (emboldened above) are F and B, respectively. Since $F >' B$ (see (16)), therefore $F \leq' B$ does not hold, thus pattern (9e) is not monotonic. This is shown in (19) below.

(19) An example for a non-monotonic assignment function: the ill-formed pattern $\langle BFBFF\rangle$

hierarchy of harmonic stem types: [B] < [BN] < [N] < [FN] < [F]

hierarchy of harmonic values: B <′ F

The implication between the well-formedness of harmonic patterns and the monotonicity of assignment functions can be stated as in (20) below.

(20) **Monotonicity of harmonic patterns** (“ordering version”)
A well-formed harmonic pattern has a monotonic assignment function.

The constant patterns $\langle FFFFF\rangle$ and $\langle BBBBB\rangle$ (see 14) are also monotonic (and contiguous). They are unattested and we do not consider them well-formed harmonic patterns because the values assigned to the prototypical contexts should be B for [B] and F for [F] concurrently in a pattern. Consequently, a well-formed pattern cannot be constant. Apart from the constant patterns, all of the monotonic patterns are attested harmony systems as we have seen in (9).

3.4 Monotonicity and similarity

Contiguity can also be formalised in another way, on the basis of a more general view which does not assume the inherent directedness of a harmonic scale (8), which (18) crucially refers to. In this formalisation we only refer to the adjacency relations between the harmonic contexts in the scale. These relations can be captured with reference to the general notion of *similarity* between contexts and between values.

The notion of similarity plays an important role in analogy-based theories (cf. Blevins et al. 2009a). Very generally, analogical interaction between units of language can be conceptualised as strong connections being established between similar forms. For two or more forms in an analogical relationship the following holds: if these forms are similar in one property, then they are similar in another property or other properties, too (e.g. Albright 2009; Blevins et al.

2009b; Bybee 2001). *Paradigm uniformity* is a pertinent example (Kenstowicz 2005; Rebrus 2012; Rebrus and Törkenczy 2005; Steriade 2000, among others): if word forms are similar in that they have the same lexical meaning (i.e. they belong to the same extended paradigm), then they are (partially) similar in their phonological forms, too: they usually have (near-)identical roots (for example *dog, dogs, dog's, doggie*). The same is true for the case where the forms are similar in their (partially) identical morphosyntactic specifications: in this case the forms will be similar in their (near-)identical affixes (e.g. plurals: *keys, pills, ribs*, etc.). This view is also applicable to the description of phonologically motivated *allomorphy*: if two or more forms have phonological similarities in their stems, then their suffixes will be similar/identical (e.g. plural of sibilant-final nouns in English: if stems are similar in that they end in a sibilant, then their plural forms are similar in that they end in the sequence [ɪz]).

These observations carry over to vowel harmony in suffixes: stems that are similar to one another in some respect combine with suffixes that are similar to one another in some respect. In the case of front/back harmony, the similarity is essentially based on the front/back feature(s) of the vowels. The similarity of suffixes is straightforward (disregarding harmonically invariant suffixes): since suffix vowels are either back or front, suffixes are either similar or not similar harmonically. Our main question is what similarity means between stems.

The frontness/backness scale of harmonic contexts as in (8) defines similarity relations between the harmonic contexts arranged in the scale. (The measure of) this similarity is defined as follows: taking a point of the scale (a harmonic context) as a point of departure, if we move in one direction (either "up" or "down" in the given order), the more steps (i.e. transitions between two adjacent points of the scale) are taken to reach another point (another harmonic context), the less similar the harmonic context at this latter point is to the harmonic context at the point of departure. If the ordering (scale) $X<Y<Z$ holds, then the ordering of similarities (as indicated by $\preccurlyeq$, whose strict version is $\prec$) is the opposite:[23] $\mathrm{sim}(X,\boldsymbol{X})\succ\mathrm{sim}(X,\boldsymbol{Y})\succ\mathrm{sim}(X,\boldsymbol{Z})$ and $\mathrm{sim}(Z,\boldsymbol{Z})\succ\mathrm{sim}(Z,\boldsymbol{Y})\succ\mathrm{sim}(Z,\boldsymbol{X})$. It is important to note that the relation between $\mathrm{sim}(X,Y)$ and $\mathrm{sim}(Y,Z)$ is undefined since $\mathrm{sim}(X,Y)$ and $\mathrm{sim}(Y,Z)$ are uncomparable. This is because it is impossible to reach the other two arguments (X and Z), from the common argument of the two similarities (Y) by moving in one direction only in the scale. In the case of a scale $X<Y<Z<V$, the following ordering relations hold for similarities (setting aside identity as an instance of maximal (total) similarity): $\mathrm{sim}(X,\boldsymbol{Y})\succ\mathrm{sim}(X,\boldsymbol{Z})\succ\mathrm{sim}(X,\boldsymbol{V})$ holds true while $\mathrm{sim}(X,Y)$ and $\mathrm{sim}(Y,Z)$ are uncomparable, and $\mathrm{sim}(X,Y)$

23 Here we assume that similarity is a symmetric function, i.e. $\mathrm{sim}(X,Y)=\mathrm{sim}(Y,X)$, and the function sim is maximal iff its arguments are identical, i.e. $\mathrm{sim}(X,X)\succcurlyeq\mathrm{sim}(X,Y)$ for all X,Y.

and sim(Y,V) are uncomparable, too. Furthermore, two similarities which do not have a common argument are also uncomparable: therefore sim(X,Y) and sim(Z, V) are uncomparable, too.[24]

Given this notion of similarity, we can define the property of monotonicity based on similarity. Informally, monotonicity in this sense requires that if we have two (or more) units of language that are similar along two dimensions of properties, then, when similarities increase along one dimension, they have to increase along the other dimension too. This makes it possible to characterise vowel harmony patterns: where one dimension is the similarity of harmonic contexts and the other dimension is the similarity of harmonic values. It can be shown that a harmonic pattern is well-formed (contiguous) if and only if the following holds.

(21) **Monotonicity of harmonic patterns** ("similarity version")
If context X is *more similar* to Y than to Z, then the same is true of the values assigned to them (allowing for the similarity of identical values).

That is, formally

$$\mathrm{sim}(X,Z) \preccurlyeq \mathrm{sim}(X,Y) \quad \Rightarrow \quad \mathrm{sim}'(f(X),f(Z)) \preccurlyeq' \mathrm{sim}'(f(X),f(Y))$$

where X, Y, Z are harmonic stem types (contexts); sim is the similarity between contexts as defined above, $\preccurlyeq$ is the partial ordering between similarities of contexts; f is the assignment function with values F or B; and sim′ is the similarity between these two values such that identical harmonic values are more similar than different ones (this new ordering is indicated by $\preccurlyeq'$), i.e. $\mathrm{sim}'(F,B) = \mathrm{sim}'(B,F) \prec' \mathrm{sim}'(F,F) = \mathrm{sim}'(B,B)$. For the sake of simplicity we can choose discrete values for these similarities: $\mathrm{sim}'(F,F) = \mathrm{sim}'(B,B) = 1$ and $\mathrm{sim}'(F,B) = \mathrm{sim}'(B,F) = 0$,[25] thus the ordering $\prec'$ is identical to the ordering between numbers: $0 < 1$.

Let us look at some examples in more detail. The five relevant contexts are the following: [B]<[BN]<[N]<[FN]<[F], which gives us $5 \times 4/2 = 10$ different pairs of contexts and 5 identical pairs, which make up the set partially ordered by the

24 Formally, the similarity function sim is a function of two variables defined on the ordered set of harmonic contexts $\langle \mathrm{H}, \leq \rangle$ and its values are in a (partially) ordered set $\langle \mathrm{S}, \preccurlyeq \rangle$, i.e. sim: $\mathrm{H} \times \mathrm{H} \rightarrow \mathrm{S}$, such that $\mathrm{sim}(X,Y) \succcurlyeq \mathrm{sim}(Z,V)$ iff $[X = Z \wedge (X \geq Y \geq V \vee X \leq Y \leq V)] \vee [X = V \wedge (X \geq Y \geq Z \vee X \leq Y \leq Z)] \vee \vee [Y = Z \wedge (Y \geq X \geq V \vee Y \leq X \leq V)] \vee [Y = V \wedge (Y \geq X \geq Z \vee Y \leq X \leq Z)]$. In our case the ordering $\preccurlyeq$ on the set of similarities is not total, but a partial ordering. Thus, this notion of similarity is not a similarity metric, because the values of a metric (which often plays an important role in analogical theories) are real numbers, whose set is totally ordered.

25 That is, similarity between harmonic values equals Kronecker's delta function: $\mathrm{sim}'(X,Y) = \delta(X,Y)$, where $X,Y \in \{F,B\}$.

relation ≼. This is shown in the Hasse-diagram in (22), where the similarity decreases from top to bottom. In the diagram the numbers represent the harmonic contexts: 1 = [B]_, 2 = [BN]_, 3 = [N]_, 4 = [FN]_, 5 = [F]_.

(22) Poset of similarities induced by the linear ordering of five members in the scale $1 < 2 < 3 < 4 < 5$

sim(1,1) sim(2,2) sim(3,3) sim(4,4) sim(5,5)	(similarities of identical items)
sim(1,2) sim(2,3) sim(3,4) sim(4,5)	similarities of neighbouring items
sim(1,3) sim(2,4) sim(3,5)	similarities of second neighbours
sim(1,4) sim(2,5)	similarities of third neighbours
sim(1,5)	similarity between 1st and last member

This partially ordered set (poset) is assigned another poset containing all the similarities between the harmonic values *F* and *B*: sim′(*F*,*B*)≺′sim′(*B*,*B*) = sim′(*F*,*F*). If *difference* is represented as 0 and *identity* (complete similarity) as 1, then the assignment between contexts and values can be represented if we write 0 or 1 in every node of poset in (22) according to the similarity (i.e. difference or identity) between the values in the harmonic contexts identified by the node in question.

First let us examine a well-formed pattern as in (17) and an ill-formed pattern as in (19). The assignment function *f* between the two linearly ordered sets (the set of harmonic contexts and the set of harmonic values) then is represented by the labelling of the nodes of the poset in (22) with 0 or 1 according to the similarity of values assigned to them. Each harmonic pattern determines its labelled poset. It is easy to determine whether a particular labelled poset represents a monotonic pattern according to (21) or not: it is monotonic if the numerical values of the labels increase from bottom to top between the linked nodes in the Hasse diagram, i.e. a pattern is non-monotonic iff its poset contains at least one instance of a node labelled 1 with a higher linked node labelled 0. For example, the well-formed pattern ⟨*BBFFF*⟩ has the labelled poset (23a): it can be seen that this labelling conforms to monotonicity. By contrast, an ill-formed pattern such as ⟨*BFBFF*⟩ is in violation of monotonicity (21) because its labelled poset (23b) contains (several) linked nodes such that a node labelled 0 is higher than a node labelled 1 (these nodes labelled 0 are emboldened in (23b)). (Here and in the following labelled posets, the first row of the posets showing the similarities of values of identical positions is omitted because it is redundant: the values of these nodes are always necessarily 1.)

(23) Labelled similarity posets of several five-member patterns

a. well-formed pattern ⟨*BBFFF*⟩ and its dual pattern ⟨*FFBBB*⟩

```
1   0   1   1
 \ / \ / \ /
  0   0   1
   \ / \ /
    0   0
     \ /
      0
```

b. ill-formed pattern ⟨*BFBFF*⟩ and its dual pattern ⟨*FBFBB*⟩

```
0   0   0   1
 \ / \ / \ /
  1   1   0
   \ / \ /
    0   1
     \ /
      0
```

c. constant patterns ⟨*FFFFF*⟩ and ⟨*BBBBB*⟩

```
1   1   1   1
 \ / \ / \ /
  1   1   1
   \ / \ /
    1   1
     \ /
      1
```

d. other non-contiguous patterns with wrong prototypes ⟨*FFBFF*⟩ and ⟨*BBFBB*⟩

```
1   0   0   1
 \ / \ / \ /
  0   1   0
   \ / \ /
    1   1
     \ /
      1
```

It is important to note here that the mapping between harmonic patterns and their labelled similarity posets is not one-to-one. Harmonic patterns uniquely determine their labelled posets, but the same labelled poset always belongs to two different patterns. This is because labelling is not sensitive to whether *F* or *B* is assigned to a harmonic context in a pattern, it is only sensitive to whether the harmonic (not necessarily immediate) contexts being compared have identical harmonic values assigned to them (*F-F* or *B-B*) or different ones (*F-B*). Hence a pattern and its *dual* pattern (which we get by substituting *F* for all instances of *B* and *vice versa*) both map onto the same labelled poset. Consider, for instance, (23a), which is the labelled poset of both ⟨*BBFFF*⟩ and ⟨*FFBBB*⟩ and (23b), which is the labelled poset of both ⟨*BFBFF*⟩ and ⟨*FBFBB*⟩.

