

# CONTROL STRATEGIES OF BRAKE DYNAMICS FOR RAILWAY VEHICLES EQUIPPED WITH ANTI-SLIP-DEVICE

István ZOBORY and Elemér BÉKEFI

Budapest University of Technology and Economics  
Faculty of Transportation Engineering  
Department of Railway Vehicles  
H-1521 Budapest, Hungary

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## ABSTRACT

From the point of view of railway vehicle braking processes it is an important requirement to maintain the rolling-motion-state of the wheels, i.e. the macro-sliding-free rolling contact between the vehicle wheels and the rail-head, since in the course of rolling in the presence of creep-dependent contact forces, the braking force transmittable on the vehicle through the rail/wheel contact patch is considerably greater, than the braking force transmittable on the vehicle in the case of wheel blocking and on the railhead continuously sliding non-rotating wheels. In this study some control strategies will be introduced that have been elaborated for developing anti-slip-device systems of railway vehicles, and that meet the requirement of treating the always present uncertainties in the force transfer conditions through the wheel/rail rolling contact by a stochastic model. With such models it is possible to develop anti-slip-devices having advantageous braking effectiveness in the operation also in case of uncertain tribological characteristics along the track.

## 1. INTRODUCTION

In the course of braking processes of railway vehicles it is very important to maintain the macroscopic sliding-free continuous rolling motion of the wheels of the vehicle, because the maximum force that can be transmitted in the presence of creep-dependent rolling contact conditions is considerably greater than the sliding friction force transmitted between the wheel and the rail in case of rotation-free, blocked wheel motion. On the one hand avoiding wheel sliding is very important for ensuring a more intensive brake-effect – shorter stopping distance -, but in this way also the phenomenon of wheel-flattening can be avoided, the latter phenomenon can cause very annoying service problems and leads to additional costs, as well. It is true that different anti-slip devices are applied in railway vehicles for a long time giving certain solutions to the problems described above, still, with the knowledge of the new scientific results achieved in contact mechanics of rolling contacts, the in the field of stochastic modelling of the wheel/rail force-connection coefficient, as well as the results of the modern control theory a considerable development can be made in the field of anti-slip devices. In the present paper starting from the system model of the anti-slip-device, more newly developed control strategies will be introduced, together with their mathematical description. The new control strategies can be advantageously applied when developing and designing new anti-slip-devices with increased effectiveness.

## 2. SYSTEM MODEL OF THE ANTI-SLIP-DEVICE

It is enough to introduce the flow-chart representing the operation conditions of the anti-skid device of a railway vehicle equipped with disc-brake only for the case of a single wheel-set. The two counteracting moments determining the rotatory motion of the braked wheel-set are on the one hand the friction moment exerted on the brake disc of the wheel-set by the resultant of the brake-disc/brake-pad sliding friction forces acting at a radius  $r$ , and the moment exerted on radius  $R$  by the tangential creep-force arising in the wheel/rail contact forced by the above mentioned friction moment, on

the other. The situation can be seen in Fig. 1, where also the evolution of the forces and moments influencing the translatory- and rotatory motion of the wheel-set can be followed with attention.

It can be read of the flow-chart, that for the definite appearance of macroscopic sliding in the course of braking it is necessary, that friction moment  $M_f$  exerted by the brake-discs on the wheel-set considerable exceed the moment  $M_r$  of the creep-forces depending with a saturation character on the creepage realising in the wheel/rail contact.

In such a situation, the absolute value of the momentary creepage realising in the wheel/rail contact is considerably greater than the absolute value of the creepage belonging to the extremum value of the force-connection coefficient function, so for increasing absolute values of the momentary creepages belong decreasing absolute values of the force-connection coefficient.

If there occurs no sufficiently fast decreasing intervention into the value of the friction moment  $M_f$  exerted by the brake-gear, then unavoidably evolves a macroscopic sliding in the wheel/rail contact at the creepage value  $-1$  belonging to it, thus the blocking caused eventually wheel flattening cannot be prevented.

