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An evolutionary-adaptive approach

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Saver types: An evolutionary-adaptive approach

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Abstract

We set up an agent-based macromodel focusing on consumption-saving without the assumption of utility maximization, but preserving certain "rational" aspects of human choice based on the idea of ecological rationality Todd et al. (2012). In this framework we address the classical problem of the efficiency of long-run capital accumulation. Three qualitatively different saving strategies are defined: 1. buffer stock saving (prudent and forward looking), 2. permanent income saving (forward looking without prudence), and 3. myopic saving (caring only about immediate consumption, and saving accidentally). In the model these types (that have subtypes depending on continuous parameters) may coexist, and we explore their respective survival chances by conducting simulations. It is found that prudent saving behavior becomes prevalent when the selection pressure is very high, but an economy comprising only prudent households tends to accumulate capital in excess of what is implied by the Golden Rule. As selection pressure is reduced, myopic consumers appear, and under very low selection pressure the distribution of the main saver types becomes almost random. A seemingly puzzling fact emerges: the economy gets close to the Golden Rule of capital accumulation via endogenous selection of subtypes in a way that can be interpreted as "perverse exploitation", i.e. the exploitation of the rich by the poor. In other words, lowering the intensity of evolutionary forces, that results in more diversity in saver types, may be socially beneficial. Crickets may be useful for society as a whole, including prudent and cautious ants.

JEL classification: C69, E21

Keywords: agent-based macromodel, bounded rationality, evolutionary learning, savings types

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Megtakarítás adaptív-evolúciós megközelítésben

Varga Gergely – Vincze János

Összefoglaló

Egy ágens alapú makromodellben vizsgáljuk a megtakarítási-fogyasztási döntést, amelyben nem tételezünk fel hasznosság maximalizálást. De megtartjuk a döntés bizonyos racionális aspektusait, amelyek az „ökológiai racionalitás” koncepcióján alapulnak. Legfontosabb kérdésünk a felhalmozás hosszú távú hatékonyságára vonatkozik. Három kvalitatívan eltérő megtakarítási stratégiát definiálunk: 1. puffermegtakarító (előrettekintő és prudens), 2. permanens jövedelemmegtakarító (előrettekintő, ám nem prudens), és 3. rövidlátó (aki csak a közvetlen fogyasztással törődik). Ezek az alaptípusok (folytonos paraméterrel indexálható) altípusokkal is rendelkeznek, és együttélésük az egyes stratégiák relatív evolúciós előnyétől függ. Szimulációs vizsgálatainkkal azt találtuk, hogy a prudens és előrelátó magatartás dominál, amennyiben a szelekciós nyomás erős, ám egy olyan gazdaság, amelyben csak prudens egyének vannak, általában túlzottan sok tőkét halmoz fel az arany szabályhoz viszonyítva. Ahogy csökkentjük a szelekciós nyomást, megjelennek először a rövidlátó fogyasztók, és végül az alaptípusok eloszlása lényegében véletlenszerűvé válik. Nem véletlenszerű azonban az altípusok evolúciója ekkor sem. Alacsony szelekciós nyomásnál olyan sajátos helyzet alakul ki, amit „perverz kizsákmányolásnak” nevezhetünk. A gazdagokat kizsákmányolják a vagyontalan szegények abban az értelemben, hogy a nettó tőkehozam o körüli (ami megfelel a társadalmilag hatékony szintnek), és a „szegények” többet fogyasztanak, mint a „gazdagok”. A tücskök léte lehet a társadalom számára hasznos, beleértve az óvatos hangyákat is.

