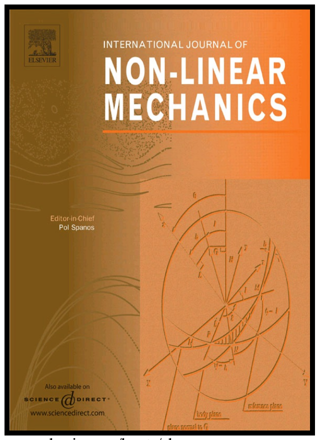

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On the impact of a rigid–plastic missile into rigid or elastic target

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Abstract

Here we carry out a systematic parametric study of a uniform cylindrical missile impacting rigid or elastic structures. We give an analytical result for the impact force in case of rigid target. A new parameter, the damage potential is introduced and it is shown that this single dimensionless combination of the parameters describes the course of the impact in this simplest case. For elastic target structures, we also show numerically that the course of the reaction force, the maximum target displacement and the duration of the impact depend primarily on the same dimensionless parameter with a secondary effect of the missile to target mass ratio and the relative stiffness of the target. The rigid target assumption is not always conservative with regard to the reaction force due to target vibration. We find a resonant effect in the maximum target displacement as the function of the missile to target mass ratio. The motivation of our work is rooted in the investigation of aircraft fuselage impact into robust structures like the containment of a

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nuclear power plant.

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1. Introduction

Analysing the consequences of potential aircraft impact into engineering structures has been an issue of high importance since September 11, 2001. The ideal situation would be to carry out substantial experimental studies, but the possibilities are limited in this direction due to the excessive expenses. We are aware of only one full-scale experiment [1, 2, 3], where a Phantom F4 fighter was impacted into a massive concrete target. This experiment is the basis for many subsequent theoretical and numerical studies in this field. Due to scarcity of experiments of this scale, it is important to obtain theoretical [4, 5, 6, 7, 8, 9, 10] and numerical (see e.g. [11, 12, 13, 14]) results regarding the safety of important structures, like nuclear power plants, during aircraft collisions.

Damage caused by impact can be either *local* or *global* [15]. Usually, local damage, like penetration, cracking, spalling, scabbing or perforation [16, 17, 18, 19] is caused by the impact of a hard missile into a relatively soft target. Global effects are related to the overall structural response of the target. In this paper we concentrate on global effects, like the influence of the impact of the aircraft fuselage into a relatively rigid structure, like the containment of a nuclear power plant. To investigate such soft impacts, one can follow the theoretical results obtained by Riera [5] that provide the instantaneous reaction force during the impact based on the assumption of a perfectly rigid target and a rigid–plastic aircraft fuselage as the missile. Another approach is a complex, detailed, coupled target–missile model, usually a finite element simulation, capable to include realistic parameters and provide detailed information on the course of the impact. However, it is difficult to delineate in these complex models the set of parameters with real influence on the outcome of the impact. It is notable to observe that even very detailed simulations [12, 13, 20, 21] use the Riera approach as a benchmark to validate the results. Hence, it is very important to have reliable theoretical results to provide solutions to aid the validation of numerical investigations.

Except for a few examples [4, 20], where simple geometry and material properties are used, theoretical approaches typically use realistic missile profiles to derive numerically the reaction force acting on the target [5, 7, 11, 12, 13, 21, 22, 23]. They either include the available aicraft data, like mass distribution and crushing force distribution of the aircraft to compute the force acting on the target [5, 7, 11, 22, 23], or use a full-scale finite element analysis [12, 13, 21]. While this is practically important and motivated, these missile models can be described by a multitude of parameters, like the mass and crushing force distribution along the length of the missile. As a consequence, in many cases the effect of an individual parameter on the reaction force or on the structural response is not clear. Some papers even question whether the assumptions of the Riera model result in a conservative estimate concerning the safety of target structures like nuclear power plants [11, 13]. Hence, in this paper, we make a step back and investigate the simplest case of a uniform missile impacting either a rigid or a one-degree-of-freedom elastic target. This way we can shed light on the relative importance of the various parameters and look for a range of parameters where the assumption of a rigid target, in the spirit of the Riera model, may lead to an underestimation of the reaction force during impact.

Following this approach, we write the governing differential equations into a dimensionless form to acquire information on the relevant combinations of the parameters that describe either the missile or the target. We find that among the obtained dimensionless parameter combinations there is one seemingly more important than the others, which we will call the *damage potential*. Beside the impact velocity, this parameter includes the mass and length of the missile, and its characteristic crushing strength. For the simplest case, when a uniform missile impacts a rigid target, we find analytical solution for the governing differential equations and we find that only the dimensionless damage potential appears in the solution. For the case of a uniform missile impacting an elastic structure [24], using numerical results, we show that the same parameter is enough to characterize the essential behavior during the impact. We find that the course of the reaction force, the maximum target displacement and the duration of the impact all depend mainly on the damage potential.