Note that the duals of the well-formed patterns in (9a–d) are not well-formed: a well-formed harmonic pattern assigns "proper prototypical values" to prototypical contexts (*B* to [B] and *F* to [F]), but its dual pattern does it in the opposite way (i.e. it is therefore a non-attested language type with "counter-harmony", see footnote 18). These patterns can be easily identified by their first and last values: the values assigned to the extreme contexts of the "well-formed" patterns (and also the "ill-formed" ones) in (9) are ⟨*B*...*F*⟩ while those assigned to the extreme contexts of their dual patterns are ⟨*F*...*B*⟩. Thus, monotonicity defined by similarity (21) allows the dual patterns of well-formed patterns, which are not well-formed.

Furthermore, the two (ill-formed) constant patterns ⟨*FFFFF*⟩ and ⟨*BBBBB*⟩ which assign identical harmonic values in all the harmonic contexts are also monotonic by similarity, see (23c), but they also have the "wrong" values in one of the prototypical contexts.

The rest of the patterns (see (23d)), i.e. those which are (i) not well-formed, (ii) not duals of well-formed ones and (iii) non-constant, are non-monotonic according to (21) (and also have wrong prototypical values).

To summarise, monotonicity can be interpreted in two ways, based on (i) ordering (20) or (ii) similarity (21), the first of which is stricter than the second one. Neither capture exactly the well-formed harmonic patterns in (9) while excluding all the theoretically possible, but not well-formed ones. Both, however, *do* capture the crucially important property of well-formed harmonic patterns, contiguity. The relationship between monotonicity in its two interpretations *versus* contiguity (11) and the entailments in (12) is summarised in (25a). We compare the well-formedness of harmonic patterns as in (9) to monotonicity in its two interpretations in (25b). There we also specify the assumption that must *also* be made in order for equivalence to hold between well-formedness and monotonicity, namely, assumption (24) about proper prototypical values.

(24) Proper prototypical values
In a vowel harmony system a prototypical context for (suffix) harmony (i.e. a context in which the target of the harmony in the suffix is immediately preceded by a non-neutral vowel in the stem) must be assigned a proper prototypical harmonic value: [B]_ must be assigned *B* and [F]_ must be assigned *F*.

(25) Equivalence of monotonicity criteria
a. monotone by ordering (20) ⇔ at least one of entailments (12a,b) holds
monotone by similarity (21) ⇔ contiguous (11)
b. well-formed ⇔ monotone by ordering & proper prototypical values
well-formed ⇔ monotone by similarity & proper prototypical values

Table (26) fleshes this out by summarising how the theoretically possible $2^5 = 32$ harmonic patterns (which result from the free combination of the harmonic values *B*, *F* with the five contexts of the harmonic scale (8)) fare on the criteria of well-formedness:[26] monotonicity by ordering, monotonicity by similarity, and

26 Attested harmonic patterns tautologically satisfy well-formedness since (following Kiparsky and Pajusalu) we have assumed that only the patterns in (9a–d) are attested and they are all well-formed.

the requirement for proper prototypical values (24). In (26) we have arranged the patterns into the groups we have discussed above: attested patterns, their duals, constant patterns, the unattested patterns of (9e–h) and the other conceivable harmonic patterns that do not fit into any of these groups. '✓' indicates that a given criterion permits a type of pattern and ✗ shows that it excludes the pattern.

(26) Types of the theoretically possible invariable patterns

pattern types: examples: criteria:	attested (9a-d) ⟨*BFFFF*⟩ ⟨*BBFFF*⟩ ⟨*BBBFF*⟩ ⟨*BBBBF*⟩	constant ⟨*FFFFF*⟩ ⟨*BBBBB*⟩	duals of attested ⟨*FBBBB*⟩ ⟨*FFBBB*⟩ ⟨*FFFBB*⟩ ⟨*FFFFB*⟩	unattested (9e-h) ⟨*BFBFF*⟩ ⟨*BFFBF*⟩ ⟨*BFBBF*⟩ ⟨*BBFBF*⟩	others ⟨*FBBFF*⟩ ⟨*BFBFB*⟩ ⟨*FBFFB*⟩ etc.
monotonicity (ordering) = entailments	✓	✓	✗	✗	✗
monotonicity (similarity) = contiguity	✓	✓	✓	✗	✗
proper prototypical values	✓	✗	✗	✓	✗
number of patterns (sum = 32)	4	2	4	4	18

(26) shows that the *two interpretations of monotonicity differ* because the ordering interpretation is stricter in that it excludes everything the similarity interpretation does and it also excludes the duals. This comes closer to capturing the well-formed harmonic patterns and excluding the ill-formed ones. Nevertheless, for a perfect fit, we also have to require proper prototypical values under both interpretations to ban the constant patterns both interpretations of monotonicity permit. Since the proper prototypical values requirement (24) excludes the duals by itself in addition to excluding the constant patterns, we can combine it with either the ordering interpretation of monotonicity or the similarity interpretation and we will capture the attested patterns while excluding all the unattested ones. This means that *in the case of the patterns of harmony we are discussing*, it makes no difference how we interpret monotonicity.

The reason for theoretically considering the similarity version of monotonicity, which permits "dual" patterns (which are unattested as harmonic patterns), is that in some other phenomena in which monotonicity also plays a central role (e.g. allomorphy patterns in paradigms, see Rebrus and Törkenczy 2008, Rebrus and Törkenczy 2011) such a “looser” interpretation of monotonicity is needed, because these phenomena do not presuppose an ordering relation such as the harmonic scale in (8). In these cases the ordering version of monotonicity (20) is too strict.

4 Variation

In this section, we will show how the principles of monotonicity (20, 21) and prototypical values (24) that constrain the patterns of (dis)harmony and distinguish the well-formed patterns from the ill-formed ones also apply to *disharmony patterns displaying variation*. Specifically, we will discuss variation in anti-harmony and transparency. In order to be able to do this, we will have to go into some descriptive detail about anti-harmony and transparency in Hungarian, a harmony system in which variation in harmony has been studied in detail (Ringen and Kontra 1989; Hayes and Cziráky Londe 2006; Benus and Gafos 2007; Hayes et al. 2009; Kálmán et al. 2012; Rebrus et al. 2012; Blaho and Szeredi 2013; Törkenczy et al. 2013; Rebrus and Szigetvári 2013; Forró 2013).

4.1 Variation in anti-harmony

We have seen in Section 2 that anti-harmony, a type of weak disharmony, can occur when a stem containing only neutral vowels (an “[N]-stem”) takes a harmonically alternating suffix with a back harmonic vowel: [[N]B]. Because of monotonicity, this can only happen in a well-formed harmony system in combination with transparency ⟨*BB***B***FF*⟩ (9c) or anti-opacity ⟨*BB***B***BF*⟩ (9d). It can also happen that in a language not *all* [N]-stems take the back alternant of harmonically alternating suffixes, only *some* of the [N]-stems do depending on the stem. In this case it is the lexical property of a stem that determines whether it takes the front or the back alternant of a harmonic suffix: [[N]B/F] – see (3). We have exemplified *lexical variation* of this kind in Hungarian with the morphemes *hír* ‘news’ and *sír* ‘tomb’ (e.g. *hír-ben* /hiːrbɛn/ vs. *sír-ban* /ʃiːrbɒn/ ‘INESSIVE’). This state of affairs is conceivable as a harmonic pattern in which two harmonic values, *B* and *F*, are assigned to one harmonic context, which, in our case, is the harmonic context [N]. This is shown in the middle row of (27) below, where the occurrence of both harmonic values *B*|*F* in one cell represents lexical variation.

(27) Lexical variation in anti-harmony

stem contexts:	[B]_	[BN]_	[N]_	[FN]_	[F]_
b. transparent, no anti-harmony (Finnish)	*B*	*B*	*F*	*F*	*F*
b:c. transparent, variable anti-harmony (Hungarian)	*B*	*B*	***B|F***	*F*	*F*
c. transparent, anti-harmony (Uyghur)	*B*	*B*	*B*	*F*	*F*

As can be seen in (27), this type of variation is a ‘mixture’ in which two patterns (language types) combine in the following way: if in a given context the two patterns have identical values, then the value in that context is the same in the “mixed” pattern too; if, however, the values in a given context are different in the two patterns, then there is variation between the two values in that context in the mixed pattern. In our case, there are two patterns without variation (henceforward we will call such patterns *invariable patterns*) and these two invariable transparent patterns, which only differ in anti-harmony ((9c = 27c), have anti-harmony and (9b = 27b) does not) combine into a pattern with variation (henceforward a *variable pattern*) in the harmonic context [N]. Thus, the pattern with variable anti-harmony ⟨*BB**V**FF*⟩ is the mixture of the invariable patterns ⟨*BB**F**FF*⟩ and ⟨*BB**B**FF*⟩.[27]

Anti-harmony in Hungarian is constrained in several ways: (i) lexically, (ii) vocalically, (iii) prosodically and (iv) morphologically. In what follows we will discuss these limitations in more detail.

(i) Lexical limitation means that monomorphemic *anti-harmonic stems belong to a closed lexical class*. This means that anti-harmony is not productive in the sense that if an all-neutral word is a recent borrowing, a proper name or a nonce word whose neutral vowel is /i/, /iː/, then productive suffixation always involves the front harmonic suffix allomorph, e.g. *nick-em* /nikːɛm/ ‘my nick(name)’, *geek-ek* /giːkɛk/ ‘geek-PL’, *Lviv-ben* /lvivbɛn/ ‘in Lviv’, *Krím-ben* /kriːmbɛn/ ‘in Crimea’. This fact, however, does not mean that harmonic back suffixation after all-neutral stems is rare

27 We are assuming here that there are languages in which anti-harmony is invariable. Kiparsky and Pajusalu (2003) list two such systems and note that one of them, Uyghur, shows some degree of variation in anti-harmony (though anti-harmony is the productive pattern).

(measured in type frequency). *More than one fourth* of monosyllabic monomorphemic free stems that are suffixable and contain /i iː/ (monosyllabic [i(ː)] stems) are anti-harmonic (26.3% = 45/171). If we focus on *verbal* monosyllabic [i(ː)] stems, 76.7% of them are anti-harmonic (23/30). In the light on these facts about frequency, the traditional assumption (e.g. Szépe 1969; Vago 1980) that anti-harmony in Hungarian is simply an irregular/exceptional phenomenon which is not essentially relevant to the overall (dis)harmony pattern of Hungarian is not tenable.

(ii) The vowels of anti-harmonic stems in Hungarian are constrained too: they almost always are /i/ or /iː/, or very rarely /eː/. The other front vowels /y, yː, ø, øː/ do not occur, i.e. *[[F]B]. Anti-harmony with back stem vowels does not occur either: there are no back vowel stems which are consistently suffixed by harmonically alternating front vowel suffixes: *[[B]F].

(iii) The third restriction on anti-harmony is prosodic. All the monomorphemic anti-harmonic stems are monosyllabic, i.e. [i(ː)] stems with more than one syllable get front harmonic suffixes obligatorily, e.g. *kilincs-ek/*-ok* /kilintʃɛk/ 'door handle-PL', *bíbic-nek/*-nak* /biːbitsnɛk/ 'pewit-DAT', *ribizli-vel/*-val* /ribizlivɛl/ 'redcurrant-INST'.[28] we will call this effect (this division in the lexical distribution of anti-harmonic roots) the *Polysyllabic Split*.

(28) **Polysyllabic Split**
Polysyllabic stems are not anti-harmonic.

This difference of harmonic behaviour requires that we should assume two harmonic contexts as distinct positions on the frontness/backness scale for Hungarian (see Section 3.1 and note 16): one for monosyllabic, and another for polysyllabic all-neutral stems: [N] and [NN$^+$].[29] We will hypothesise that these two positions are in a strict linear order on the scale: [BN]<[N]<[NN]<[FN]. Since a new stem context ([NN]) is introduced, now we have six harmonic contexts to which values are assigned, hence we will get harmonic patterns with 6 positions on the scale. This is shown in (29) below, where the Hungarian pattern ⟨*BB**V**FFF*⟩ (*V* in the third position stands for variation between *B* and *F*)

28 Potential counterexamples are *férfi* /feːrfi/ 'man' and *derék* /dɛreːk/ 'waist'. Both take back suffixes obligatorily in certain forms: *férfi-ak/*ek* /feːrfiɒk/ 'man-PL' and *derek-am/*em* /dɛrɛkɒm/ 'waist-1SG.POSS'. With other suffixes, however, vacillation occurs: e.g. *férfi-val/vel* /feːrfivɒl % feːrfivɛl/ 'with (a) man', *derék-ban/ben* /dɛreːkbɒn % dɛreːkbɛn/ 'in waist'.