Tárgyszavak: ágens alapú makromodell, korlátozott racionalitás, evolúciós tanulás, megtakarítás

JEL kódok: C69, E21

Saver types: An evolutionary-adaptive approach*

Gergely Varga[†], János Vincze[‡]

Abstract

We set up an agent-based macromodel focusing on consumption-saving without the assumption of utility maximization, but preserving certain "rational" aspects of human choice based on the idea of ecological rationality Todd et al. (2012). In this framework we address the classical problem of the efficiency of long-run capital accumulation. Three qualitatively different saving strategies are defined: 1. buffer stock saving (prudent and forward looking), 2. permanent income saving (forward looking without prudence), and 3. myopic saving (caring only about immediate consumption, and saving accidentally). In the model these types (that have subtypes depending on continuous parameters) may coexist, and we explore their respective survival chances by conducting simulations. It is found that prudent saving behavior becomes prevalent when the selection pressure is very high, but an economy comprising only prudent households tends to accumulate capital in excess of what is implied by the Golden Rule. As selection pressure is reduced, myopic consumers appear, and under very low selection pressure the distribution of the main saver types becomes almost random. A seemingly puzzling fact emerges: the economy gets close to the Golden Rule of capital accumulation via endogenous selection of subtypes in a way that can be interpreted as "perverse exploitation", i.e. the exploitation of the rich by the poor. In other words, lowering the intensity of evolutionary forces, that results in more diversity in saver types, may be socially beneficial. Crickets may be useful for society as a whole, including prudent and cautious ants.

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1 Introduction

Saving is one of the central themes in economics. In growth theory it is through savings that nations become rich, although it was recognized that too much saving can harm consumption. (It is the problem of dynamic inefficiency, see e.g. Blanchard and Fischer 1989, chap. 3) Short and medium term fluctuations in savings are an important ingredient of business cycle analysis,

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and Akerlof and Shiller (2010, chap. 10) suggested that longer term waves in macroeconomic variables may be due to unstable saving behavior in the long run. Saving behavior exhibits heterogeneity across time and nations. The existence of cross-sectionally different saving rates is an important issue in the economics of poverty (Banerjee and Duflo 2011, chap. 8.), and it is thought to have bearing on the ever changing wealth distribution within developed countries (Piketty 2014).

In this paper we want to make a theoretical contribution to the issue of low frequency interactions between savings, income and consumption. Specifically two issues are targeted: 1. What are the forces that can lead to over- or underaccumulation, where these terms are understood relative to the golden rule principle? 2. Can changes in saving behavior generate "arbitrary" long term fluctuations in the productive capabilities of an economy, as Akerlof and Shiller suggested?

When we speak of a theoretical contribution we do not have in mind the traditional optimization approach. In fact another motive for our work is to contribute to the development of agent-based macroeconomics. Agent-based macromodels have followed a radically different approach than traditional neoclassical or neoknesian macroeconomics. The deviations are multiple but an important one is how they treat behavior. Whereas in traditional models consumption-savings decisions are derived from utility maximization agent-based macromodels directly postulate saving rules, without deriving them from first (or rather more basic) principles. This state of affairs is somewhat unsatisfactory, as agent-based modellers probably do not believe that human behaviour is completely automatic, but rather that it is adaptive, and not deprived wholly of "rationality". What agent-based modellers do not want to follow is the joint hypotheses of stable preference orderings and the ability of economic agents to achieve the optimal outcome. Our goal in this paper is to suggest a way for improving on this situation, and to develop a model that, while falling very short of treating agents as "fully rational", gives room for goal directed adaptation of savings behaviour. Our proposal consists in modelling saving as a set of rules of decision in a specific evolutionary framework, where the rules change in time endogeneously.

Section 2 offers a look at the literature on low frequency capital accumulation and saver types, and outlines our approach for answering the two questions raised above. Section 3 presents a mathematical model, its parameterization and the simulation settings. Section 4 contains the basic findings via simulation exercises. Section 5 summarizes.