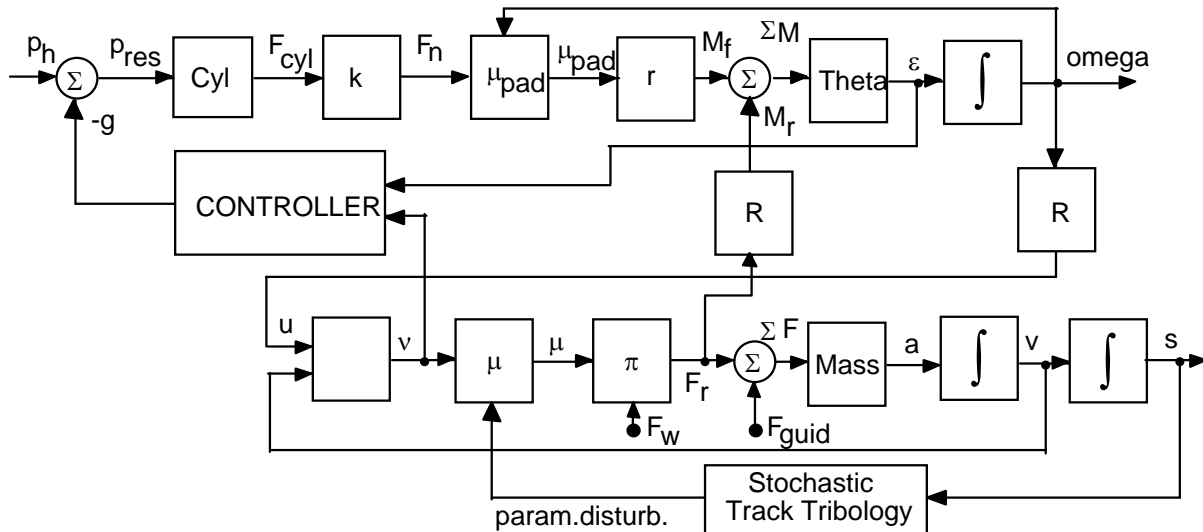


Fig.1 Flow chart of the control of a wheel-set equipped with anti-slip-device

In the flow chart plotted in Fig. 1 the block “stochastic track tribology” mapping the uncertainty of the tribological characteristics of the wheel/rail connection is indicated. The role of block in question in the flow-chart can be identified as a stochastic feed-back in the track-vehicle system, that makes the process-structure stochastic in the whole brake-system.

The increased wheel-tread-sliding and from the latter eventually evolving damaging and from more points of view becoming dangerous process of full blocking is prevented by the negative feed-back realised by the block “controller”, that is well identified in the flow-chart.

As it will be seen in the following from the control strategies to be treated, that the basic principle can be formulated by the requirement, that with the knowledge of the wheel-tread creepages and/or that of the angular deceleration of the wheel-set, a preventive measure can be taken for decreasing the friction moment  $M_f$ , viz. by the suffi-

ciently intensive and possibly fast decrease of the distributor-valve controlled brake cylinder pressure  $p_h(t)$  in the brake cylinders belonging to the wheel-sets that inclines to wheel slipping

Before beginning the detailed treatment of the control strategies, it is important to point out, that in accordance with the objective of the present study it was our effort to emphasise the most decisive effect from among the effects having role in causing wheel-sliding, thus in this study the excitation effect of the tribology-caused disturbances were analysed and built into the dynamical model of the braked vehicle, only.

Nevertheless, it is to be mentioned, that as far as the design or operation problem a real anti-slip-device concerned, there are at least two not negligible disturbing effects influencing the longitudinal creep-forces on the wheel-tread, that should be built in into the system-flow-chart. The two disturbing effects (excitation sources) are as follows:

1. The excitation effect caused by the unevennesses of the railway track [1], [2] lead to excited vertical, pitching and rolling vibrations, and due to the vibrations mentioned, time dependent fluctuations in the vertical wheel force  $F_n = F_n(t)$  should be reckoned with. The fluctuations in the vertical wheel force will be transferred into fluctuations of the longitudinal creep-forces, and these variations can be related with initiating further wheel-sliding processes.
2. The wheel-sliding conditions of a railway vehicle in the course of braking are influenced also by the longitudinal dynamical interactions between the adjacent vehicles in the train. In the background of the interactions mentioned there is an energy transfer between the adjacent vehicles through the draw- and buffer gears. Certain amount of kinetic energy of the adjacent vehicles should be dissipated in the brake system of the vehicle in the middle.