We also find a resonant effect in the maximum displacement of the elastic target as a function of the ratio of the mass of the missile to that of the target. At a certain value of this ratio the displacement of the structure is found to be the highest. For a simple case we give an estimate for the resonant mass ratio. We also show that the maximum reaction force can be higher than that for rigid target, hence the rigid target based Riera approach may not always lead to conservative estimation of the highest reaction force.

Next, in Sec. 2 we review the Riera model [5] for elastic target [24] and cast the equations into a dimensionless form to find the relevant combination of the parameters. For a uniform missile impacting a rigid target, in Sec. 3 we derive an analytical formula for the reaction force as a function of time, and show that this only depends on the damage potential. In Sec. 4 we present numerical results for the case of an elastic target, and show that the details of the impact can be characterized by the same dimensionless combination of the parameters, by the damage potential. Finally, in Sec. 5 we draw our conclusions.

2. Riera model with elastic target

2.1. Governing equations

A commonly used analytic model to determine the impact force acting on a sufficiently rigid structure has been developed by Riera [5]. In this model the missile, impacting the target in normal direction, is assumed to be a deformable rod of rigid–perfectly plastic material, and the structure is assumed to be perfectly rigid. It is also assumed that the missile crushes only at the cross-section adjacent to the target. Therefore, the missile consists of two parts: an uncrushed part of length $x(t)$ and of mass $m(t)$ time t after the start of the impact, and an infinitesimally small part of mass $(-dm) > 0$ that crushes in the next time instant, see Fig. 1a. Note that $dm < 0$ means there is a loss of mass concerning the missile during a short time dt. The instantaneous velocity of the intact part is $v(t) = dz/dt$ with $z(t)$ as the displacement of the intact part since the start of the impact. The impact force to be determined is $F(t)$, while the force acting between the intact and the crushing parts is the crushing force $P(x)$ which depends on the actual intact length $x(t)$ of the missile. In principle, $P(x)$ depends on the load bearing capacity of the cross-section at a distance x measured from the rear of the missile and also on the possible dynamic buckling that occurs during the impact.

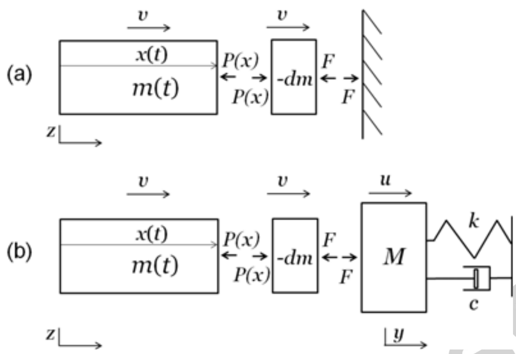

Figure 1: (a) Original and (b) elastic Riera model.

This model has been extended by Wolf *et al.* [24] to include a one degree of freedom damped, elastic system modeling the flexibility of the target, see Fig. 1b. The mass of the target is M, the spring constant is k, the damping is c. The displacement and velocity of the target are $y(t)$ and $u(t) = dy/dt$, respectively. The main goal of this paper is to evaluate the parameter dependence of this model to see whether the elasticity of the target plays an important role.

First, we briefly recall the governing equations of this model. Since $z+x = L+y$, L being the original length of the missile, see Fig. 1, we find the velocity dx/dt of the crushing as the velocity difference between the target and the uncrushed part of the missile:

$$\frac{dx}{dt} = u - v = \frac{dy}{dt} - \frac{dz}{dt}. \tag{1}$$

Introducing $\mu(x)$ as the mass per unit length at x, we find

$$\frac{dm}{dt} = \mu(x)\frac{dx}{dt} = \mu(x)\left(\frac{dy}{dt} - \frac{dz}{dt}\right). \tag{2}$$

At time t, crushing force $P(x)$ acts on the intact part of the missile and breaks mass $(-dm) > 0$ off the missile, cf. Fig. 1. The balance of momentum right before and after the break off of $(-dm)$ is

$$-P(x(t))dt + m(t)v(t) = [m(t) + dm][v(t) + dv] - dm[v(t) + dv], \tag{3}$$

or, after simplifying it:

$$-P(x(t)) = m(t)\frac{dv}{dt} = m(t)\frac{d^2z(t)}{dt^2}. \tag{4}$$

Force $P(x)$ and reaction force $F(t)$ act on mass $(-dm)$ that slows from velocity $v(t)$ to $u(t)$ during time dt, see Fig. 1b. The balance of momentum gives

$$[P(x(t)) - F(t)]dt - dm \cdot v(t) = -dm \cdot u(t), \tag{5}$$

leading to

$$P(x(t)) - F(t) = -\frac{dm}{dt} \cdot \frac{dx}{dt} = -\mu(x)\left(\frac{dx}{dt}\right)^2. \tag{6}$$