29 The subdivision of a context of variation is not necessarily phonologically natural (e.g. Hayes et al. consider the Polysyllabic Split an unnatural constraint). Note that in our approach, the Polysyllabic Split is not (entirely) unnatural since it is a subdivision of a scale which is based on a phonologically motivated relationship, similarity.

is in the second row (29b:c′).[30] This pattern can be considered the mixture of two patterns. One of them is the transparent type without anti-harmony $\langle BB\boldsymbol{FF}FF\rangle$ (29b), which is essentially the same as the five-position pattern $\langle BB\boldsymbol{F}FF\rangle$ (9b) that only contains one all-neutral position. The other one is the invariable pattern $\langle BB\boldsymbol{BF}FF\rangle$, which does not correspond to any of the well-formed patterns in (9a–d) (i.e. none of the patterns of (9a–d) subsumes $\langle BB\boldsymbol{BF}FF\rangle$). This pattern assigns different values to the harmonic contexts [N] and [NN]: the former is *B*, the latter is *F* – this is the pattern in (29c′). We do not know a language that productively shows this pattern, but, given the assumed ordering between the harmonic contexts [N] and [NN$^+$] in the frontness/backness scale, it is well-formed (monotonic by both versions of monotonicity). This is a pattern with the Polysyllabic Split but, unlike Hungarian, without variation: the only difference between this pattern and the categorical anti-harmonic one (29c = 10c) is the different values assigned to the harmonic context [NN$^+$]. The last row in (29c) shows the attested language type (29c = 10c) whose harmonic pattern (in which the two all-neutral harmonic contexts have an identical value *B*) is close to the hypothesised well-formed pattern (29c′).

(29) Patterns of anti-harmony (AH) and the Polysyllabic Split (PS):

stem contexts:	[B]_	[BN]_	[N]	[NN$^+$]	[FN]_	[F]_
b. no AH (Finnish)	*B*	*B*	*F*		*F*	*F*
b:c′. variable AH & PS (Hungarian)	*B*	*B*	***B|F***	*F*	*F*	*F*
c′. invariable AH & PS	*B*	*B*	*B*	*F*	*F*	*F*
c. invariable AH & no PS (Uyghur)	*B*	*B*	*B*		*F*	*F*

(iv) The fourth restriction on Hungarian anti-harmony is morphological: there is a difference in the way in which anti-harmony works after monomorphemic roots and morphologically complex stems. Anti-harmony after suffixed stems is subject to a paradigm uniformity constraint, *Harmonic Uniformity* as a result of which polymorphemic all-neutral stems may behave anti-harmonically, see Törkenczy et al. (2013), Rebrus and Szigetvári (2013).

30 The Finnish stem-internal pattern (cf. Kiparsky and Pajusalu 2003) seems to be identical to the Hungarian suffixed pattern (29b:c′).

These suffixed anti-harmonic forms violate the Polysyllabic Split in (28). As the resulting subpattern does not violate monotonicity, in the following discussion we abstract away from this complication and focus on monomorphemic stems.

In the next section we will examine another kind of variable harmonic pattern: the variable transparency of neutral vowel sequences in Hungarian, which manifests itself in vacillation.

4.2 Variation in transparency

It is well-known (e.g. Vago 1980; Siptár and Törkenczy 2000) that in Hungarian weakly disharmonic mixed roots containing a (series of) back vowel(s) and one neutral vowel ([B^+N]) count as harmonically back. This transparent behaviour is virtually invariable in the case of i/iː and shows very little variability in the case of eː. If, however, the root contains more than one of these final neutral vowels ([B^+N^+]), then massive variability occurs, which is manifested in a high degree of vacillation (intraspeaker variation) as after the root types [Bi(ː)i(ː)] (e.g. *alibi-val/-vel* 'with alibi'), [Bi(ː)eː] (e.g. *klarinét-nak/-nek* 'clarinet-DAT') and [Beːi(ː)] (e.g. *protézis-ek/-ok* 'denture-PLUR').[31] This phenomenon, the quantitatively sensitive transparency/opacity of neutral vowels is called the *Count Effect*, see Ringen and Kontra 1989; Hayes and Cziráky Londe 2006.[32] The Count Effect has a consequence for the harmonic pattern: the original subsumption of the context we labelled [BN] as in (8) has to be split into [BN] containing *exactly* one N and [BNN^+] containing more than one. This is shown in (30) below (where the monosyllabic neutral context [N] is omitted for the sake of simplicity).

31 Type [Beːeː] is virtually non-existent, the only examples are rare proper names, e.g. *Athéné* [ɒteːneː] 'Athena'. There are very few words with three neutral vowels in this position ([BNNN], e.g. *horribilis* 'horrible'), and words with more than three are non-existent.
32 The Count Effect we discuss in this section is not the only source of variation in transparency in Hungarian. We disregard here the Height Effect, the tendency that more open neutral vowels are more variably transparent than less open ones (even when only one occurs in a [B^+N] stem), see Hayes and Cziráky Londe 2006. E.g. there is a high degree of variability in the case of ɛ but there is virtually no variability after i(ː).

(30) Patterns of transparency (TP) and the Count Effect (CE):

<table>
<tr><td>stem contexts:</td><td>[B]_</td><td>[BN]_</td><td>[BNN$^+$]</td><td>[NN$^+$]_</td><td>[FN]_</td><td>[F]_</td></tr>
<tr><td>a. no TP (E. Khanty)</td><td>B</td><td colspan="2">F</td><td>F</td><td>F</td><td>F</td></tr>
<tr><td>b′. invariable TP & CE (?)</td><td>B</td><td>B</td><td>F</td><td>F</td><td>F</td><td>F</td></tr>
<tr><td>b′:b. variable TP & CE (Hungarian)</td><td>B</td><td>B</td><td>B%F</td><td>F</td><td>F</td><td>F</td></tr>
<tr><td>b. invariable TP & no CE (Finnish)</td><td>B</td><td colspan="2">B</td><td>F</td><td>F</td><td>F</td></tr>
</table>

It can be seen in (30) that the disharmonicity of the actual pattern of Hungarian (third row in (30)) is ‘in between’ the invariable opaque pattern (first row: 30a) and the invariable transparent pattern (last row: 30b). The behaviour of [BN]-stems and [BNN$^+$]-stems differs, i.e. Hungarian displays the Count Effect and variation in transparency simultaneously.[33] This pattern (third row: 30b′:b) can be considered a mixture of the invariable totally transparent pattern (last row: 30b) and another pattern which is invariable and also shows the Count Effect (second row: 30b′). Both latter patterns are monotonic, and (30b) is attested (e.g. Finnish). We do not know of a language which shows exactly the same pattern as (30b′) – but, since it is monotonic it *could* exist.

Table (31) shows the overall Hungarian pattern (31b′:c′) with variation in transparency and in anti-harmony in the new harmonic contexts we have distinguished resulting in a seven-position scale. It can be seen that when variation is added to the picture the Hungarian overall pattern is really a “mixed” pattern between two monotonic patterns: one of them has invariable transparency with the Count Effect without anti-harmony (see 30b′ = 31b′) and the other has invariable transparency without the Count Effect together with invariable anti-harmony and the Polysyllabic Split (see 29c′ = 31c′).

33 Similarly to anti-harmony, the Count Effect and therefore the variation in transparency also interacts with Harmonic Uniformity (see Section 4.1 (iv)), which we abstract away from for the same reason as before.

(31) Variable transparency and anti-harmony and the closest invariable patterns

stem context:	[B]_	[BN]_	[BNN]	[N]_	[NN]	[FN]_	[F]_
b′. invariable TP, CE, no AH	*B*	*B*	*F*	*F*		*F*	*F*
b′:c′. variable TP, CE, AH, PS	*B*	*B*	***B/F***	***B/F***	*F*	*F*	*F*
c′. invariable TP, no CE, AH, PS	*B*	*B*		*B*	*F*	*F*	*F*

In the following sections we extend our definitions of monotonicity to variation and will show that this complex variable pattern is monotonic.

4.3 The monotonicity of patterns with variation: ordering

In the previous two sections, we discussed two examples of variable harmony patterns in detail (one with lexical variation and the other with vacillation). We have pointed out that these variable patterns (and we suggested that variable patterns in general) must conform to the same constraints or principles of organisation as invariable patterns, namely monotonicity (and proper prototypical values). We have argued that this is true and the examples make our argument plausible, but we have not examined how our definitions of monotonicity (20, 21) apply to variation. The existence of harmonic patterns with variation makes it necessary to revise the definition of monotonicity since the variation of harmonic values may be interpreted in various ways and the different interpretations have different consequences.

We can interpret variation in a particular context of a pattern as an overlap in that context between the (adjacent) domains of the values assigned to the contexts, i.e. there is (at least) one context which is assigned two values (in our case both *F* and *B*). This is not the interpretation of monotonicity that we will adopt (and explore) for two reasons (i) it is incompatible with the ordering interpretation of monotonicity (20),[34] and (ii) we would run into problems when we include quantification in our model, see Section 4.5.

34 Under such an interpretation *uniqueness* does not hold for the assignment, thus it cannot be a function.

A simple alternative approach is to interpret *variation as a harmonic value*, i.e. we stipulate a third harmonic value "variation" (V) which is ordered in between the two prototypical values B and F by the ordering relation $<'$ of harmonic values. This can be seen in (32) below.

(32) The mapping between harmonic stem types (contexts) and the harmonic values B, V, F

hierarchy of harmonic stem types:	[B] < [BN] < [N] < [FN] < [F]
assignment function	f: {harmonic stem types} $\rightarrow$ {harmonic values}
hierarchy of harmonic values:	$B \;<'\; V \;<'\; F$

The definition of a monotonic pattern is the same as before: the assignment function has to be monotonic (20) and the definition of a monotonic assignment function is the same as in (18), which is repeated below in (33).

(33) Monotonic function
f is *monotonic* iff $[X] \leq [Y] \;\Rightarrow\; f([X]) \leq' f([Y])$ for all harmonic contexts [X], [Y]

For example, the assignment in the case of the pattern ⟨*BBVFF*⟩ (which we encountered when we discussed anti-harmony – see (27b:c)) is monotonic as shown in (34) below.

(34) An example for a monotonic assignment function: the pattern ⟨*BBVFF*⟩

hierarchy of harmonic stem types: [B] < [BN] < [N] < [FN] < [F]

hierarchy of harmonic values: $B \;<'\; V \;<'\; F$

By contrast, the pattern ⟨*BVBFF*⟩ has a non-monotonic assignment function, because the second context ([BN]) is assigned the harmonic value V and the third context ([N]) is assigned the value B. [BN] is "lower" than [N] in the scale of harmonic contexts ([BN]<[N]), but the same is not true for the relevant harmonic values since $V>'B$. This is shown in (35).

(35) An example for a non-monotonic assignment function: the pattern ⟨*B**V**B**FF*⟩

hierarchy of harmonic stem types: [B] < [BN] < [N] < [FN] < [F]

hierarchy of harmonic values: $B \;<'\; V \;<'\; F$

However, there are well-formed patterns that show *variation in more than one harmonic context*. This raises the question of how we should model patterns with two or more positions of variation. Should we assume that variation is always the assignment of the *same value* V, even when it occurs in more than one context? Or should we assume *different values of variation* V_1, V_2, etc. for the contexts in which variation occurs? We will refer to the former approach as the *single V-value model* and the latter as the *multiple V-value model*.

If we adopt the former approach, and assume only one value for variation (V), then the value V behaves formally like the other two values B and F, i.e. it is possible for the same value V to be assigned to two (or more) different harmonic contexts. In (36) below we show such a monotonic pattern.

(36) An example for a monotonic assignment function (single V-value model): $\langle B\boldsymbol{VV}FF\rangle$

hierarchy of harmonic stem types: [B] < [BN] < [N] < [FN] < [F]

hierarchy of harmonic values: B <′ V <′ F

The other approach is that we have more than one (potentially non-identical) value for variation. Let us take two values V_1 and V_2 for the sake of simplicity and assume that in the ordered set of harmonic values they are ordered in the following way: V_1<′V_2. In this case – in contrast to the other model with a single value for variation – the order of the two V-values in the pattern matters: the pattern $\langle BV_1V_2FF\rangle$ is monotonic by the ordering version of monotonicity (20), but $\langle BV_2V_1FF\rangle$ is not, as (37a, b) shows.