### 3. APPLICABLE CONTROL STRATEGIES FOR ANTI-SLIP-DEVICES

In the following, some control strategies will be introduced, that can practically be applied in the operation of anti-slip-devices. In the course of discussing the strategies, two essentially different variation-classes will be introduced. Control strategies belonging to the first variation-class realise a state feedback determined by a non-linear function of the deviations from the commanded value for decreasing the brake-cylinder pressure, and in this way to reduce the friction moment  $M_f$ .

Control strategies belonging to the second variation-class are model-based strategies. Here it is evaluated in every simulation step, if the value of the actual wheel-tread-creepage falls into that creepage interval, which is located just over the peak of the force connection coefficient, and therefore the event of wheel sliding can easily occur. The feedback intervention into the brake-cylinder pressure (i.e. brake-cylinder pressure reduction) takes place in every computation step coming after another, if the actual wheel-tread-creepage falls into the "critical creepage interval".

In the overwhelming majority of the control strategies to be introduced here the parameter vector  $\mathbf{q}$  characterising the feedback function will be regarded as a vector containing con-

stant components. However it did not avoid the attention of the authors that the commanded longitudinal creepage value and the limit value specified for the angular deceleration of the wheel taking role in parameter vector  $\mathbf{q}$  – as a value pair assigning the thresholds of starting with the feed-back intervention - can be defined as a function the parameter vector  $\mathbf{p}$  giving the tribological inhomogeneity of the wheel/rail force-connection coefficient by the two parameter vector-valued stochastic field  $\mathbf{p}(\nu, x, w)$ . Here in the parenthesis  $x$  stands for the locus of the track,  $\nu$  for the longitudinal creepage, and  $w$  for the random elementary event. The control process realising in the latter case becomes an adaptive process, matching to the stochastic variations in the tribological characteristics of the wheel/rail rolling contact along the railway track.

### 3.1 Control strategies based on the increase in the absolute value of wheel angular acceleration and of the creepage realising in the rolling contact

For the realisation of the control strategy introduced above more versions of bivariate feedback functions were elaborated. Each variation operates with non-linear functions, but their smoothness is different, therefore the “gradualness” of the evolution of the brake-cylinder pressure reducing feedback effect is essentially different in case of the variations in question.

#### 3.1.1 Piece-wise linear bivariate feedback function

The elaborated piece-wise linear feed-back function is defined in a close-form expression depending on the motion characteristics, namely on the longitudinal wheel-tread creepage and the angular deceleration by the following min/max formula:

$$g(\nu, \varepsilon, \mathbf{q}) = \min\{p_{h_{\max}}, \max\{\max\{0, \lambda_\nu(\nu - \nu_0)\}, \max\{0, \lambda_\varepsilon(\varepsilon - \varepsilon_0)\}\}\}.$$

As it can be read off, together with the two kinematical variables, namely with the actual longitudinal creepage and the actual deceleration of the wheel, also parameter vector  $\mathbf{q}$ , necessary for the actual numerical characterisation, is indicated. The coordinates of parameter vector  $\mathbf{q}$  are the always negative valued commanded creepage  $\nu_0$  and limit angular deceleration  $\varepsilon_0$ , furthermore the also negative valued slopes (direction tangents)  $\lambda_\nu$  and  $\lambda_\varepsilon$ , characterising the reduction-rate in brake-cylinder pressure, brought about by the feed-back effect.

The value of the maximum brake-cylinder pressure reduction can be selected in form  $p_{h_{\max}} = c \cdot 3.8$ , where  $0 \leq c \leq 1$  is an adjusting constant factor. So the pressure value is obtained in unit of measure bar. In Fig 2. the performance surface of the piece-wise linear bivariate feed-back function is plotted, which is built up - in an obvious way - from plane peaces, continuously connecting with each other.