Reaction force F acts on the target, which is a linear vibrating system:

$$F(t) - ky(t) - c\frac{dy}{dt} = M\frac{d^2y}{dt^2}. \tag{7}$$

From Eqs. (1), (4), (6) and (7) we obtain the differential equations

$$\frac{d^2x}{dt^2} = \frac{P(x(t))}{m(t)} + \frac{P(x(t))}{M} + \frac{\mu(x(t))}{M}\left(\frac{dx}{dt}\right)^2 - \frac{c}{M} \cdot \frac{dy}{dt} - \frac{k}{M}y(t), \tag{8}$$

$$\frac{d^2y}{dt^2} = \frac{P(x(t))}{M} + \frac{\mu(x(t))}{M}\left(\frac{dx}{dt}\right)^2 - \frac{c}{M}\cdot\frac{dy}{dt} - \frac{k}{M}\cdot y(t) \tag{9}$$

with initial conditions

$$x(0) = L, \quad \frac{dx}{dt}(0) = -v_0, \quad y(0) = 0, \quad \frac{dy}{dt}(0) = 0, \tag{10}$$

where v_0 is the impact velocity, that is, the velocity of the missile at the start of the collision. It is set of nonlinear ordinary differential equations. Reaction force $F(t)$ can be expressed from (6) as

$$F(t) = P(x(t)) + \mu(x(t))\left(\frac{dx}{dt}\right)^2, \tag{11}$$

which can directly be computed once $x(t)$ is obtained.

2.2. Dimensionless form

It is worth casting the governing equations into dimensionless form. This way we expect to find the essential combinations of the parameters that determine the course and the final outcome of the impact.

Using the original length L of the missile as the unit for distances, we can define the dimensionless actual length $\tilde{x}$ and target displacement $\tilde{y}$ as

$$\tilde{x} = x/L, \quad \tilde{y} = y/L. \tag{12}$$

We use P_0, the characteristic crushing force, as the force unit so that

$$P(x) = P_0\vartheta(\tilde{x}), \tag{13}$$

with $\vartheta(\tilde{x})$ characterizing the shape of $P(x)$. Then we can define the dimensionless time variable $\tilde{t}$ using $\sqrt{Lm_0/P_0}$ as the time unit so that

$$\tilde{t} = \frac{t}{\sqrt{\frac{Lm_0}{P_0}}}, \tag{14}$$

where $m_0 = m(0)$ is the total original mass of the missile. The distributed mass $\mu(x(t))$ can also be transformed to dimensionless form as

$$\tilde{\mu}(\tilde{x}) = \frac{L}{m_0}\mu(x). \tag{15}$$

Using these new, dimensionless variables, Eqs. (8) and (9) can be rewritten as

$$\frac{d^2\tilde{x}}{d\tilde{t}^2} = \frac{\vartheta(\tilde{x})}{\tilde{m}(\tilde{x})} + \varepsilon\left[\vartheta(\tilde{x}) + \tilde{\mu}(\tilde{x})\left(\frac{d\tilde{x}}{d\tilde{t}}\right)^2 - \gamma\frac{d\tilde{y}}{d\tilde{t}} - \kappa\tilde{y}\right], \tag{16}$$

$$\frac{d^2\tilde{y}}{d\tilde{t}^2} = \varepsilon\left[\vartheta(\tilde{x}) + \tilde{\mu}(\tilde{x})\left(\frac{d\tilde{x}}{d\tilde{t}}\right)^2 - \gamma\frac{d\tilde{y}}{d\tilde{t}} - \kappa\tilde{y}\right], \tag{17}$$

where the dimensionless actual and initial mass of the uncrushed part of plane are, respectively,

$$\tilde{m}(\tilde{x}) = \int_0^{\tilde{x}} \tilde{\mu}(\hat{x})d\hat{x}, \tag{18}$$

$$\tilde{m}_0 = \tilde{m}(0) = \int_0^1 \tilde{\mu}(\hat{x})d\hat{x} = 1. \tag{19}$$

In case of a uniform missile, $\vartheta(\tilde{x}) \equiv 1$ and $\tilde{\mu}(\tilde{x}) \equiv 1$, we find that $\tilde{m}(\tilde{x}) = \tilde{x}$. The following dimensionless parameters have been introduced:

$$\varepsilon = \frac{m_0}{M}, \quad \gamma = \sqrt{\frac{c^2L}{m_0P_0}}, \quad \kappa = \frac{kL}{P_0}. \tag{20}$$

Parameter γ gives the strength of damping, in this paper we take $\gamma = 0$ meaning no structural damping during the short duration of the impact. Parameter κ gives the stiffness of the target relative to the crushing force of the missile. Parameter ε is the ratio of the mass of the missile to that of the target. In case of the original Riera model, when the target is rigid, we have $\varepsilon = 0$ simplifying governing Eqs. (16) and (17) to $d^2\tilde{x}/d\tilde{t}^2 = \vartheta(\tilde{x})/\tilde{m}(\tilde{x})$ and $d^2\tilde{y}/d\tilde{t}^2 = 0$.