(37) Examples for assignment functions (multiple V-value model):

a. monotonic pattern $\langle B\boldsymbol{V_1V_2}FF\rangle$ (V_1<′V_2)

hierarchy of harmonic stem types: [B] < [BN] < [N] < [FN] < [F]

hierarchy of harmonic values: B <′ V_1 <′ V_2 <′ F

b. non-monotonic pattern $\langle B\boldsymbol{V_2V_1}FF\rangle$ (V_1<′V_2)

hierarchy of harmonic stem types: [B] < [BN] < [N] < [FN] < [F]

hierarchy of harmonic values: B <′ V_1 <′ V_2 <′ F

The question whether the single V-value model or the multiple V-value model is more appropriate is not independent of the interpretation of the ordered set of harmonic values. If the exact "positions" of the two (or more) V-values relative to the other values do not matter, i.e. the distinction is empirically unnecessary, then the single V-value model is sufficient. But if the "proximity" (and also the similarity) of the multiple V-values to the prototypical values F and B is relevant, then the multiple V-value model is the adequate one. A plausible interpretation of this proximity/similarity is a measure which is sensitive to how many forms there are with front suffixes vs. back suffixes in the relevant set of forms. An appropriate measure of this is the *relative token* frequency[35] of forms with front[36] harmonic suffixes. In this case, if the relative frequency of forms in the set whose harmonic suffix is B equals 0 (i.e. there are no such forms), then the relative frequency of forms whose harmonic suffix is F equals 1 (all the forms are of this kind). Thus V is represented by a number between them: $0 < p < 1$. Therefore $V_1 < V_2$ means that $p_1 < p_2$, i.e. the relative token frequency of forms suffixed with a front harmonic suffix vowel in the first variation site (the harmonic context that is assigned the value V_1) is smaller than in the second variation site (V_2). In other words: the set of forms associated with the harmonic position that is assigned the value V_2 contains relatively more front harmonic forms (in percentage) than the set associated with the position whose value is V_1. We will return to this issue in Section 4.5.

Let us set aside for the time being the possible implementation of quantitative differences in variation in a multiple V-value model and let us approach the problem of the monotonicity of variable harmonic patterns in a single V-value model. The properties of variable patterns we discuss in this section are valid in both models.

Consider (38), where we use the label *B/F* instead of V to highlight the fact that variation is with the values B and F (note that *B/F* does not distinguish between lexical variation $B|F$ and vacillation $B\%F$). The table in (38) shows all the monotonic patterns containing variation in any of their non-prototypical harmonic contexts assuming five harmonic contexts[37] and the fixed proper

35 It is possible that there are other adequate quantifications which assign numbers as values to contexts in such a way that the values assigned to variable contexts are in between the values that F and B can take.

36 Equally, we could have chosen the relative token frequency of *back* suffixes. This choice is immaterial and does not affect the results about monotonicity.

37 The overall Hungarian variable pattern requires seven contexts (cf. 31), here in this general discussion we use five for the sake of simplicity. The general conclusions we draw, however, are valid for patterns with any number of contexts.

values (*B* and *F*, respectively) of the prototypical contexts [B] and [F]. There are 6 patterns of this kind: out of the six patterns there are 3 patterns with one variation site, there are 2 with two variation sites, and there is one which shows variation in all of the three non-prototypical contexts. Note that three of these patterns actually occur in Hungarian as harmony patterns of different neutral vowels i/iː, eː and ɛ. Recall that in this paper we only discuss the first of these in detail – this is the pattern meant when we refer to “the Hungarian pattern”.

(38) Monotonic variable patterns

stem contexts:	[B]_	[BN]_	[N]_	[FN]_	[F]_
with one variation site					
a:b. opacity–transparency variation; no anti-harmony (Hung. ɛ)	*B*	*B/F*	*F*	*F*	*F*
b:c. transparent; variable anti-harmony (Hungarian i/iː)	*B*	*B*	*B/F*	*F*	*F*
c:d. transparency–anti-opacity variation; anti-harmony	*B*	*B*	*B*	*B/F*	*F*
with two variation sites					
a:c. variation in opacity–transp. & anti-harmony (Hungarian eː)	*B*	*B/F*	*B/F*	*F*	*F*
b:d. variation in transparency–anti-opacity & anti-harmony	*B*	*B*	*B/F*	*B/F*	*F*
with three variation sites					
a:d. variation always after N	*B*	*B/F*	*B/F*	*B/F*	*F*

Monotonic variable patterns can always be decomposed into two well-formed invariable patterns, i.e. each of the patterns in (38) is a mixture of a pair of well-formed patterns in (9a–d) – this is shown by their labels in (38): a:b, b:c, c:d, etc. The first pattern (38a:b) is a mixture of the opaque (9a) and the transparent (9b) types without anti-harmony. The harmony pattern of the vowel ɛ in Hungarian is a pertinent example here: there are no anti-harmonic stems with ɛ and the harmonic context [Bɛ] shows robust variation in transparency (Ringen and Kontra 1989; Hayes and Londe 2006). The next type (38b:c) is the mixture of two patterns with transparency, the one without anti-harmony (9b), and the one with anti-harmony (9c). This pattern is exemplified by the behaviour of Hungarian i/iː: disregarding the Count Effect in [BNN$^+$]-type stems and the Polysyllabic Split in [NN$^+$]-type stems, see (27b:c)). The third pattern (38c:d) is a hybrid of the pattern with transparency (9b) and the pattern with anti-opacity (9c), i.e. back harmonic suffixes occur after weakly disharmonic [BN] stems, but variation takes place after non-disharmonic stems of the [FN] type (in formulae: [BN]*B* and [FN]*B/F*). There is very little information available about languages with anti-opacity, and we know of no language that is an example for pattern (38c:d). Nevertheless this pattern is well-formed by monotonicity.

Patterns with more than one variation site (we will refer to these as multiply variable patterns) are also allowed. (38a:c) is the type which shows variation in opacity–transparency and also in anti-harmony: this is a mixture of the opaque pattern without anti-harmony (9a) and the transparent pattern with anti-harmony (9c). This type is exemplified by the harmony pattern of the vowel eː in Hungarian: there are a few anti-harmonic stems with eː and some [Beː] stems display variation in transparency. Type (38b:d) shows variation in transparency without anti-harmony and anti-opacity (it is the same as the variable pattern (38c:d) with an extra element of variation in anti-harmony too). The last type (38a:d) shows variation of the most extreme extent: it has variation in all the three non-prototypical harmonic contexts. This type of language has variation after every stem which ends in a neutral vowel independently of the previous vowel. This is a well-formed harmony pattern according to monotonicity but we do not know of any natural language examples for it.

The table in (39) shows some non-monotonic variable patterns,[38] specifically, those that would be monotonic if variable cells were ignored.

38 Note that there are many more: the number of all the theoretically possible variable and invariable five-context patterns with proper prototypical values is $3^3 = 27$. Of these, 10 patterns are monotonic: 4 invariable ones (see table (9a–d)) + 6 variable ones (see table 38). Thus, the number of non-monotonic patterns with proper prototypical values is 17 – of which there are 4 invariable (see (9e–h)) and 13 variable ones (10 of these are shown in 39).

(39) Non-monotonic patterns (that are monotonic for *F* and *B*)

stem contexts:	[B]_	[BN]_	[N]_	[FN]_	[F]_
with one variation site					
j. invariable opacity; variable anti-harmony	*B*	*F*	*B/F*	*F*	*F*
k. invariable anti-opacity; variable anti-harmony	*B*	*B*	*B/F*	*B*	*F*
l. opacity–transparency variation; invariable anti-harmony	*B*	*B/F*	*B*	*F*	*F*
m. transparency–anti-opacity variation; no anti-harmony	*B*	*B*	*F*	*B/F*	*F*
n. anti-opacity–anti-transparency variation; (anti-harmony)	*B*	*B/F*	*B*	*B*	*F*
p. opacity–anti-transparency variation; (no anti-harmony)	*B*	*F*	*F*	*B/F*	*F*
with two variation sites					
q. opacity–anti-transparency & anti-harmony variation	*B*	*F*	*B/F*	*B/F*	*F*
r. anti-opacity–anti-transparency & anti-harmony variation	*B*	*B/F*	*B/F*	*B*	*F*
s. everything is variable but no anti-harmony	*B*	*B/F*	*F*	*B/F*	*F*
t. everything is variable but anti-harmony	*B*	*B/F*	*B*	*B/F*	*F*

A comparison of (38) and (39) reveals that *in the monotonic variable patterns*, (i) variation sites are contiguous (as defined in 11) and (ii) are located in between cells that are assigned different values for *F* and *B*; and furthermore that the statements (i) and (ii) are not true for non-monotonic patterns. Thus, the following prediction can be made about the possible locus of variation in harmony[39]

39 Crucially, as throughout this paper, no distinction is made between lexical variation and vacillating variation. We consider them essentially the same, which is in agreement with Hayes et al.'s concept of "zone of variation" (Hayes et al. 2006: 829).

(which conforms to observations about variation in other phonological and morphological phenomena, Kálmán et al. 2012; Rebrus and Törkenczy 2011).

(40) **Locus of variation**
Given a fixed ordering of (harmonic) contexts and different values assigned to the contexts, variation between two values can only occur in a context or a contiguous sequence of contexts that is at the border of contexts which are assigned non-identical values, i.e. in a (sequence of) context(s) flanked by contexts that are assigned different invariable values.

This is a direct consequence of monotonicity, and it is also compatible with the contiguity assumption (11) and the entailments between harmonic values (12) we made when we discussed monotonic patterns without variation. We repeat the entailments here as (41):

(41) **Entailments between harmonic values**
a. If a context is assigned the value of *F*, then all the contexts in the scale which are "more front", i.e. closer/more similar to the prototypical context [F], also have the value *F*
b. If a context is assigned the value of *B*, then all the contexts in the scale which are "more back", i.e. closer/more similar to the prototypical context [B], also have the value *B*

We have seen that a *harmony pattern without variation* only contains contiguous sequences of values (*B*, *F*) if *either* of the entailments holds (in this case, when one of the entailments in (41) holds, then the other necessarily holds too). Equivalently, if a harmony pattern without variation is monotonic (assuming proper prototypical values (24)), then both of the entailments hold.

It is not necessary to reformulate the entailments in (41) or to supplement them if we also include *harmony patterns with variation*, i.e. introduce a third value ($B/F = V$). The crucial difference is that in order to handle patterns with variation, we have to require that *both* (41a) and (41b) should hold. If both entailments hold, then it follows that all the sequences of all the three values are contiguous and the pattern is monotonous (and well-formed). Note that it is not necessary to supplement (41ab) with a statement about what the occurrence of the values *B/F* entails in a given context within a monotonic pattern since, if both of them hold, then the sequence of *B*'s and the sequence of *F*'s each is contiguous, and then it follows that the sequence of harmonic contexts whose value is *V* must be contiguous, too. Again, monotonicity (together with the Proper prototypical values assumption (24)) is equivalent to (41) and equivalent to the contiguity of *B*-value sequences, *F*-value sequences and *V*-value sequences.

Given (41), we can make predictions about the value(s) assigned to (a) given context(s) on the basis of the values assigned to other contexts in monotonic harmony patterns. If the value assigned to a context is *B*, then all the contexts "on its left" (i.e. closer to [B]_, the prototypical context for *B*) must be *B* too. This prediction is borne out in the monotonic pattern (38b:c) <*BBVFF*>. This pattern is also well-behaved when we consider the prediction about values we can make on the basis of an *F* value assigned to a context: *viz.* all the contexts "on its right" (i.e. closer to [F]_, the prototypical context for *F*) must be *F* too. (41) also predicts that if both (41a) and (41b) are met, then, if there is more than one variation site in the pattern, the variation sites will be contiguous too. This prediction is borne out in the monotonic pattern (38a:c) <*BVVFF*>, for instance (and in all the other multiply variable patterns in (38)). The non-monotonic pattern (39m) <*BBFVF*> shows that *both* (41a) and (41b) must be met in order for a pattern to be monotonic. Since *only* (41b) is met in (39m), it is not well-formed.