2 Lessons from the literature

2.1 Savings in the long run: efficiency and stability

The possibility of dynamic inefficiency, i.e. accumulating capital in excess of the Golden Rule level (where the net marginal product of capital equals the growth rate), emerged as a possible feature of overlapping generations models (see Blanchard and Fischer 1989). Whereas in models with infinitely living maximizing agents impatience drives savings below the level consistent with long-run consumption maximization, when agents have finite lives and must provide for themselves in their old-age, they may find rational to keep a stock of capital with a net return below the growth rate of the economy. If the forces of the necessity of self-provision ("supported" by low capital productivity) outweigh that of impatience the equilibrium outcome

may not be Pareto-efficient, despite market clearing and individual rationality. Thus either under- or over-saving can be the outcome in traditional models, attributable to the interplay between short-sightedness and "induced" prudence. Empirical work conducted in the 1980s (see Abel et al. 1989) seemed to indicate that of the two theoretically possible regimes the undersaving one has been realized in the major economies, but Geerolf recently raised doubts about this conclusion in an unpublished paper (Reassessing dynamic efficiency. manuscript, Toulouse School of Economics, 2013), claiming that "Japan and South Korea have unambiguously overaccumulated capital".

A closer inspection of national savings rates hints at puzzling facts about savings in the long run. Akerlof and Shiller (2010, chap. 9) pointed out a feature that they call the "arbitrariness" of saving. Across nations savings rates as high as 1/3 coexist with negative ones. Capriciousness can be detected in time, too. The personal saving rate in the US was reduced from 10% in the early 1980s to negative rates in the XXIst century. Akerlof and Shiller notice also the variability of savings rates across individuals resulting in huge differences in retirement wealth, and emphasize the inability of the traditional theory to account for this fact. Their preferred explanation is "Animal Spirits", but they do not offer a clue for what it means exactly. They muster observations indicating that savings decisions are made on "irrelevant cues", and their anecdotal description of the role of stories (culture) seems plausible. But how irrelevant cues can play a role in such an important decision, and how saving culture is shaped and changed are left unanswered. Traditional economics reduces the explanation of behavior on exogenous variation of preferences and technology (including demographics). Therefore when apparently large changes are detected in saving rates, explanations are looked for along these lines. For instance, Dobrescu et al. (2012) proposed models to account for the large declines in saving rates in developed countries, and concluded that the cause must be shifts in social preferences. This explanation, naturally, leaves open the question why preferences have been unstable.

2.2 Individual heterogeneity in savings

The traditional utility maximizing theory of saving, originating in Fisher (1930), developed several models for rational households. Early theories disregarded non-diversifiable income risks, therefore risk-taking did not play a significant role in them. A general result was that the marginal utility of consumption follows a random walk, and its response to permanent shocks is much stronger than to temporary ones. There were empirical studies that seemed to confirm that "certainty equivalent" households indeed exist (e.g. Hall and Mishkin 1982). Later researchers developed theories emphasizing non-diversifiable labour income risk, and studied its interaction with prudent preferences. From this literature the buffer-stock theory of saving was born (see Carroll and Summers 1991). The main result here was that households' behavior can be characterized by a desired savings/income ratio. Buffer stock savers adjust consumption in order to get as close as possible to this target. This kind of behavior leaves more room for the effects of non-permanent income, and seems to correspond at least partly to available evidence (see Carroll 1997).

Empirical modelling of saving had to face the problem of more individual heterogeneity than that implied by utility maximizing agents. There is a long empirical tradition that separates people into two groups: those whose behaviour can be safely described as "fully rational", and those who seem to behave in a purely current consumption oriented manner, largely disregarding the future consequences (see for instance Hall and Mishkin 1982). Whether the latter group

behaves rationally but under constraints, or suffer from "myopia", has always been an open question, but the distinction between these two fundamentally different types of consumers was born on empirical grounds. A modern variant that makes use of psychological findings about hyperbolic time discounting was rendered in its now usual format by Laibson (1997). Here the distinction is between rational long-run maximizers, and naive agents who have time-inconsistent preferences. Some more recent theories are based on other — more complete — psychological theories according to which people have a dual-self, and sometimes are unable to behave in their best long-run interests (Fudenberg and Levine 2006). A related observation is made in Banerjee and Duflo (2011, chap. 8) where the authors consider the baffling fact that though many poor people over the world might increase their well-being substantially with a little bit of more saving, there are only a few who accomplish this. Banerjee and Duflo enlist several explanations, but their favourite one rests on the lack of self-control coupled with positive biological feedback. Of their description we underline one major point: people say frequently that money at home cannot be kept, because there will be some purpose to spend it in no time. Our interpretation is that many people decide on spending without much regard to the future. All of these theories rely on a persistent coexistence of rational and irrational behaviour with respect to saving, while ignore the problem why people cannot learn to act rationally, or why evolution cannot eradicate an apparently inferior behavioral pattern.