The initial conditions in dimensionless form are:

$$\tilde{x}(0) = 1, \quad \tilde{y}(0) = 0,$$
$$\frac{d\tilde{x}}{d\tilde{t}}(0) = -v_0\sqrt{\frac{m_0}{LP_0}}, \quad \frac{d\tilde{y}}{d\tilde{t}}(0) = 0. \tag{21}$$

We define the *damage potential* as

$$D = \frac{\frac{1}{2}m_0 v_0^2}{LP_0} \tag{22}$$

This dimensionless parameter is the ratio of the initial kinetic energy of the missile to the work required to crush it. With this new parameter, the dimensionless initial condition for $d\tilde{x}/dt$ can be written as $d\tilde{x}/dt(0) = -\sqrt{2D}$.

The total length of the impact is determined by either one of the following conditions. Either the whole missile crumbles (that is, $\tilde{x} = 0$ is reached) or the crushing stops (that is, $d\tilde{x}/d\tilde{t} = 0$ occurs). In either case we consider the impact finished.

The dimensionless form of the reaction force is

$$f(\tilde{t}) = \frac{F(t)}{P_0} = \vartheta(\tilde{x}(\tilde{t})) + \tilde{\mu}(\tilde{x}(\tilde{t}))\left(\frac{d\tilde{x}}{d\tilde{t}}\right)^2. \tag{23}$$

Once $\tilde{x}(\tilde{t})$ is computed, $f(\tilde{t})$ is readily obtained from this equation.

3. Simplest case: Uniform missile impacting a rigid target

The simplest special case of (16) and (17) is a rigid target $\varepsilon = 0$ hit by a uniform missile $\vartheta \equiv 1$, $\tilde{\mu} \equiv 1$. In this case the equations simplify to

$$\frac{d^2\tilde{x}}{d\tilde{t}^2} = \frac{1}{\tilde{x}}, \quad \tilde{y} \equiv 0, \tag{24}$$

with initial conditions

$$\tilde{x}(0) = 1, \quad \frac{d\tilde{x}}{d\tilde{t}}(0) = -\sqrt{2D}. \tag{25}$$

Even this simplest case forms a nonlinear ordinary differential equation for $\tilde{x}(\tilde{t})$. Note that Eq. (24) contains no parameter, its solution does *not* depend on any parameter, e.g., properties of the missile. Only the damage potential enters the solution, and even that only through the initial conditions. Note also that the dimensionless damage potential depends on the properties of the missile, see (22), but only this special combination of the impact velocity v_0, the missile mass m_0, length L and crushing force P_0 determines the overall behavior of a uniform missile hitting a rigid wall. The fact that parameters of the missile do not enter (24) means that solution curves $\tilde{x}(\tilde{t})$ are the same for such impacts, it is only the initial point along the curve that is determined by the damage potential. We also note that our dimensionless parameter D is very similar to Johnson's damage number (see, e.g., Ref. [25]), but in our case the parameters of the missile appear instead of the properties of the target.

In fact, Eq. (24) can be solved analytically. Integrating it once results in

$$\frac{d\tilde{x}}{d\tilde{t}} = \pm\sqrt{2\ln\tilde{x} + C_1}, \tag{26}$$

where $C_1 = 2D$ is fixed from the initial conditions (25). In the right-hand side of (26), the negative sign is physically relevant, because the actual length of the missile decreases hence $d\tilde{x}/d\tilde{t} \leq 0$. This leads to [20]:

$$\frac{d\tilde{x}}{d\tilde{t}} = -\sqrt{2\ln\tilde{x} + 2D}. \tag{27}$$

This equation, apart from the factor 2 under the square root, is very similar to the equation derived by Tate [4] using a hydrodynamical approximation. In our case, however, the target is rigid, hence the hydrodynamic approximation does not hold.

Integrating (27) again, we find

$$\tilde{t} + C_2 = \sqrt{\frac{\pi}{2}} i e^{-D} \operatorname{erf}\left(i\sqrt{\ln \tilde{x} + D}\right), \tag{28}$$

where i is the imaginary unit, and $\operatorname{erf}(z)$ is the Gauss error function [26]

$$\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-\xi^2} d\xi.$$

In Eq. (28), $C_2 = i\sqrt{\pi/2}\exp(-D)\operatorname{erf}(i\sqrt{D})$ can be fixed from the initial conditions. After rearrangement, we find

$$\tilde{x}(\tilde{t}) = e^{-D} e^{-\left\{\operatorname{inverf}\left[-i\sqrt{\frac{2}{\pi}}e^{D}t + \operatorname{erf}\left(i\sqrt{D}\right)\right]\right\}^2}, \tag{29}$$

where $\operatorname{inverf}(z)$ is the inverse function of $\operatorname{erf}(z)$. Despite i appearing in these formulae, the result is real at all physical values of $\tilde{t}$.