If a given harmonic context is a variation site (i.e. the value *V* is assigned to it), it follows from (41) that "on its left" (i.e. closer to [B]_, the prototypical context for *B*) (i) either all the contexts are assigned *B* or (ii) there are one or more contiguous variation sites with one or more contiguous contexts that are assigned the value *B* "on the left" of the variation sites. The mirror image of this is found "on the right" (i.e. closer to [F]_, the prototypical context for *F*) of a variation site: (i) either all the contexts are assigned *F* or (ii) there are one or more contiguous variation sites with one or more contiguous contexts that are assigned the value *F* "on the right" of the variation sites. These predictions are borne out, e.g. in the monotonic variable pattern (38b:d) <*BBVVF*> since the first variation site (i) only has *B* values "on the left", and (ii) has another variation site 'on its right' which only has contexts that are assigned the value *F* "on its right"; and the second variation site (i) only has *F* values "on the right", (ii) has another variation site "on its left" which only has a contexts that are assigned the value *B* "on its left". The variable pattern (39q) <*BVVBF*> is non-monotonic (and ill-formed) because the second variation site (the one closest to [F]_, the prototypical context for *F*) has a context that is assigned a value other than *F*.

The generalisation about sandwiched contexts (12) also holds true of all the three values (*B*, *F*, *V*) in monotonic variable patterns (in which the entailments in (41) are true):[40]

(42) Harmonic values in sandwiched contexts (with variation)
- i. $F...X...F \Rightarrow X = F$
- ii. $B...X...B \Rightarrow X = B$
- iii. $V...X...V \Rightarrow X = V$

The entailments in (41) and the generalisations about harmonic values in sandwiched contexts (42) (which follow from (41)) make cross-linguistic predictions about possible harmony systems with variation.

For example, a harmony system that is *invariably opaque* (i.e. the harmonic contexts [BN]_ and [FN]_ are both assigned the value *F*) *cannot also have anti-harmony*, variable or invariable. This is because the harmonic context [N]_ (which would take the values *B* or *V* in the case of invariable or variable anti-harmony, respectively) is sandwiched by the contexts [BN]_ and [FN]_ in the frontness/backness scale ([BN]<[N]<[FN]), and therefore cannot be assigned a value other than *F* (in accordance with (42)): $^*\langle B\boldsymbol{F}B\boldsymbol{FF}\rangle$ or $^*\langle B\boldsymbol{F}V\boldsymbol{FF}\rangle$. Conversely, a harmony system that is *invariably anti-harmonic* (i.e. the harmonic context [N]_ is assigned the value *B*) *cannot be variably or invariably opaque* at the same time because the context [BN]_ is sandwiched by [B]_ and [N]_ in the frontness/backness scale ([B]<[BN]<[N]) and therefore [BN]_ cannot have a value other than *B*: $^*\langle \boldsymbol{B}F\boldsymbol{B}FF\rangle$ or $^*\langle \boldsymbol{B}V\boldsymbol{B}FF\rangle$.

Naturally, predictions can be made on the basis of the value *V* too: e.g. *a variably opaque* system in which the harmonic contexts [BN]_ and [FN]_ are assigned the value *V must be variably anti-harmonic* because the harmonic context [N]_ is sandwiched by [BN]_ and [FN]_ ([BN]<[N]<[FN]), so $\langle B\boldsymbol{V}V\boldsymbol{V}F\rangle$ is well-formed but $^*\langle B\boldsymbol{V}B\boldsymbol{V}F\rangle$ or $^*\langle B\boldsymbol{V}F\boldsymbol{V}F\rangle$ is not.

We will now examine the monotonicity of patterns with variation under the similarity interpretation of monotonicity.

4.4 The monotonicity of patterns with variation: similarity

We have seen in Section 3.3 that the monotonicity of patterns can be defined on the basis of similarity, too. Such a definition presupposes two concepts: (i) the

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40 Recall that these contexts are not necessarily strictly adjacent in the scale of harmonic contexts.

similarity poset of harmonic contexts induced by the linear ordering of these contexts and (ii) the *linear ordering of harmonic values*. Concept (i) has been introduced in (22) as a partially ordered set of similarities based on neighbouring (not necessarily strictly adjacent) contexts in the scale. Concept (ii) has also been defined as simply as possible. If we only examine invariable patterns, we only have two values, F and B. We have assumed the ordering $<'$ between the two values: $B<'F$, cf. (16). Then we define the similarities between all of the possible combinations of the values F and B, namely sim′(F,F), sim′(B,B), sim′(F,B) and sim′(B,F). The similarity function sim′ is assumed to be symmetric i.e. sim′(F,B) = sim′(B,F) and therefore these two similarities need not be distinguished. Thus, a similarity poset can be generated for the harmonic values also, which is shown on the left in (43) below, where the first row contains the two similarities of the identical elements and the last row contains the only similarity of non-identical elements. In Section 3 this simple poset is represented by numbers: the maximal element (the similarities between the identical values) as 1, and the minimal element (the similarities between the non-identical values) as 0. This could be done without loss of generality because the only significant piece of information relevant is that the similarity between identical values is greater than that between non-identical ones in accordance with the partial ordering $\preccurlyeq'$. This is shown on the right in (43) where 0 and 1 represent the minimal and the maximal element, respectively.

(43) Similarity poset induced by the linear ordering of the harmonic *values*: $B<'F$

sim′ (B,B) sim′ (F,F)	similarities of identical items	1
sim′ (B,F)	similarities of neighbouring items	0

The definition of the similarity version of monotonicity is based on whether the mapping between the two similarity posets (the one induced by the context scale and the one by the ordering of values) is a monotonic function or not, see (21). For a specific harmonic pattern this can be easily checked if the nodes of the similarity poset induced by the harmonic context scale are labelled with the members of the similarity poset induced by harmonic values. In this way, we get a labelled similarity poset in which the latter similarities are represented as 0 and 1, see (23).

Now consider the variable patterns. Since the scale of harmonic contexts is the same, the similarity poset induced by this ordering is the same too. However, now we have three harmonic values B, F, and V ordered $B<'V<'F$. We can construct the similarity poset of harmonic values induced by this ordering. In order to do this we have to define non-extremal similarities between the maximal and the minimal similarities. The maximal similarity is the identity case: sim′(*F*,*F*) = sim′(*B*,*B*) = sim′(*V*,*V*) = 1 and the minimal one is when the different prototypical values are compared (i.e. the similarity between the first and last member of the linear ordering of values): sim′(*F*,*B*) = sim′(*B*,*F*) = 0. Under the single *V*-value model of variation (introduced in Section 4.3), the two new similarities are between the values *B* and *V* and between the values *V* and *F*: let us represent these similarities as p and q where p = sim′(*V*,*F*) and q = sim′(*B*,*V*). This is shown in (44) below where the poset on the left shows the original posets of similarities between the values and the numbers on the right show its simplified version in which the maximal similarities are merged into a largest element 1.

(44) Similarity poset induced by the linear ordering of the harmonic *values*: *B*<′*V*<′*F*

sim′ (*B*,*B*) sim′ (*V*,*V*) sim′ (*F*,*F*)	similarities of identical items	1
sim′ (*B*,*V*) sim′ (*V*,*F*)	similarities of first neighbours	$0 < q, p < 1$
sim′ (*B*,*F*)	similarities of second neighbours	0

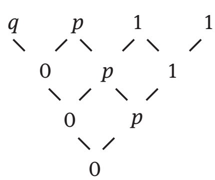
In order to check whether a variable harmony pattern is monotonic, first we should label the nodes of the similarity poset of *contexts* (22) with the numbers 1, q, p, 0. The resulting labelled poset is similar to those that we have used for the invariable patterns (see (23)), but in addition to 0 and 1, labels can be q and p, too (where $0 < q < 1$ and $0 < p < 1$). If a pattern is monotonic according to (21), then the numbers in the labelled poset, which represent similarities between harmonic values, *increase* from bottom to top, i.e. there is no pair of nodes n_1 and n_2 in the labelled poset (of contexts) such that $n_1 \lessdot n_2$ while the opposite holds for their labels.

We can see this in (45) below where the (five-member) variable patterns (45a,b) have one variation site. Pattern (45a) fulfils the monotonicity criterion discussed above, but pattern (45b) does not. In the latter case there are two nodes labelled with q (emboldened in the diagram) above nodes labelled with 1, and there are two nodes labelled with p below nodes labelled with 0 (emboldened in the diagram). By the monotonicity requirement (21), the following must hold for these labels: $1 \leq q$ and $p \leq 0$. This can only be true in the case when $q = 1$ and $p = 0$, which means that sim′(*B*,*V*) = sim′(*B*,*B*) and sim′(*V*,*F*) = sim′(*B*,*F*). Thus the second position in the pattern ⟨*BVBFF*⟩ (which we assumed is a *V*-value) is in fact not a variable position, but it has the non-variable value *B*, i.e. the *variable* pattern (45b) is not monotonic. The dual patterns of these patterns are also shown in (45).[41] The monotonicity of a dual pattern (and its labelled poset) is identical to its counterpart.

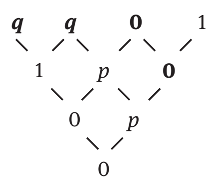
(45c,d) exemplify patterns containing two variation sites. Monotonicity holds for pattern (45c) (and its dual pattern). Pattern (45d) would be monotonic only if $q = 1$ (for the same reasons as in the case of (45b) discussed above), but this would mean that the second and the fourth positions of the pattern $\langle BVBVF\rangle$ (i.e. the variation sites) would have to be assigned the value B (and therefore the pattern would be invariable). Consequently, the variable pattern (45d) (and its dual pattern) is non-monotonic.

(45) Labelled similarity posets of several five-member patterns

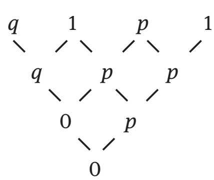
a. monotonic pattern $\langle BVFFF\rangle$ (if $0 \leq p,q \leq 1$) and its dual pattern $\langle FVBBB\rangle$



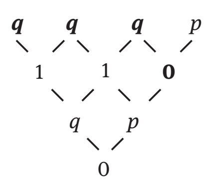
b. non-monotonic pattern $\langle BVBFF\rangle$ and its dual pattern $\langle FVFBB\rangle$



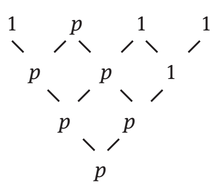
c. monotonic pattern $\langle BVVFF\rangle$ (if $0 \leq p,q \leq 1$) and its dual pattern $\langle FVVBB\rangle$



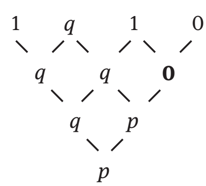
d. non-monotonic pattern $\langle BVBVF\rangle$ and its dual pattern $\langle FVFVB\rangle$



e. monotonic(!) pattern $\langle VVFFF\rangle$ (if $0 \leq p \leq 1$) and its dual pattern $\langle VVBBB\rangle$



f. non-monotonic pattern $\langle VVBBF\rangle$ and its dual pattern $\langle VVFFB\rangle$



41 Note that a dual pattern substitutes p for every q and vice versa. This, however, does not influence the monotonicity of the patterns.

Patterns (45a–d) have proper prototypical values in accordance with (24), i.e. the first position of the pattern has the value *B* (this is the position of the context [B]_), and the last one has the value *F* (the context [F]_). Dual patterns in (45a–d) have these assignments in the opposite way: value *F* is in the first, and value *B* is in the last position. Patterns (45e,f), however, do not follow either of these assignments: their first positions have the value *V* (instead of *B* or *F*). Nevertheless, there are monotonic patterns of this type, too, e.g. (45e) is monotonic as its labelled poset shows. Pattern (45f) is, however, non-monotonic because there is a 0 above the label *p* (which means that for it to be monotonic $p = 0$ must hold, which means that this pattern cannot be variable).

Note that pattern (45e) contains only the values *V* and *F*, while pattern (45f) contains all the values *V*, *B* and *F*, but in the wrong order: the variable sites are not between contexts that are assigned different invariable values *F* and *B*, as (40) requires for the locus of variation. The entailment restrictions (41) do not hold in (45f) either because in $\langle VVBBF \rangle$ there exist sites that have values other than *B* (the 1st and 2nd positions have the value *V*) left of a context that is assigned the value *B* (the 3rd and 4th positions). However, this criterion is met in the pattern (45e). In fact, $\langle VVFFF \rangle$ satisfies both of the entailment restrictions of (41). Consequently, the pattern (45e) is monotonic according to the ordering interpretation of monotonicity (cf. (18)), but (45f) is not. The same is true of the monotonicity of these patterns according to the similarity interpretation defined in posets above (see (21)).