2.3 Savings and agent-based macroeconomics

Behavioral economists have striven to build realistically founded theories of the consumption function. While the behavioral economics literature departs significantly from the traditional model, it does not give up the idea that people (or their selves) try to maximize some preference functional—in a non-traditional sense—over their lifetime. We believe that there exist observations that are difficult to fit into any maximizing model. For instance, the extensive literature on 401(k) accounts (Madrian and Shea 2001) would hardly be made compatible with any preference-based theory, therefore it seems reasonable to search for non-maximizing types of explanation.

Agent-based models have been established in economics for at least 20 years now. There exist several definitions and characterizations of agents-based modelling in general (see for example Gilbert 2008), and in economics, in particular (see for instance Tesfatsion 2001, 2006; LeBaron 2006). Agent-based modeling can be considered as a technology that is well-suited for certain types of theoretical approaches in economics. We think that when someone subscribes to an evolutionary-adaptive view of economic behavior, and wishes to develop the implied notions into a mathematical model it is almost inevitable that the elected framework will be of the agent-based type.

Though agent-based models usually do not include agents who straightforwardly and faultlessly maximize, they have room for learning or adaptation, which lends some sort of rationality (or goal-orientedness) to behavior. Learning in the agent-based setting was several times reviewed (Brenner 2006; Duffy 2006). For instance, learning can occur continuously when we make decisions, then observe the outcomes and the environment, and modify our world view based on our experience. Another type of learning or adaptation mimics biological evolution, when from the pool of available behavioral patterns the more successful ones tend to be selected (i.e. survive with higher probability than the less successful), while the pool itself changes according to chance events (mutation) or to some higher-order cognitive process. Evolutionary

learning attracted special attention in some economic applications (see Arifovic 2000). Both types of learning presuppose the existence of some criterion (fitness or success), underlying the role of goal-directed behaviour, but without making strong assumptions concerning the consistency or completeness of goals, and the ability of achieving the goals.

However, in the case of agent-based macroeconomic models one can find that these tend to rely almost exclusively on the empirical literature when modelling savings, and avoid theoretical considerations with respect to the basis of behavior. For instance, Dosi et al. (2013) assume simply that consumers want to consume in each period as much as possible, with the justification that consumption traces current income rather closely empirically. Another well-known empirical regularity is the concavity of the consumption function. A concave consumption function is specified in the flagship model of Delli Gatti et al. (2011). It seems that more and more frequently agent-based models invoke a parametrized form of the buffer-stock theory (see Delli Gatti et al. 2011; Deissenberg et al. 2008). However, to our best knowledge, no agent based model has mixed behavioural types, and, in consequence, has asked about the relative advantages of different behavioral types. In other words agent-based macromodels have distanced themselves from traditional macroeconomics a bit too much, and only the shade of rationality remained in them.

2.4 Summary

Looking at the accumulated empirical evidence on savings it is fair to say that heterogeneity is the rule, not only in quantitative, but also in qualitative terms. There seems to exist people with precautionary motives, but also those who save "too little". There appears consumption smoothing, implying forward looking behavior, but at the same time excessive short-termism or myopia does not seem to be alien to many of us.