#### 3.1.2 Quadratic bivariate feedback function joining smoothly to the zero plane

The quadratic feedback function connecting to the basic zero plane in a continuously differentiable way, a function that depend on the wheel motion characteristics, namely on the wheel-tread creepage and the angular deceleration is specified by the following close-form min/max formula, elaborated straight for this purpose:

$$g(\nu, \varepsilon, \mathbf{q}) = \min\{p_{h_{\max}}, \max\{\max\{0, \lambda_\nu(1 - U(\nu - \nu_0))(\nu - \nu_0)^2\}, \max\{0, (1 - U(\varepsilon - \varepsilon_0))\lambda_\varepsilon(\varepsilon - \varepsilon_0)^2\}\}\}.$$

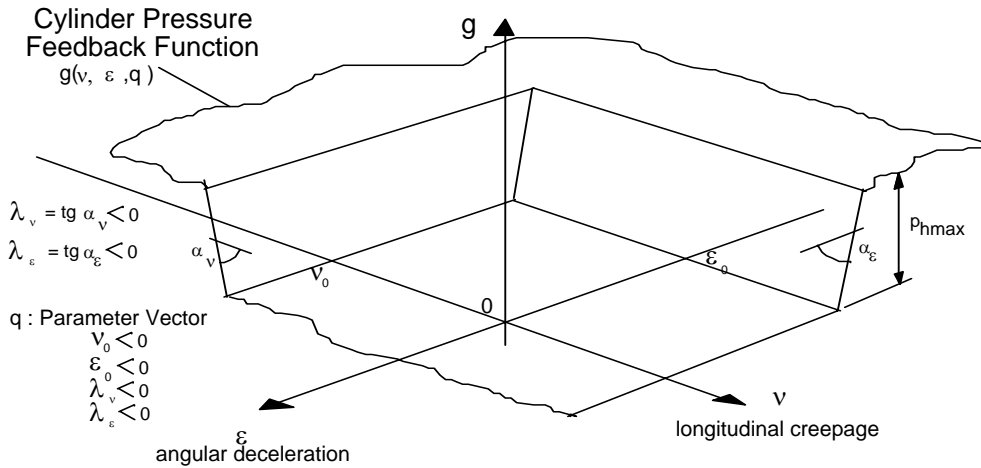


Fig.2 Performance surface of a piece-wise linear bivariate feedback function

In the above formula  $U(\cdot)$  stands for the unit-jump (Heaviside) function. As it can be read off, together with the two kinematical variables, namely with the actual longitudinal creepage and the actual deceleration of the wheel, also parameter vector  $\mathbf{q}$ , necessary for the actual numerical characterisation, is indicated.

The co-ordinates of this latter parameter vector are the always negative valued commanded creepage  $v_0$  and limit angular deceleration  $\varepsilon_0$  belonging to the “coming into action” threshold of the controller, furthermore the positive valued slope-adjusting constants  $\lambda_v$  and  $\lambda_\varepsilon$ , characterising the reduction-rate in brake-cylinder pressure, brought about by the feed-back effect. The value of the maximum brake-cylinder pressure reduction can again be selected in form  $p_{hmax} = c \cdot 3.8$ , where  $0 \leq c \leq 1$  is an adjusting constant factor. So the pressure value is obtained in unit of measure bar. In Fig 3. the performance surface of the piece-wise from parabolic cylinder surface sections and constant level plane sections built up continuous bivariate feed-back function is plotted.