Differentiating (29) with respect to time, one obtains the velocity of crushing as a function of time:

$$\frac{d\tilde{x}}{d\tilde{t}} = i\sqrt{2}\operatorname{inverf}\left[-i\sqrt{\frac{2}{\pi}}e^{D}t + \operatorname{erf}\left(i\sqrt{D}\right)\right]. \tag{30}$$

Substituting it into (23) we find the dimensionless reaction force as a function of time:

$$f(\tilde{t}) = 1 - 2\left\{\operatorname{inverf}\left[-i\sqrt{\frac{2}{\pi}}e^{D}t + \operatorname{erf}\left(i\sqrt{D}\right)\right]\right\}^2. \tag{31}$$

We note again that the solution only depends on the dimensionless damage potential D, a specific combination of the parameters of the missile, as given by (22). The dimensionless reaction force as a function of time for various different impact velocities is shown in Fig. 2.

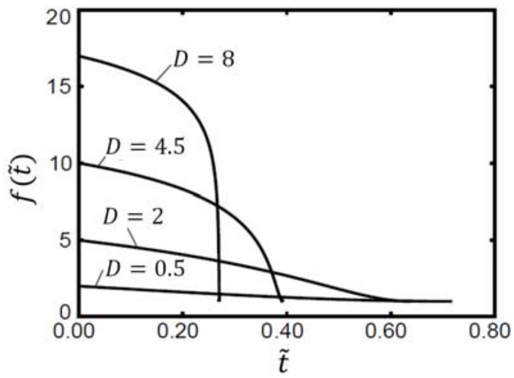

Figure 2: Dimensionless reaction force functions $f(\tilde{t})$ for various values of the dimensionless damage potential D in case of rigid target.

In fact, in this model, for a uniform missile impacting a rigid target, the maximum reaction force (23) arises at the beginning of the impact, when the speed of the missile is the highest, see Fig. 2. The values obtained for the maximum reaction force look contradicting to the statement in Ref. [27] that the maximum force would depend exponentially on the impact velocity. Rather, Eq. (23) suggests that the maximum force depends on the square of the impact velocity.

We found these results for a uniform missile impacting a rigid target. However, we show in the next section that the important parameter characterizing the properties of the impact of a uniform missile is the same dimensionless combination of the parameters, the damage potential $D = v_0^2 m_0 / 2LP_0$, independently of the properties of the target.

4. Uniform missile impacting an elastic target

4.1. The role of the damage potential

The solutions of (16) and (17) in case of an elastic target, that is, with $\varepsilon > 0$, are obtained numerically. We use the 4th order explicit Runge-Kutta method with absolute error tolerance 10^{-8}. We neglect damping ($\gamma = 0$) since damping is expected to play a minor role during the short duration of the impact. We survey the behavior during the impact as a function of the remaining three independent dimensionless parameters D, κ and ε.

Figure 3 shows some representative reaction force curves for a wide variety of parameter values. Initially, the reaction force vs. time curves oscillate around a roughly horizontal plateau due to the elasticity of the target.

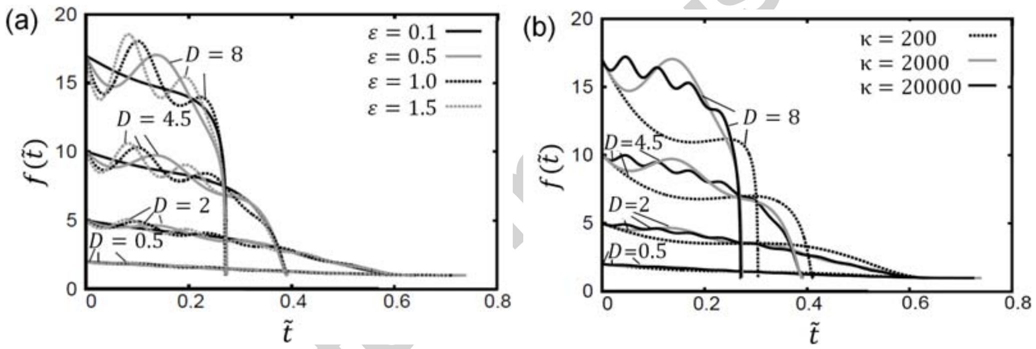

Figure 3: Dimensionless reaction force function $f(\tilde{t})$ for various parameter values. (a) $\kappa = 2000$, D and ε are as indicated; (b) $\varepsilon = 0.5$, κ and D are as indicated.