Our guess is that disparate savings behavior must be one key to explain arbitrariness of savings, and may elucidate the under or over-accumulation issue, too. The appropriate modeling framework must be an agent-based model where evolutionary forces (selection, mutation) are at work. In such a model one can ask whether different behavioral types can coexist, and what circumstances result in the prevalence or the extinction of some of them. In order to set up such a model we will define a few "basic" behavioral patterns. In accordance with the literature we will distinguish three types of savers: 1. buffer stock savers (prudent and forward looking), 2. permanent income savers (forward looking without prudence), and 3. myopic savers (caring only about immediate consumption, and saving accidentally). These types will have subtypes, also. For instance, to fully define a buffer stock saver we have to fix the target buffer that controls his behavior. In a classical model the "optimal" buffer is some function of the model's fundamentals, such as the utility function, the income process etc. Having no utility functions we have to find a way for buffer stock savers to settle on an "ecologically rational" level of the target buffer. On the concept of ecological rationality in general see Todd et al. (2012). For our purposes ecological rationality can be defined as goal-directed behavior equipped with the ability to learn, and adapt to changing circumstances. We characterize the behavior of myopic savers with the strength of the drive to emulate others, and that of permanent income savers with the degree of optimism they regard their future income prospects. All of these subtype-defining characteristics are "inheritable" or "imitable", and undergo modifications due to evolutionary forces. To apply an evolutionary reasoning we need a measure of fitness, identified with long-term consumption, which in a certain sense operates

in our model as "objective" (intersubjective) utility. We believe that we do not unduly deviate from the spirit of traditional economics in that respect.

One warning sentence: our saver types, though similar in spirit, cannot be directly compared with agents in traditional rational expectation models. They emanate from distinct philosophical approaches: rational versus adaptive expectations, rationality as maximization versus ecological rationality.

3 The model

As we focus on long-term issues, income and its primary distribution are determined on the supply side. The model has a simple production-distribution framework based on neoclassical assumptions regarding production, labour and physical capital services markets. Agents plan their consumption level in each period in accordance with their types (and subtypes), but, like in other agent-based macromodels, there is no in-built equilibrating mechanism, thus plans may turn out to be infeasible. Therefore there must exist an (imperfect) credit market that intermediates between plans and actual consumption. The order of subsections follows the order of events in a given period: 3.1. production and the distribution of primary income, 3.2. planning of consumption, 3.3. the opening of the credit market and the determination of consumption, borrowing and physical capital accumulation. 3.4. evolutionary learning. Then in subsection 3.5 the basic simulation setting is presented.

3.1 The production-distribution side

The production–primary income distribution side of the model follows the model in Aiyagari (1994), a version of the Bewley-type models (see Ljungqvist and Sargent 2004, chap. 17), which have been studied to find out how incomplete markets influence equilibria in dynamic general equilibrium frameworks. Saving opportunities include private loans and physical capital. Our setting differs from traditional Bewley-type models in the way asset markets clear, and, most importantly, in the behavior of households.

There exist N *ex ante* identical, infinitely living dynasties of households. The labor supply of each dynasty (from now on dynasties are called either agents or households) is characterized by the same two-state Markov-chain, with the transition matrix:

	$L1$	$L2$
$L1$	p	$1 - p$
$L2$	$1 - q$	q

where $L1 < L2$.

Individual labour supplies are independent, thus, if the number of agents is large, aggregate labor supply uncertainty is almost nil, though individual uncertainty can still be substantial.

The economy's aggregate production function is of the Cobb-Douglas type, with aggregate labour and capital as its arguments, and α as labour's share:

$$\begin{aligned}
Y_t &= K_t^{1-\alpha} L_t^\alpha, \\
L_t &= \sum_{k=1}^N L_{tk}, \\
K_t &= \sum_{k=1}^N K_{tk},
\end{aligned}$$

where L_t is aggregate labour, L_{tk} is labour supply of household k , K_t is aggregate physical capital, K_{tk} is capital owned by household k , and Y_t is aggregate output. Capital depreciates at rate δ per period.