Recall that in Section 3.3 when we discussed the monotonicity of invariable harmonic patterns, we found that the only two cases which are monotonic according to both definitions of monotonicity ((20) and (21)) but do not meet the proper prototypical values requirement (24) are the constant patterns $\langle FFFFF \rangle$ and $\langle BBBBB \rangle$. The other invariable patterns that are monotonic according to both definitions are the patterns that we called well-formed: they have proper prototypical values and are attested harmony systems in languages, see (26). The patterns that are non-monotonic by ordering (20) and monotonic by similarity (21) are those that are dual patterns of the well-formed patterns.

The case of variable patterns is similar, but slightly different, as shown in (46) below (compare (46) with (26)). If patterns with five contexts are assumed, there are $3^5 = 243$ theoretically possible (variable and invariable) patterns of harmony. We have the well-formed patterns: they can be invariable (4 patterns, see (9a–d)) or variable (6 patterns, see (45a–f)). These patterns are monotonic according to both (20) and (21), *and* they have proper prototypical values (see column (i) in (46)). In addition to the well-formed ones, we have those patterns that are also monotonic according to both definitions of monotonicity, but they do not have proper prototypical values: these are (a) the 3 possible constant patterns (the 2 invariable ones and the pattern ⟨*VVVVV*⟩, see (46iii)), and (b) patterns that we have not considered before because they do not arise as invariable patterns. The patterns of type (b) are those that only contain values *V* and *B* or *V* and *F* in the appropriate order. There are 8 patterns of this type: ⟨*VFFFF*⟩, ⟨*VVFFF*⟩, ⟨*VVVFF*⟩, ⟨*VVVVF*⟩ or ⟨*BVVVV*⟩, ⟨*BBVVV*⟩, ⟨*BBBVV*⟩, ⟨*BBBBV*⟩; they are monotonic by both interpretations of monotonicity, but one of their prototypical harmonic contexts does not have the proper prototypical values F or B: it has the value *V* instead. The column in which this new pattern type appears and which only contains variable patterns is shaded in grey in (46ii) below. There are additional patterns which are monotonic only in the broad sense (i.e. monotonic only by the looser, similarity interpretation of monotonicity) and non-monotonic in the narrow sense (i.e. non-monotonic by the stricter, ordering interpretation of monotonicity). These are the duals of the patterns discussed above (other than the constant patterns), see (46iv). The rest of the possible patterns are non-monotonic under any interpretation of monotonicity see (46v), which contains the ill-formed patterns in (9e–h) and in (39), which have proper prototypical values, and also those patterns that do not have proper prototypical values.

(46) Types of the theoretically possible (variable and invariable) patterns

pattern types: examples: criteria:	(i) attested /predicted ⟨*BFFFF*⟩ ⟨*BVFFF*⟩ ⟨*BBFFF*⟩ ⟨*BBVFF*⟩ ...	(ii) 'semi- harmonic' ⟨*VFFFF*⟩ ... ⟨*VVVVF*⟩ ⟨*BVVVV*⟩ ...	(iii) constant ⟨*FFFFF*⟩ ⟨*BBBBB*⟩ ⟨*VVVVV*⟩	(iv) duals of (i-ii) ⟨*FBBBB*⟩ ⟨*FVBBB*⟩ ... ⟨*VBBBB*⟩ ...	(v) unattested, others ⟨*BVBFF*⟩ ⟨*FVVFF*⟩ ⟨*FBVBB*⟩ etc.
monotonicity (ordering) = entailments	✓	✓	✓	✗	✗
monotonicity (similarity) = contiguity	✓	✓	✓	✓	✗
proper prototypical values	✓	✗	✗	✗	✓/✗
number of patterns (sum = 243)	10	8	3	18	204

In our analysis, patterns (46i) are the well-formed ones; however, the patterns in (46ii) and (46iii) are also monotonic under both interpretations of monotonicity. These latter patterns are not well-formed since they violate the proper prototypical values requirement (24). Note that in our system it is the proper prototypical values requirement that expresses vowel harmony, so the patterns in (46ii) and (46iii) are not vowel-harmony systems (although they may constrain the occurrence of vowels in suffixes in various ways). The patterns in (46ii) may be called *semi-harmonic* because they only require a proper prototypical value, either *F* or *B*, in one of the prototypical harmonic contexts but not in the other.[42] The patterns in (46iii) do not display any harmony at all – the constant patterns can be seen as an overall phonotactic restriction on suffix vowels independently of the harmonic properties of the stem. The patterns in (46iv), which are non-monotonic according to the strict interpretation of monotonicity, are counter-harmonic (i.e. they assign the opposite values to prototypical harmonic contexts). This type of dissimilation harmony is unattested to the best of our knowledge.

4.5 The monotonicity of patterns with variation: quantification

In the previous sections we discussed variable harmony patterns in the single *V*-value model (see Section 4.3). In this section we will explore variation in harmony in the multiple *V*-value model, in which multiple variation within a harmony pattern may involve the assignment of *different values of variation* (V_1, V_2, etc.) to the harmonic contexts in which variation occurs. We pointed out in Section 4.3 that this makes it possible to quantify variation. Such a quantification has to be sensitive to the ratio of harmonic suffix alternants in each variable harmonic context. As suggested in Section 4.3, a possible way of quantification is to identify the harmonic values with the *relative token frequency* of the forms with a *front suffix allomorph* in a given harmonic context, i.e. the value is the number of forms with a *front* suffix allomorph divided by the number of forms with *all* the harmonic (front or back) suffix allomorphs. Thus, harmonic contexts we have labelled so far with an invariant *F* will be assigned the number 1 as a

42 Such asymmetrical harmony patterns are 'more harmonic' than the constant patterns (46iii) and the dual patterns (46iv) which are monotonic at least by the looser version. We do not know of patterns of variation like this in vowel harmony (where for instance only back stems require back suffix alternants while the front ones may take both harmonic alternants of suffixes). Outside vowel harmony, however, there exist variation patterns of this type where two patterns compete, but only one of them occurs without variation (see Rebrus and Törkenczy 2008). This pattern of variation seems to differ from the U-shape pattern discussed by Zuraw (2015).

value and those with an invariant *B* will be assigned the number 0 as a value. Variation sites will be assigned the number *p* such that $0<p<1$. For example, the pattern $\langle BVFFF \rangle$, which has a single variation site, is quantified as the five-number series $\langle 0\ p\ 1\ 1\ 1 \rangle$ and the pattern $\langle BV_1V_2FF \rangle$, which has more than one, as $<0\ p_1\ p_2\ 1\ 1>$, etc.

Under the ordering interpretation these patterns are monotonic if the numbers *increase* in the series. In the former example this is always true since $0<p<1$, and in the second example it is true if $0<p_1 \leq p_2<1$. Those patterns that are non-monotonic in the single *V*-value model are also non-monotonic in the multiple *V*-value model, e.g. pattern $\langle BVBFF \rangle$ is quantified as the series $\langle 0\ \boldsymbol{p}\ \mathbf{0}\ 1\ 1 \rangle$ which is non-monotonic if $p>0$ since then the second and third sites have numbers in a decreasing order (the relevant positions are emboldened). Similarly, the pattern $\langle BV_1FV_2F \rangle$ is non-monotonic (independently of the relation between p_1 and p_2), because its series $\langle 0\ p_1\ \mathbf{1}\ \boldsymbol{p_2}\ 1 \rangle$ has successive numbers 1 and p_2 in a decreasing order (if $p_2<1$). Therefore, the multiple *V*-value model (and this quantification) yields a stricter version of monotonicity: all the patterns that are monotonic in the quantified model are also monotonic in the single *V*-value model, but not the other way round. Consequently, the criteria of monotonicity discussed in the previous section must also be satisfied here. For instance, the requirements about sandwiched contexts (42i–iii) are valid as well: the value between two identical harmonic values (*F*...*F*, *B*...*B* or *V*...*V*) agrees with the flanking values. The last criterion (42iii) can be made stronger in the quantified model: in a monotonic pattern if the sandwiched context is between two *V*-values p_1 and p_2, then the value in the middle has to be between p_1 and p_2. This is also valid for the non-variable values *F* and *B*, and for those sandwiched contexts where the flanking values are different. Thus, we can state the following generalisation.

(47) Generalized sandwiched contexts
In a monotonic pattern if *Y* is sandwiched by *X* and *Z*, then the quantified value of *Y* is between that of *X* and *Z*,
i. e. $\langle ...X...Y...Z... \rangle \quad \Rightarrow \quad p_X \leq p_Y \leq p_Z \quad$ (where $0 \leq p_X,\ p_Y,\ p_Z \leq 1$)

(47) has the criteria in (42) as its direct corollaries: sandwich type *F*...*F* (42i) is a special case when $X=Z=F$, thus $p_X=p_Z=1$ and therefore $p_Y=1$ (i.e. $Y=F$); the same holds for the sandwich type *B*...*B* (42ii) when $X=Z=B$: if $p_X=p_Z=0$ then $p_Y=0$ (i.e. $Y=B$). In the case of the sandwich type *V*...*V* (42iii), however, $0<p_X,\ p_Z<1$ holds, which entails that $0<p_Y<1$ (i.e. $Y=V$) by (47).

Let us now examine the similarity interpretation of monotonicity in the multiple *V*-value model. As before (see Sections 3.3 and 4.4), we have (i) the similarity poset induced by the linear ordering of the harmonic *contexts*, (ii) the similarity poset induced by the linear ordering of the harmonic *values* and (iii) an assignment between the nodes of poset (i) and poset (ii), which is determined by the assignment function *f* between contexts and values (given in (16/32)). The assignment (iii) is defined between the similarities represented by the nodes of the two similarity posets (i) and (ii) in the following way.

(48) Assignment between the two similarity posets

nodes of the similarity poset of contexts	→	nodes of the similarity poset of values (numbers)
sim (X,Y)	→	sim′$(f(X),f(Y))$

The poset of contexts ((i) above) is the same as in the two other cases we have examined above, see (22) for the five-context scale. However, in the multiple *V*-value model, the poset of values ((ii) above) is different in that the similarity of two *V*-values is not necessarily maximal since in this model two *V*-values *may* be different and their similarity is only maximal if their values are identical (like the similarities between the pairs of values *B*–*B*, *F*–*F*, and two V's in the single V-value model). Therefore, now the similarity poset induced by the linear ordering of values is more complex. An example with two *different* V-values is shown in (49), where the similarity between the two V-values (sim′(V_1,V_2)) is not maximal (emboldened in (49) below).

(49) Similarity poset induced by the linear ordering of the harmonic *values*: $B<'V_1<'V_2<'F$

sim′(B,B) sim′(V_1,V_1) sim′(V_2,V_2) sim′(F,F)	similarity of identical positions	1
sim′(B,V_1) **sim′$(\mathbf{V_1},\mathbf{V_2})$** sim′$(V_2,F)$	similarity of neighbouring positions	q_1, ?, p_2
sim′(B,V_2) sim′(V_1,F)	similarity of second neighbours	q_2, p_1
sim′(B,F)	similarity of first and last positions	0

The question arises how to quantify these similarities. Clearly, the similarity of identical values is maximal (see the first row in (49)) and can be quantified as the number 1. The minimal similarity (see the last row in (49)) is between the two prototypical harmonic values (*F* and *B*) and can be quantified as 0. The real issue is the quantification of similarities between a variable value and some other (non-identical) value (see the intermediate rows in (49)), whose value must be between the minimal value 0 and the maximal value 1. It is natural to assume that the similarity between the value F and a variable value V can be identified with a number that expresses the proportion of F values in V. This is equal to p, the number we have assigned to a harmonic context in which this variation occurs according to the relative token frequency of word forms whose harmonic suffix occurs in its *front* allomorph in this context, i.e. $\text{sim}'(V,F)=p$. Similar considerations apply to the similarity between B and V, whose similarity (denoted by q in (44/45)) is the relative token frequency of forms with the *back* allomorph of a harmonic suffix, i.e. $\text{sim}'(B,V)=1–p$. The last type of similarity to be considered is the similarity between two different V-values, i.e. $\text{sim}'(V_1,V_2)$. This similarity can be captured if we compare the relative frequency of front suffixed forms in V_1 with the relative frequency of front suffixed forms in V_2 (i.e. p_1 and p_2) and also the relative frequency of back suffixed forms in V_1 with the relative frequency of back suffixed forms in V_2 (i.e. $1–p_1$ and $1–p_2$). A suitable measure of comparison is to take the smaller of the two numbers for front suffixed forms and add it to the smaller of the two numbers for back suffixed forms, as shown in (50) below.[43]

(50) $\text{sim}'(X,Y)=\min(p_X,p_Y) + \min(1–p_X, 1–p_Y)$ (where $0 \leq p_X,\ p_Y \leq 1$)

Notice that if X or Y are prototypical values F *or* B, then the formula in (50) gives the expected numbers: e.g. $\text{sim}'(F,B)=\min(1,0)+\min(0,1)=0$; $\text{sim}'(F,F)=\min(1,1)+$ $+\min(0,0)=1$; and $\text{sim}'(V,F)=\min(p,1)+\min(1–p,0)=p$; $\text{sim}'(B,V)=\min(0,p)+$ $+\min(1,\ 1–p)=1–p$. In the case of two V values, the similarity in (50) can be obtained as follows: $\text{sim}'(V_1,V_2)=1–|p_2–p_1|$. This shows that the similarity is between 0 and 1 (as the poset in (49) requires), and the similarity is greater if the difference between the two relative frequencies ($p_2–p_1$) is smaller, and it is smaller if the difference is greater. In an extreme case, if $p_1=p_2$ then the similarity is maximal ($=1$), which means that the two V-values are not different ($V_1=V_2$).