In the later part of the impact, as time passes, the reaction force starts to decline rapidly, see Fig: 3. The shape of the reaction force curve depends quite strongly on the damage potential D. However, we can see in Fig. 3 that the overall shape of the $f(\tilde{t})$ curves is very similar, independent of the stiffness κ of the structure and the mass ratio ε for the same, fixed values of D. Comparing the results with those presented in Fig. 2 we see that the damage potential D has a major effect on the impact. That is, mainly the combination $D = v_0^2 m_0/2LP_0$ of the parameters determines how the impact affects the structure.

It is important to observe, however, that the elasticity of the target can also play a role in the maximum reaction force during the impact. As shown in Fig. 3, as the flexibility of the missile increases (ε becomes larger or κ decreases) oscillations of the reaction force increase, which results in higher peaks of $f(\tilde{t})$ than the maximum reaction force for a rigid target occurring at the start of the impact. This means that the target can only be assumed rigid if its mass is more than twice the mass of the missile ($\varepsilon < 0.5$). Otherwise the Riera model, based on the rigid target assumption, may not be conservative. Note that $\varepsilon < 0.5$ typically holds for robust structures like containment buildings of nuclear power plants even in case of large aicraft fuselages as missiles.

We also investigate how the duration of the impact depends on the parameters κ, ε and D. In Fig. 4, with color coding, the duration of the impact is visualized as a function of ε and D (Fig. 4a) and κ and D (Fig. 4b). We see that the impact time essentially does not depend on ε and κ, however, it does depend on D, the damage potential. This is further illustrated in Fig. 6a, where the dimensionless impact time is shown as a function of the damage potential for various values of the other parameters. We see that the impact time is determined by D, the elasticity of the target plays only a very minor role. We also see that the impact time has a maximum around D close to 1, independent of the values of ε and κ.

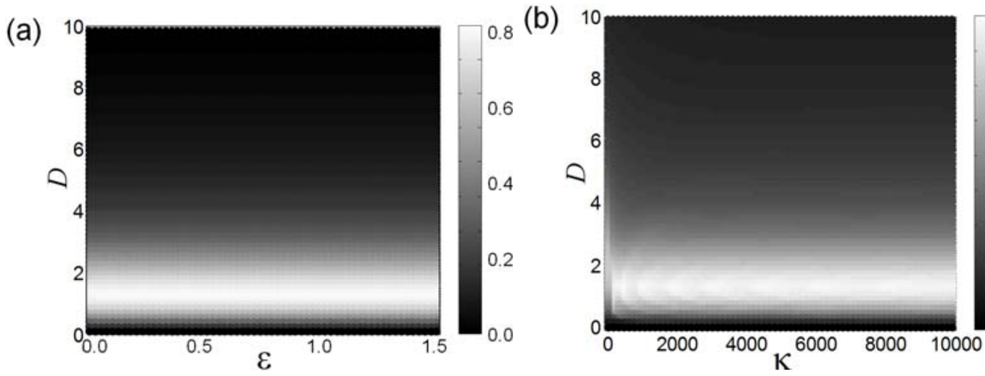

Figure 4: Colour coding of the dimensionless impact time as a function of (a) ε and D ($\kappa = 2000$ fixed), and (b) κ and D ($\varepsilon = 0.5$ fixed).

The length of the part of the missile crushed during the impact can also be used to characterize the impact. In Fig. 5, with colour coding, we show how the crushed length of the missile depends on parameters ε and D (Fig. 5a), and κ and D (Fig. 5b). We see that there is a quite sharp transition between the regime where the full length of the missile is crushed during the impact and the regime where a part of the missile remains intact after the impact. The transition seems to depend only on the value of the damage potential D, it is in the range of D between 0.5 and 2. This is also visible in Fig. 6b, where the dimensionless crushed length is shown as a function of the damage potential for various values of the other parameters. We see that the crushed length is determined by D, the elasticity of the target plays only a very minor role.

Comparing Fig. 4 to Fig. 5, or Fig. 6a to Fig. 6b, we see that the value of the damage potential D is the same at the maximum of the impact time and at the transition between cases of fully crushed (crushed length is 1) and partially crushed missiles at the end of the impact.