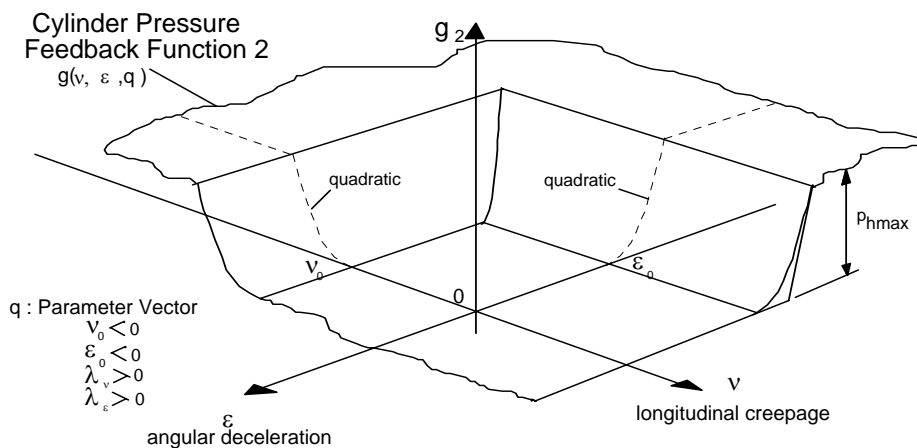


Fig. 3 Performance surface of the bivariate feed-back function containing quadratic parabola generated cylinder surfaces between the zero and maximum levels

### 3.1.3 Bivariate feedback function of square-root character

Over the two above described feedback functions the authors of the present paper elaborated further feedback functions of similar structures on the basis of that heuristic

recognition, that the faster starting of the anti-slip interfering effect – by ensuring a “first stage pressure decrease” – and after certain progress in brake-cylinder pressure reduction process the decrease in the rate of the further pressure reduction process gives rise to a control process having advantageously smoother qualitative performances, i.e. smaller overshoot and better stability behaviour.

More versions of feedback functions of “root-character” can come into question, since the selection of the root-exponent can mean a further action parameter for the optimisation. Though in the following the shape of the feedback function will be introduced for the “square-root” case, it is to be emphasised, that other root-exponent greater than 1 can also be imagined. Keeping in force the designations applied above, the following min/max formula was elaborated:

$$g(v, \varepsilon, \mathbf{q}) = \min\left\{p_{hmax}, \max\left\{\max\left\{0, \lambda_v(1-U(v-v_0))\sqrt{-(v-v_0)}\right\}, \max\left\{0, \lambda_\varepsilon(1-U(\varepsilon-\varepsilon_0))\sqrt{-(\varepsilon-\varepsilon_0)}\right\}\right\}\right\}.$$

As it can be read off, together with the two kinematical variables, namely with the actual longitudinal creepage and the actual deceleration of the wheel, also parameter vector  $\mathbf{q}$ , necessary for the actual numerical characterisation, is indicated. The co-ordinates of parameter vector  $\mathbf{q}$  are the always-negative valued commanded creepage  $v_0$  and limit angular deceleration  $\varepsilon_0$ , belonging to the “coming in action” threshold of the controller, furthermore the positive valued slope-adjusting numerical values  $\lambda_v$  and  $\lambda_\varepsilon$ , determining the intensity of the pressure decrease brought about by the feedback applied.

Among the co-ordinates of vector  $\mathbf{q}$  can enter also the eventually applied non-quadratic root-exponent. In Fig. 4 the performance surface of the feedback function consisting of parabolic cylinder surfaces generated by square-root constituent carrier curves and limiting horizontal plane-sections is shown.