Now we can create the labelled posets of harmonic contexts (discussed in (23)/(45) for the non-quantified models) where the labels are numbers between 0 and 1 and derive from the poset (48) and the formula (50). This is shown in (51) below, where we use the same patterns as examples as for the non-quantified model in (45).

43 In the special case when the cardinality of the two sets of forms that occur in the contexts that X and Y are assigned to is equal, this function counts the number of pairs of forms in the two sets which have harmonically identical suffixes and divides the result with the number of all pairs (including the pairs of non-identical values). In a more refined model this similarity can be defined with reference to the *mutual information* of the two sets of forms, which is based on the (conditional) entropy of the two sets (one version of such a metric is the shared information distance).

(51) Labelled similarity posets of several five-member patterns in the quantified model

a. monotonic pattern $\langle BVFFF\rangle$ if $0 \leq p \leq 1$ and its dual pattern $\langle FVBBB\rangle$

$$\begin{array}{ccccccc}
1-p & & p & & 1 & & 1 \\
 & \backslash\ / & & \backslash\ / & & \backslash\ / & \\
 & 0 & & p & & 1 & \\
 & & \backslash\ / & & \backslash\ / & & \\
 & & 0 & & p & & \\
 & & & \backslash\ / & & & \\
 & & & 0 & & &
\end{array}$$

b. non-monotonic pattern $\langle BVBFF\rangle$ if $p>0$ and its dual pattern $\langle FVFBB\rangle$

$$\begin{array}{ccccccc}
\mathbf{1-p} & & \mathbf{1-p} & & \mathbf{0} & & 1 \\
 & \backslash\ / & & \backslash\ / & & \backslash\ / & \\
 & 1 & & p & & \mathbf{0} & \\
 & & \backslash\ / & & \backslash\ / & & \\
 & & 0 & & p & & \\
 & & & \backslash\ / & & & \\
 & & & 0 & & &
\end{array}$$

c. monotonic pattern $\langle BV_1V_2FF\rangle$ if $p_1 \leq p_2$ and its dual pattern $\langle FV_1V_2BB\rangle$

$$\begin{array}{ccccccc}
1-p_1 & & 1-|p_2-p_1| & & p_2 & & 1 \\
 & \backslash\ / & & \backslash\ / & & \backslash\ / & \\
 & 1-p_2 & & p_1 & & p_2 & \\
 & & \backslash\ / & & \backslash\ / & & \\
 & & 0 & & p_1 & & \\
 & & & \backslash\ / & & & \\
 & & & 0 & & &
\end{array}$$

d. non-monotonic pattern $\langle BV_1BV_2F\rangle$ if $p_1 > 0$ and its dual pattern $\langle FV_1FV_2B\rangle$

$$\begin{array}{ccccccc}
\mathbf{1-p_1} & & \mathbf{1-p_1} & & 1-p_2 & & p_2 \\
 & \backslash\ / & & \backslash\ / & & \backslash\ / & \\
 & 1 & & 1-|p_2-p_1| & & \mathbf{0} & \\
 & & \backslash\ / & & \backslash\ / & & \\
 & & 1-p_2 & & p_1 & & \\
 & & & \backslash\ / & & & \\
 & & & 0 & & &
\end{array}$$

e. (non-well-formed) monotonic $\langle V_1V_2FFF\rangle$ if $p_1 \leq p_2$ and its dual pattern $\langle V_1V_2BBB\rangle$

$$\begin{array}{ccccccc}
1-|p_2-p_1| & & p_2 & & 1 & & 1 \\
 & \backslash\ / & & \backslash\ / & & \backslash\ / & \\
 & p_1 & & p_2 & & 1 & \\
 & & \backslash\ / & & \backslash\ / & & \\
 & & p_1 & & p_2 & & \\
 & & & \backslash\ / & & & \\
 & & & p_1 & & &
\end{array}$$

f. non-monotonic $\langle V_1V_2FBB\rangle$ if $p_2<1$ and its dual pattern $\langle V_1V_2BFF\rangle$

$$\begin{array}{ccccccc}
1-|p_2-p_1| & & p_2 & & \mathbf{0} & & 1 \\
 & \backslash\ / & & \backslash\ / & & \backslash\ / & \\
 & p_1 & & 1-p_2 & & \mathbf{0} & \\
 & & \backslash\ / & & \backslash\ / & & \\
 & & 1-p_1 & & 1-p_2 & & \\
 & & & \backslash\ / & & & \\
 & & & 1-p_1 & & &
\end{array}$$

The conditions on monotonicity are the same here as in the single *V*-model illustrated in (45). The one crucial difference is that here the similarity between two *V* positions is not always maximal as in the single *V*-model (cf. 45), but is determined by the formula in (50). For example, in patterns (51c, e), which contain two consecutive variable positions, the similarity of V_1 and V_2 equals $1-|p_2-p_1|$, see the first row of their labelled posets. It is clear from the posets that these patterns are monotonic only if the quantified value of V_2 is larger than that of V_1, i.e. $p_1 \leq p_2$. This is because there are nodes in these posets with labels p_2 above nodes with p_1 (but no nodes in the reverse "ranking") and according to the monotonicity requirement (21), the numbers labelling the nodes of the poset must increase from bottom to top.[44] Since p_1 and p_2 are not just parameters in the labelled posets, but express the rate of "frontness" of forms belonging to the variation sites in the patterns, the requirement has a direct effect on the patterns, too. Namely, V_2 must be "more front" than $V_{1.}$ This result is identical to the one we discussed in the case of the ordering version of monotonicity (see the text at the beginning of this section). Therefore, we can state that in the quantified version of the multiple *V*-value model discussed here the two versions of monotonicity are equivalent, just as in the other two models (the invariable model and single *V*-value model).[45]

4.6 The Hungarian pattern in the quantified model

In Sections 4.3–4.5, we have used a five-context scale in our discussion of the properties of variable patterns. We have shown in Sections 4.1–4.2 that a seven-context scale is needed for a proper description of the Hungarian harmony pattern. At the end of Section 4.2 we summarised the Hungarian harmony pattern, which shows variation in anti-harmony and transparency, see (31b′:c′). We have identified the variation sites and have shown that the Hungarian pattern conforms to monotonicity, but we did not quantify the variation. In this section we will apply the quantified model we have developed to the Hungarian variable pattern. The Hungarian pattern, which also appears in (31b′:c′), is repeated in (52) below.

(52) The Hungarian variable pattern with seven contexts

contexts	[B]_	[BN]_	[BNN$^+$]	[N]_	[NN$^+$]_	[FN$^+$]_	[F]_
harmonic values	*B*	*B*	*B/F*	*B/F*	*F*	*F*	*F*

It can be seen from (52) above that both the Count Effect and the Polysyllabic Split (28) are in operation: the former manifests itself in the difference between the values in the second and the third columns (contexts [BN]_ vs. [BNN$^+$]_), and the latter in the difference between the values in the fourth and the fifth

44 This is also true of all the pairs of nodes in (51c, e) since if $p_1 \leq p_2$, then $1-p_2 \leq 1-p_1$ and $p_1 \leq 1-|p_2-p_1|$ and $1-p_2 \leq 1-|p_2-p_1|$.

45 Naturally, the equivalence is valid only if we assume the proper prototypical values requirement, since dual patterns (46 iv) are monotonic by similarity, but non-monotonic by ordering in this model, too.

columns (contexts [N]_ vs. [NN$^+$]_). Note that as in our discussion of the Hungarian pattern before, we only consider harmony in suffixes after *monomorphemic* stems as contexts because Harmonic Uniformity (cf. Section 4.1 (iv.)) would result in complex subpatterns if suffixed stems are also considered as harmonic contexts.[46]

Now consider the frequency data about harmony in the harmonic contexts relevant here. We have carried out a corpus search to discover the "frontness ratio" of variable sites, i.e. the relative token frequencies of forms suffixed by the front alternant in given types of contexts. We used the Hungarian webcorpus *Szószablya* (Halácsy et al. 2004; Szószablya 2014), which contains 541 million tokens of words. The table in (53) shows the results, where the frontness ratio (i.e. the number *p*) is set in large bold-face, and the number of all harmonically suffixed forms (in thousand tokens) appears in subscript. To make the search easier, we have only counted words with maximally bisyllabic roots as harmonic contexts and exactly monosyllabic suffixes. The one context where searching for longer roots was necessary is [BNN] (the third column in (53)), where we counted forms with exactly trisyllabic roots. Since longer monosyllabic roots are much rarer (in type and token frequency), this simplification does not change the results significantly. The neutral vowels in (53) are always only /i/ or /iː/. The suffixes in the forms counted are always inflectional, because inflectional forms can be obtained from the corpus we used with higher reliability. The table (53) is organised in the same way as table (52) and contains quantified information about the same harmonic patterns.

(53) The "frontness ratios" of harmonically suffixed forms by token frequency (N = i/iː, only inflectional monosyllabic harmonising suffixes)

stem context:	[B(B)]_	[BN]	[BNN]	[N]_	[NN]_	[FN]_	[F(F)]_
harmonic values as numbers	**0** $_{29,760}$	**0** $_{1,041}$	**0.08** $_{18}$	**0.70** $_{2,595}$	**1** $_{50}$	**1** $_{509}$	**1** $_{16,535}$

The numbers in (53) indicate the harmonic values in the Hungarian pattern, i.e. the percentage of forms with a front harmonic suffix compared to the total number of suffixed forms with a given harmonic context. If the number is 0 then all the forms have a *B*-suffix, if it is 1, then all the forms have an *F*-suffix. In variable contexts the higher the number is, the higher the relative number of front suffix forms. The numbers in variable sites suggest that that the Count

46 All of the resulting subpatterns are also monotonic, cf. Rebrus and Törkenczy (2015).

Effect only has a statistically small impact on the relative token frequencies as shown by the number 0.08 (see column 3), i.e. the cumulative ratio of front suffixed tokens of [BNN]-stems is only 8%, which means that there *is* variation, but there is a strong preference for back harmony.[47] By contrast, the Polysyllabic Split has a striking effect on relative token frequencies: compare 0.70 and 1 in the contexts [N] and [NN], respectively. This means that while 70% of the forms with an [N]-context have an *F*-suffix and 30% have a *B*-suffix, *all* the forms with an [NN]-context have *F*-suffixes (see 4th and 5th columns).

Crucially, despite the above effects and the great differences between the number of tokens in the various harmonic contexts,[48] the seven-place harmonic pattern in (53) is monotonic even according to the quantified model, the strictest of the models we have examined (both under the ordering and similarity interpretation). Thus, in these patterns (i) the occurrence of variation with respect to the invariable contexts (i.e. the position of the contexts with variable values) is appropriate in that it conforms to the requirement about the locus of variation (49), contiguity (11) and the entailments in (12)/(50); and (ii) the series of variable values is increasing in accordance with the ordering of contexts.

The quantified multiple *V*-value model requires that variable values in a monotonic pattern must conform to (ii) above. Such a requirement also makes a prediction about the quantified values (numbers) that can occur in variation sites, which makes it possible to make more fine-grained predictions than are possible in a non-quantified model. Recall that in a non-quantified model, monotonicity predicts that anti-harmony is only possible in a system that has transparency (cf. Section 3.2). A quantified model makes the same prediction, but also predicts that in a possible harmony system that has variable transparency and variable anti-harmony, *the effect of anti-harmony must be weaker than the effect of transparency*, i.e. the ratio of back suffix alternants in the context of [N]_ should be lower that in the context of [BN]_. In other words, in a well-formed five-member pattern $\langle BV_1V_2FF \rangle$ the p values that belong to V_1 and V_2 are such that $p_1 \leq p_2$. This is exactly what we see in the seven-member Hungarian pattern $\langle BBV_1V_2FFF \rangle$ (53), where the relevant variation sites are [BNN]_ (3rd position) and [N]_ (4th position). Here the degree of transparency is very high (i.e. the p-value is low, $p_1 = 0.08$) and the degree of anti-harmony is lower (i.e. the p value is higher, $p_2 = 0.70$). The first site is subject to the Count

47 Interestingly, this is in disagreement with descriptive statements about variation in Hungarian harmony in the literature which usually suggest that variation is typical in a [BNN] context (see Törkenczy 2011 for an overview).