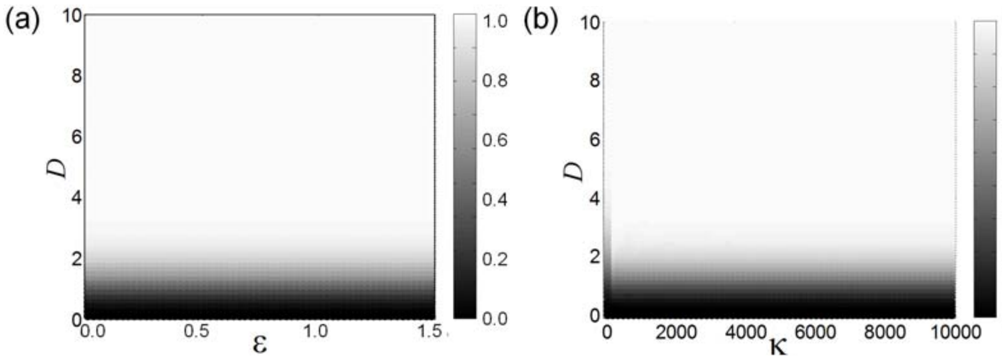

Figure 5: Colour coding of the dimensionless crushed length as a function of (a) ε and D ($\kappa = 2000$ fixed), and (b) κ and D ($\varepsilon = 0.5$ fixed).

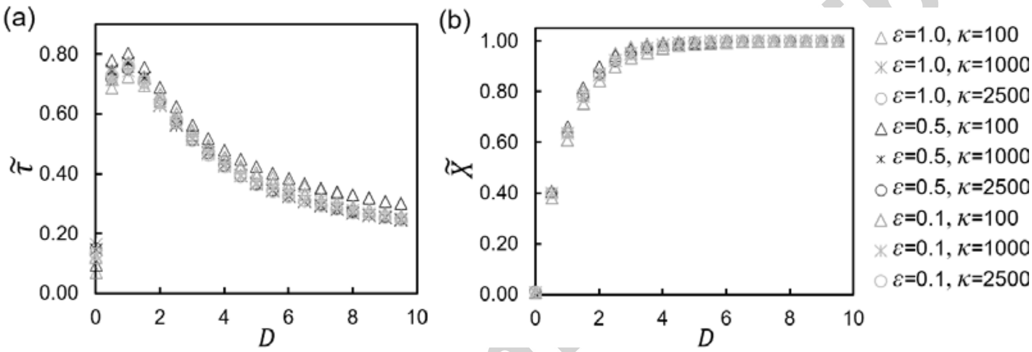

Figure 6: Dimensionless (a) impact time and (b) crushed length as a function of the damage parameter D for various values of parameters ε and κ.

We note that this critical D value between 0.5 and 2 seems to be close to the limit set by Rambach *et al.* [20] for an impact to be hard. They state that impacts are hard when $\beta = 2P_0/\mu v_0^2 > 1$. Since from (22) we find $D = 1/\beta$, the limit for hard impacts in terms of the damage potential becomes $D < 1$. The limit value $D = 1$ is precisely in the range where the impact time has its maximum and where the crossover between partially and fully crushed missile regimes is found. Indeed, if the damage potential is below this limit, for example, if the crushing strength P_0 of the missile is large, the impact can be considered hard, hence only a part of the missile is crushed. In case of such hard impacts, when a relatively rigid missile collides with the structure, local damage effects might need to be considered, and the target cannot be modelled as rigid or elastic.

4.2. Resonant behavior of the target

Figure 7 shows the maximum displacement of the target as a function of the mass ratio ε for fixed values of the dimensionless target stiffness κ and impact velocity D. We see that there is a peak in the maximum target displacement $y_{\max}$ at a finite mass ratio ε. This is not very surprising. On the one hand, for small values of ε the mass of the target is large, hence its displacement is small as a consequence of the impact by a missile of relatively small mass. On the other hand, for large values of ε the mass of the target is small, hence its natural frequency is high. This has the consequence that the target starts to move backwards, towards the missile, during the impact, hence the crushing becomes faster, more intense. This implies that the loss of energy increases due to crushing, and hence less kinetic energy remains for target displacement. In the intermediate range, there is a value for ε where the maximum displacement $y_{\max}$ of the target is largest. This can be considered as a resonant effect, at this mass ratio ε the natural frequency of the target is such that it results in maximum displacement.

One can give an estimation for the resonant mass ratio ε as follows. The natural circular frequency of the target is $\omega = \sqrt{k/M}$. We find that the duration of the impact is $\tau = \pi/\omega = \pi\sqrt{M/k}$ assuming that the maximum displacement occurs when the whole impact takes place during half of the

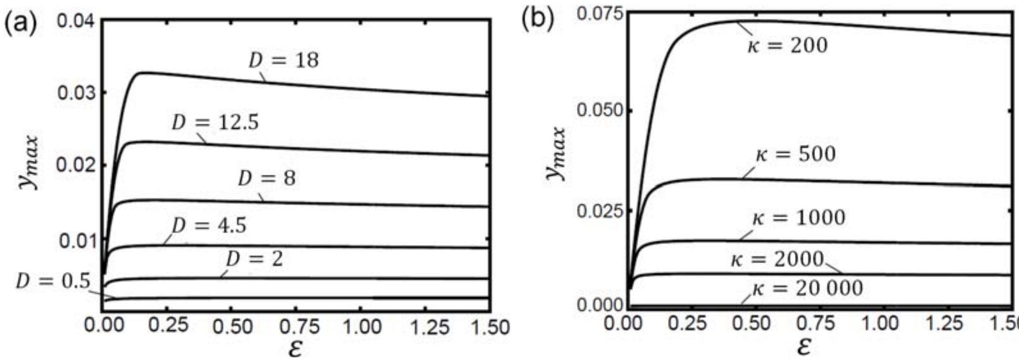

Figure 7: Maximum displacement $y_{\max}$ as a function of the mass ratio ε for various values of (a) the damage potential D ($\kappa = 2000$ fixed) and (b) the dimensionless target stiffness κ ($D = 4.5$).