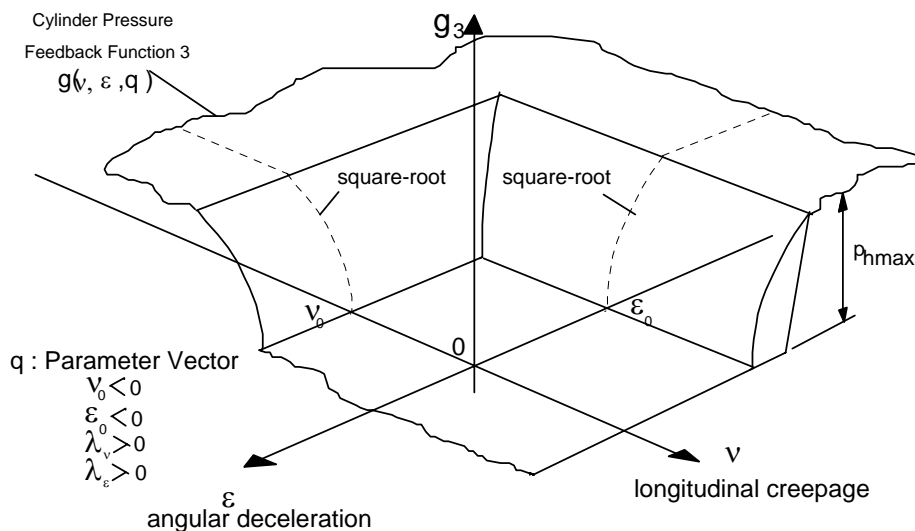


Fig. 4. Performance surface of the bivariate feedback function containing square-root-parabola generated cylinder surfaces between the zero and maximum levels

### 3.1.4 Bivariate feedback function of exponential character

On the basis of the train of thoughts applied also in case of the previous root-function profiles, an exponential transition profile based version of the bivariate feedback func-

tions has been elaborated. Here, the transition between the lower and the upper constant levels was solved by using cylindrical surface sections of exponential carrier curves. Preserving the designations applied before, and introducing the slope adjusting constants  $T_v$  and  $T_\varepsilon$ , the following min/max formula was developed:

$$b(v, \varepsilon, \mathbf{q}) = \min\{p_{h\max}, \max\{\max\{0, \lambda_v(1-U(v-v_0))(1-\exp((v-v_0)/T_v))\}, \max\{0, \lambda_\varepsilon(1-U(\varepsilon-\varepsilon_0))(1-\exp((\varepsilon-\varepsilon_0)/T_\varepsilon))\}\}\}$$

As it can be read off, together with the two kinematical variables, namely with the actual longitudinal creepage and the actual deceleration of the wheel, also parameter vector  $\mathbf{q}$ , necessary for the actual numerical characterisation, is indicated. The coordinates of parameter vector  $\mathbf{q}$  are the always negative valued commanded creepage  $v_0$  and limit angular deceleration  $\varepsilon_0$ , assigning the “coming into action” threshold of the controller, furthermore the positive valued slope-adjusting numerical values  $\lambda_v$  and  $\lambda_\varepsilon$ , as well as  $T_v$  and  $T_\varepsilon$ , determining the intensity of the pressure decrease brought about by the exponential section of the feedback applied.

In Fig. 5 the performance surface of the feedback function consisting of cylinder surfaces generated by exponential constituent carrier curves and limiting horizontal plane-sections is shown.

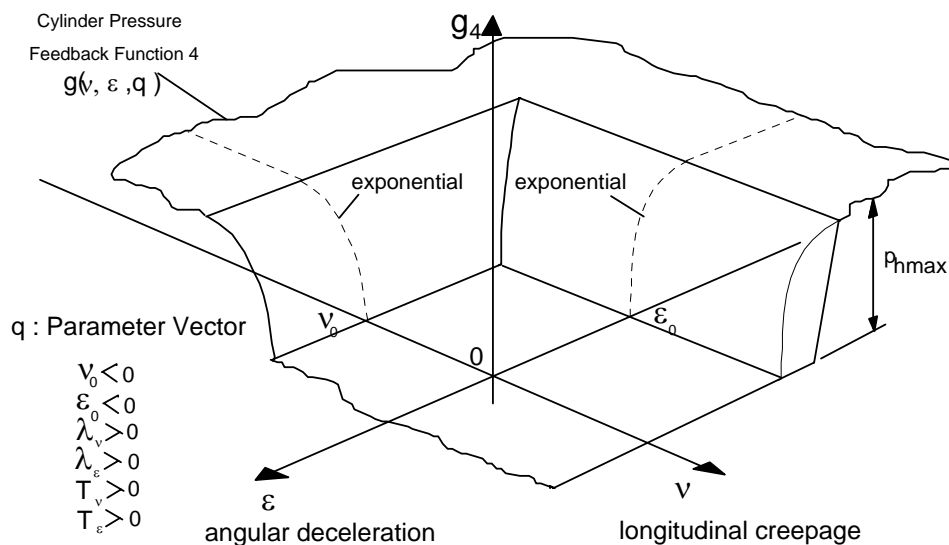


Fig.5 Performance surface of the bivariate feedback function containing exponential function generated cylinder surfaces between the zero and maximum levels