48 Consider the subscript numbers in table (53) show: the greatest token number ([B(B)]-stems: about 30 million tokens) is 1600 times the smallest ([BNN]-stems: about 18 thousand tokens).

Effect (which disfavours transparency to a small degree), but nevertheless transparency is still stronger than anti-harmony ($p_1 = 0.08$, $p_2 = 0.70$), just as monotonicity requires in the quantified model. This model then predicts a correspondence between the Count Effect and the effect of anti-harmony: assuming a given degree of variation in anti-harmony, the Count Effect cannot be strong enough to suppress transparency to a degree (p_1) which is greater than the harmonic effect in anti-harmony (p_2).[49]

The pattern in (53) can be examined with respect to the similarity interpretation of monotonicity in the quantified multiple *V*-value model too. If we generate the quantified labelled similarity poset of the 7-member Hungarian pattern $\langle BBV_1V_2FFF \rangle$ of (53) as shown in (54) below, it is clear that the pattern is also *monotonic according to the similarity* interpretation since the similarity values calculated according to (49) increase towards the top. The first row of figure (54) shows the *p* values of the pattern from (53), the second row gives the seven harmonic contexts in linear order, and the labelled similarity poset containing all the similarity values assigned to all the pairs of contexts appears below.

(54) Quantified labelled similarity poset of the 7-member pattern $\langle BBV_1V_2FFF \rangle$ of (53)

0	0	0.08	0.70	1	1	1	**harmonic values** (*p*-values)
[B]	[BN]	[BNN]	[N]	[NN]	[FN]	[F]	**harmonic contexts** (in order)
							similarity poset:
1	**0.92**	0.38	**0.70**	**1**	**1**		similarity of adjacent positions
0.92	0.30	0.08	**0.70**	**1**			similarity of 2nd neighbours
0.30	0	0.08	**0.70**				similarity of 3rd neighbours
0	0	0.08					similarity of 4th neighbours
0	0						similarity of 5th neighbours
0							similarity of first and last

Interestingly, quantification, (e.g. as in the poset in [54]) can also be interpreted as expressing another property, too. As in all the labelled posets we have used, the

49 There are approaches to the Count Effect in Hungarian which propose to explain the variation in context [BNN]_ as the coexistence of two invariable patterns one of which has a Count Effect so strong that it results in total opacity after more than one N (e.g. Nevins 2010). Our analysis predicts that, assuming the existence of (some degree of) anti-harmony in the same pattern, such a pattern cannot be a well-formed (monotonic) subpattern. In such a pattern $p = 1$ would be the value in the context [BNN]_ and, assuming that there is variation in the context [N]_ (i.e. $p<1$), this would violate monotonicity.

labels (i.e. the quantified similarity values) express the similarity between harmonic suffix behaviour in the harmonic positions compared. The closer the number is to 1, the more similar the compared behaviours are (measured in relative token frequency). It can be seen in (54) that the distribution of similarity values is *uneven* in the pattern whose poset (54) is: values greater than 0.5 occur in the "neighbourhoods" of the two prototypical contexts [B] and [F], i.e. in the upper left corner and the upper right corner of the poset (these are highlighted in (54)). Harmonic behaviour in the contexts in these neighbourhoods is highly similar to the harmonic behaviour in the prototypical contexts. The border between the two neighbourhoods is blurry: it is somewhere between the contexts [BNN] and [NN], either between [BNN] and [N] or between [N] and [NN]. The similarity numbers can be used to measure the harmonic affiliation of the all-neutral contexts [N] and [NN]. It is clear that statistically they belong to the neighbourhood (i.e. follow the harmonic behaviour) of the front prototypes: the similarity of the value assigned to [N] and value assigned to [BNN] (i.e. $\mathrm{sim}'(f([\mathrm{BNN}]),f([\mathrm{N}])) = 0.38$) is lower than the similarity of the value assigned to [N] and the value assigned to [NN] (i.e. $\mathrm{sim}'(f([\mathrm{N}]),f([\mathrm{NN}])) = 0.70$), see the first row in the poset. The same relation holds between the similarity-values assigned to [N] and [BN] and the similarity values assigned to [N] and [FN], too: they are 0.30 and 0.70, respectively (see the second row in the poset). This relationship also obtains if we examine the second neighbours of [N]: $\mathrm{sim}'(f([\mathrm{B}]),f([\mathrm{N}])) = 0.30$ and $\mathrm{sim}'(f([\mathrm{N}]),f([\mathrm{F}])) = 0.70$ (see the third row in the poset). This means that [N] is a member of the neighbourhood of [F] more than it is the member of the neighbourhood of [B]. This asymmetry is the most striking in the case of the context [NN]: its "similarity" to [BN] or to [B] is 0, while its "similarity" to [FN] or [F] is 1.

The fuzzy border between the two neighbourhoods is graphically presented in (55) below, where all of the similarities of (54) are arranged in tabular format (the maximal similarities between identical positions always equal 1; this redundant information is left out from the table). The shading shows the two neighbourhoods and the strength of the affiliation of their members: the greater the similarities are, the darker the cells are. The darkness of the cells highlights the fact that the context BNN is a member of the B-neighbourhood to a greater extent than NN is a member of the F-neighbourhood (their types are in cells with different shades of gray). The context N is in the middle: it is in the neighbourhood of F more (mid-grey) than of B (light-gray). Such behaviour of the all-neutral contexts in Hungarian can be related to the fact that *neutral vowels are phonetically front*. The question arises how universal this behaviour is, i.e. whether the propensity of phonetically front neutral vowels to behave like front vowels harmonically which we have found in Hungarian is manifested cross-linguistically in the attested front/back harmony patterns. In other words, the question is if there are significantly more patterns with this kind of asymmetry than the reverse.[50]

(55) The two prototypical harmonic contexts and their neighbourhoods in Hungarian

	B	BN	BNN	N	NN	FN	F
B	-	**1**	**0.92**	0.30	0	0	0
BN	**1**	-	**0.92**	0.30	0	0	0
BNN	**0.92**	**0.92**	-	0.38	0.08	0.08	0.08
N	0.30	0.30	0.38	-	**0.70**	**0.70**	**0.70**
NN	0	0	0.08	**0.70**	-	**1**	**1**
FN	0	0	0.08	**0.70**	**1**	-	**1**
F	0	0	0.08	**0.70**	**1**	**1**	-

Recall that a similarity poset contains limited information about the similarities of harmonic positions, since some harmonic positions are uncomparable, see the discussion in Section 3.4. It is important to point out that labelling a similarity poset of contexts makes similarities between *all* the harmonic contexts comparable. We have seen that in a sequence of harmonic contexts $X<Y<Z$, the similarities of Y with the two other contexts (sim(X,Y) and sim(Y,Z), respectively) are theoretically uncomparable. In a given pattern, however, when the similarities are calculated from the measured relative token frequencies, the resulting labels in the poset provide information about how similar any context is to any other context. Language specifically then the labels categorise the harmonic contexts according to how similar they are to the prototypical contexts and one another. The statistical information about the harmonic behaviour of *suffix types* in different harmonic contexts makes it possible to determine the similarities between all the harmonic positions the (*stem types*) in the universally fixed scale of harmonic contexts.

The main characteristics of patterns of similarity discussed above resemble what is usually thought about the properties of connection *by analogy* between

50 Based on the data in the harmony literature it seems that front/back harmony patterns of the latter type, where a phonetically front neutral vowel tends to behave as harmonically back, i.e. invariably anti-harmonically (cf. (9c) e.g. Uyghur and (9d) e.g. Eastern Vepsian), are rarer or less stable.

units of language. Linguistic analogy is considered to have the following general properties (see Blevins and Blevins (2009b), Bybee (2010) among others): (i) analogy holds between *all* units of language; though its strength can be extremely different, it basically depends on both (ii) the *similarity* between units of language, and (iii) the *frequency* of units participating in the analogical connection. Our quantified model utilises these properties also: (ii) the labels of the posets are the quantified similarities; (i) these similarities hold between values assigned to *all* the contexts of the pattern; and (iii) the calculated similarities in the poset crucially depend on the token frequencies of front and back suffixed forms. Therefore, it is reasonable to consider the quantified labelled poset of a specific harmonic pattern of a language as a representation of the analogical connections between the harmonic contexts (types of units). Thus, we can think of these similarity values as expressing the strength of the analogical connections between the positions compared within the specific harmony pattern whose poset we are examining.

5 Conclusion

In this paper we have examined the typology of suffix harmony, focussing on weak disharmony, in front/back vowel harmony systems and argued that the possible surface patterns of well-formed systems conform to a general principle of monotonicity. After a detailed examination of variation in transparency and anti-harmony in Hungarian, we have shown that this monotonicity requirement holds for a harmony system with variation and hypothesised that this is true for variable harmony patterns cross-linguistically.

We have given formal definitions of monotonicity (applicable to variable and invariable harmonic patterns): one based on the similarity interpretation of monotonicity and one based on the ordering interpretation. We have also proposed a requirement (related to monotonicity) about the proper prototypical values in a harmony pattern. Thus we have different types of monotonicity requirements. We have studied the predictions the combinations of these requirements impose on harmonic patterning in three models: one in which there is no variation (i.e. only invariable harmonic values occur), one where there is variation, but there is only one variable value (the single-*V* model) and one in which different variable values are permitted (the multiple-*V* model).

If we examine models that incorporate different types of monotonicity requirements we find that they have different "discriminatory power". The strongest one combines either interpretations of monotonicity (ordering or similarity) and the proper prototypical values requirement (56i), the weakest one is based on the looser (similarity) interpretation only (56iii), and the intermediate one incorporates the strict (ordering) interpretation only (56ii). We illustrate this for a five-context scale by showing the discriminatory power of the three types of monotonicity requirements in two models (expressed in the number and the percentage of well-formed patterns by each type compared to all the possible patterns). The discriminatory powers are quite high since the percentage of well-formed patterns are low, and obviously, the stricter the type of monotonicity is, the lower the percentage of well-formed patterns is according to the model in question.[51]

(56) Types of monotonicity and their percentage for a 5-member pattern

types of monotonicity	no *V*-value model		single *V*-value model	
	invariable patterns		variable & invariable patterns	
i. well-formed = monotone with proper prototypes	4	12.5 %	**10**	**4.1 %**
ii. monotonically increasing = monotone by ordering	6	18.8 %	21	8.6 %
iii. monotonically increasing or decreasing = monotone by similarity	10	31.3 %	41	16.9 %
all monotonic and non-monotonic patterns	32	100 %	243	100 %

While models based on weaker types (56ii, iii) of monotonicity may be suitable for other (morpho)phonological patterns (e.g. the phonology of paradigms), we claim that the strictest type (56i) is required for the typology of front/back harmony systems and very interesting predictions (e.g. about the co-occurrence of transparency and anti-harmony) follow from a model based on this type of monotonicity.

We have developed a quantified version of the multiple-*V* model, which makes even more fine-grained predictions about variation than a single-*V* model. All these predictions are empirically testable and to be tested, but since we know relatively much less about variation in harmony systems language-specifically and cross-linguistically than we do about (what

51 The percentages are even lower (and the discriminatory power is higher) if we look at patterns that have more than five members, e.g. only 0.96% of 7-member variable patterns (a pattern mentioned in the previous section) would be judged well-formed by a model based on the strictest type (56i) and this percentage would be as low as 0.08% if the pattern has 10 members.

we assume to be) categorical harmonic behaviour, this task is for future research.

Finally, the models we have examined are formally flexible in that they can be modified to accommodate other aspects of harmonic patterning if the need arises. For example, we have assumed that the similarities between some harmonic contexts on the frontness/backness scale are universally uncomparable. It is formally possible to modify the properties of this universally fixed scale. We could make the formal properties of the scale less strict by assuming that it is not linearly ordered. As a result the similarity poset will contain fewer comparable nodes than before. Alternatively, we could make more of the similarities (nodes) comparable, which would impose a stricter version on the well-formed patterns.

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