period $2\pi/\omega$ of natural vibration of the target.

Casting the impact time into dimensionless form $\tilde{\tau} = \tau\sqrt{P_0/Lm_0}$ we end up with

$$\varepsilon = \frac{\pi^2}{\tilde{\tau}^2\kappa}. \tag{32}$$

We find that this estimation indeed gives highest displacement when the missile is completely crushed during the half period of the target's natural vibration. This is the case when the total length L of the missile is crushed during a time period $\pi/\omega = \pi/\sqrt{M/k}$. Assuming that the missile still travels at its initial speed v_0 during this short time, we find that the whole missile is crushed if $\pi v_0\sqrt{M/k} > L$, or, in dimensionless form, if $\sqrt{\varepsilon\kappa} < \pi\sqrt{2D}$. We verified numerically that this is indeed the case: if $\sqrt{\varepsilon\kappa} < \pi\sqrt{2D}$ holds, the largest target displacement occurs for $\varepsilon = \pi^2/\tilde{\tau}^2\kappa$.

This resonant behavior is not desirable, we intend to keep the displacements of the target minimal. Hence the value of the mass ratio should be chosen not to fall close to the critical value resulting in maximum target displacement. However, it is to be noted in Fig. 7 that for higher values of ε the maximum displacement does not decrease dramatically, hence smaller values of ε are better. Smaller values of ε imply larger target mass, which is usually the case for robust structures.

5. Conclusions

The main goal of this paper is to find the important parameters that govern the response of structures during an impact. Therefore, we carry out a systematic parametric study of a uniform, cylindrical, rigid–plastic rod impacting a rigid or elastic target. The modeling assumptions for the missile are similar to those of Riera's model [5]. We believe that using dimensionless governing equations and simplified models containing only a few parameters is the approach to discover which parameters are relevant to determine the main features of the response of the impacted structure.

We indeed find that the only relevant combination of the parameters is the dimensionless damage potential defined as

$$D = \frac{\frac{1}{2} m_0 v_0^2}{L P_0}, \tag{33}$$

where v_0 is the velocity of the missile before the impact, m_0 is the initial total mass of the missile, L is its length, and P_0 is its characteristic crushing force. The damage potential is essentially the ratio of the initial kinetic energy of the missile to the work required to crush the missile. The fact that the course of the impact and the reaction force acting on the structure depend uniquely on this single parameter is a rigorous result for a uniform missile impacting a rigid target. For elastic targets, the importance of the damage potential is found using numerical simulations in a wide range of the parameter values. We find that the ratio of the missile mass to that of the target structure or the ratio of the target's stiffness to the crushing force of the missile have only secondary effect on the course of the impact. However, if the mass of the missile is more than half of that of the target, the peak reaction force can exceed the peak reaction force in case of a rigid target, which implies that the Riera model may not provide conservative results. For robust buildings similar to containments of nuclear power plants hit by an aircraft fuselage this is not an issue, but for less massive structures this effect might need to be considered.

For the simplest case of a uniform missile impacting a rigid target, we derive explicit formulae both for the course of the impact and for the reaction force acting on the target. While these are quite complicated for practical purposes, they can serve as benchmarks to validate numerical codes.

Our numerical findings are specific to the model we investigated. It is to be verified with more complex missile and target models how other parameters that appear in those models affect the behavior. We conjecture, however, that the dimensionless damage potential remains an important parameter, and other parameters only refine the details of the impact process. This conjecture is supported by the similarity of this parameter (33) to Johnson's damage number [25].

A dimensionless number, similar to our damage potential, was found to play an important role in fragmentation processes [28]. This number depends on the ratio of the initial kinetic energy of colliding solid bodies to the total energy required to disintegrate them. It has been shown that the fragmentation process of colliding solid bodies depends on this ratio [28] or on parameters that appear in this ratio [29]. A similar dimensionless number was found to characterize the dynamic response of box-shaped structures under internal blast investigated experimentally [30]. This number is the ratio of the total explosive energy to the energy required to yield one side of the container. In spirit, this number is similar to our damage potential, characterizing both the cause of the blast and the properties of the target.

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