### 3.2 “Two-tact” time-dependent feedback

As it has been mentioned, from among the control strategies proper for realising the anti-slip-measures, the following version to be discussed is a model-based strategy, which can be characterised by its feature, that the in the dynamical simulation followed process of the wheel rotation, the actual value of the wheel-tread creepage is checked in each simulation step, if the it falls into the interval of creepages critical from the point of view of evolving wheel sliding.

Then the feedback, as a reducing interference into the brake-cylinder pressure is realized in each consecutive simulation step, in which step the actual wheel-tread creepage falls into the assigned “critical” interval.

For the sake of a nearer explanation, consider Fig. 6, in which the well-known force-connection coefficient vs. longitudinal creepage function is plotted, and the creepage interval  $[v_2, v_1]$  situated on the negative half-axis of the creepage axis is also indicated, just over the negative extremum of the force connection coefficient curve. If the actual creepage value enters into the so assigned “critical “ interval, or for some time steps it remains in the interval in question, then in the wheel/rail connection the occurrence of the event of wheel sliding is more than probable, thus, the interference of the anti-slip control is to be started.

Let  $\{t_i\}_{i=1}^N$  be the equidistant time-point sequence of the dynamical simulation output, thus, for the time-points of which all the essential mechanical characteristics of the system are computed in a  $t_i$ -dependent output vector. So, sequence  $\{v(t_i)\}_{i=1}^N$  of the wheel-tread creepages is also known for the whole simulation procedure. In this situation there is no obstacle for checking the occurrence of the three disjoint events

$$v(t_i) < v_2 \quad , \quad v_2 \leq v(t_i) \leq v_1 \quad \text{and} \quad v(t_i) > v_1$$

in each simulation step.

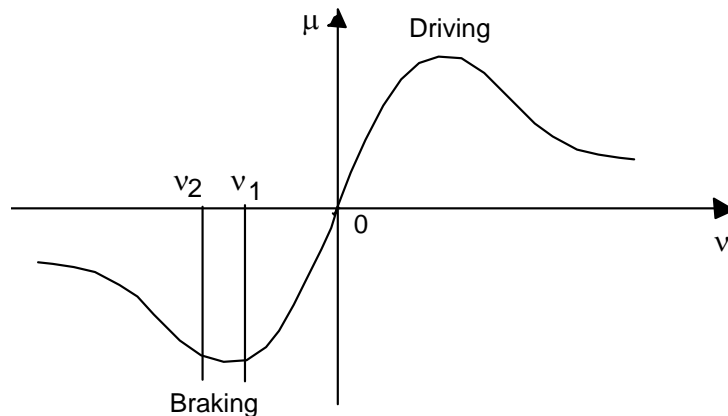


Fig.6. Assigning of the limit values of the creep-interval necessary for defining the intervention events of the „two-tact” time dependent feedback

Let  $\Delta p > 0$  be now a very small pressure-change, which the brake cylinder pressure reducing intervention of the anti-slip purpose control is based on. The elaborated control algorithm operates in each step of the dynamical model simulation, namely it is checked, that from among the above defined three events - associated with the actual value of the momentary creepage – which occurs. Then the intervention into the brake cylinder pressure then is realised as follows:

1. If  $v(t_i) > v_1$  , then the actual value of the brake cylinder pressure  $p_h(t_i)$  is not changed in the next computation step,
2. If  $v_2 \leq v(t_i) \leq v_1$  , then in next computation step the brake-cylinder pressure is reduced:  $p_h(t_{i+1}) = p_h(t_i) - c_1 \Delta p$  ,
3. If  $v(t_i) < v_2$ , then in next computation step the brake-cylinder pressure is reduced:  $p_h(t_{i+1}) = p_h(t_i) - c_2 \Delta p$  .



When applying the above “two-tact” time-dependent feedback strategy, due to our experience it is advisable to select the value of  $c_2$  about 5...7 times greater than the value of  $c_1$ .

On the basis of the above introduced control strategy, there can be elaborated also a “three-tact” time-dependent feedback strategy, the essence of which is, that an attempt is taken in case of point 1., when event  $v(t_i) > v_1$  occurs, to achieve a better utilisation of the adhesion limit, namely in the next time point  $t_{i+1}$  the following cylinder pressure value is to be set in:

$$p_h(t_{i+1}) = \min\{p_{h_{\max}}, p_h(t_i) + c_3 \Delta p\},$$

i.e. in successful case the brake-cylinder pressure can mildly be increased, so the brake tends to operate in a more effective way, without wheel-sliding. Due to numerical experiences the selection  $c_1 = c_3$  leads to a balanced operation process.

#### 4. EMBEDDING OF THE CONTROL STRATEGIES INTO THE DYNAMICAL PROCESSES OF THE BRAKED RAILWAY VEHICLE

Since the control strategies introduced in the previous chapter – by putting the inner relationships of the block “CONTROLLER” in the flow-chart of Fig. 1 concretely - determine the evolution of the rotatory and translatory motions of the wheel-sets of the braked railway vehicle, their influence obviously spread forth influencing the motion conditions of the further masses (the bogies and the car-body) of the complete vehicle.

However, concerning Fig. 1 it has already been emphasised, that the longitudinal resultant forces acting upon the wheel-sets are influenced also by the elastic-dissipative forces  $F_{\text{guid}}$  arising in the connection elements between the axle-boxes and the bogie-frames.

It is very important to recognise that the system flow-chart in Fig. 1 should be built in into the complete brake-system flow-chart of the four axle railway vehicle in four identical exemplars. To the brake-system of every wheel-set belongs its own “CONTROLLER” block, the operation of which basically depends on the tribological disturbances acting on the wheel/rail contact of the wheel-set in question, nevertheless in this respect also the elastic-dissipative axle-box-guidance connection force  $F_{\text{guid}}$  determined by the motion state of the bogie takes an un-negligible role.

The distributor valve of the braked vehicle provides the brake cylinders of all the four wheel-sets with a common brake-cylinder pressure  $p_h(t)$ , but the anti-slip-devices in accordance with the control strategies introduced, will reduce this nominally common brake-cylinder pressure if necessary, depending on the actual tribological conditions prevailing in the wheel/rail contact belonging to the considered individual wheel-set, in order to avoid the wheel sliding.

Since the tribological conditions of the wheel/rail rolling contact associated with the longitudinal force transfer is characterised by the vector-valued stochastic process  $\mathbf{p}(v, x, w)$  depending on the parameter-pair longitudinal creepage  $v$  and the track length  $x$ , it can be easily seen that the stochasticity reflecting the uncertain tribological properties of the track will be superimposed on all inner dynamical and control characteristics of the

vehicle equipped with an anti-slip-device, thus the braking performances (e.g the stopping distance of paramount importance) become stochastic, i.e. the performances mentioned will take unpredictable, more or less uncertain outcome values.

Nevertheless, it is important to emphasise, that still under the uncertainties caused by the operation of the anti-slip-device, more favourable stopping distances can be kept with a vehicle equipped with anti-slip-device, than without that device, because in the latter case the stopping distances intolerably increase due to the strongly decreased sliding friction coefficient values, and what is more inconvenient, the eventual appearance of wheel-flattenings can lead to further significant increase in the operation costs.

## 5. CONCLUDING REMARKS

Each elaborated control strategy has been tested numerically, using the data of a standard four-axle passenger carriage equipped with disc brakes. Though every introduced feedback strategy satisfied the required of anti-slip-action in case of stop braking in the presence of a 4.7 m long oil-spot on the railheads, however the best results were received with the application of the *square-root parabola generated feed-back function*, that realised a fast, “first-stage pressure reduction” in the brake-cylinder pressures, when the creepage or the angular deceleration exceeded the “coming into action” threshold limits specified. The numerical results of the numerical testing will be published in a following study.

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