The number of households and the parameters of the Markov-chain uniquely determine long-run average labor supply (L). We know that in the Cobb-Douglas case without uncertainty the Golden Rule saving rate is $1 - \alpha$, and the corresponding levels of capital and consumption can be expressed as

$$\begin{aligned}
K &= \left(\frac{1 - \alpha}{\delta} \right)^{\frac{1}{\alpha}} L \\
C &= \alpha K^{1-\alpha} L^\alpha,
\end{aligned}$$

respectively. Below we will make use of these reference values when reporting results of simulation exercises.

Labor markets always clear, and work is rewarded according to its marginal product. Therefore the wage rate (w_t) and the compensation of capital services (r_t^K) can be expressed explicitly as

$$\begin{aligned}
w_t &= \frac{\alpha Y_t}{L_t} \\
r_t^K &= \frac{(1 - \alpha) Y_t}{K_t}.
\end{aligned}$$

At the beginning of each period idiosyncratic labor supply shocks materialize, then production and the distribution of primary income take place.

3.2 Planning consumption and saving

Net wealth (A_{tk}), and current income (I_{tk}) are defined as:

$$\begin{aligned}
A_{tk} &= K_{tk} + B_{tk} - D_{tk} \\
I_{tk} &= w_t L_{tk} + (r_t^K - \delta) K_{tk} + r_t(B_{tk} - D_{tk}),
\end{aligned}$$

where r_t is the rate of interest on private loans, $B_{tk} \geq 0$ is credits to other households, and $D_{tk} \geq 0$ is debt due to other households. (About lending and borrowing see below.) We will make use of the concept of cash-in hand (TW)

$$TW_{tk} = A_{tk} + I_{tk}.$$

3.2.1 Consumption plans

Households determine their consumption plans depending on their type. At each time every household belongs to one of three main types as specified below.

Prudent type Prudent households follow buffer-stock saving behavior. They have an idiosyncratic target wealth/ (permanent) labor income ratio (i.e. how much periods of labor income they want to hold as wealth), and adjust their consumption so as to achieve this target:

$$\psi_{tk} = \frac{A_{t+1,k}}{\tilde{E}_{tk}(LW_k)}.$$

Here the target wealth/labor income ratio (ψ_{tk}) is a characteristics of prudent households (about its determination see below in the subsection on Adaptation-selection), and $\tilde{E}_{tk}(LW_k)$ is "permanent" labor income of household k in period t . Permanent income is a weighted average of previously realized idiosyncratic labour income, and it can be recursively defined as

$$\tilde{E}_{tk}(LW_k) = \sigma \tilde{E}_{t-1,k}(LW_k) + (1 - \sigma)w_t L_{tk}, 0 < \sigma < 1,$$

where σ close to 1 would mean that agents forget most of their past when projecting labor income. Taking into account that $A_{t+1,k} = TW_{tk} - C_{tk}$ the consumption plan

$$C_{tk}^P = \max \left(0, TW_{tk} - \psi_{tk} \tilde{E}_{tk}(LW_k) \right)$$

ensues, where C_{tk}^P is the planned consumption of household k , in a period when household k happens to be a buffer stock saver. (Strictly speaking we should index C_{tk}^P by type, but this can be inferred from the context, since other types do not use ψ_{tk} .)

Permanent-income type Permanent-income households intend to consume capital income plus permanent labour income, thereby keeping their total (human plus non-human) wealth intact:

$$C_{tk}^P = \max \left[0, o_{tk} \left((r_t^K - \delta)K_{t,k} + r_t(B_{tk} - D_{tk}) + \tilde{E}_{tk}(LW_{tk}) \right) \right],$$

where o_{tk} is a property of permanent-income households that reflects their optimism. (Its determination is explained in the Adaptation-selection subsection below.) As the formation of expectations is not necessarily unbiased (households are unable to calculate permanent income objectively) we allow for "feelings" or self-confidence by introducing the o_{tk} parameters. Essentially a more "optimistic" household behaves as one who adjusts its permanent income estimate upwards compared to a less optimistic household.

Myopic type Myopic households focus exclusively on consumption. Each household occupies a node in an Erdős–Rényi random graph (Erdős and Rényi 1959) interpreted as determining a neighborhood relationship among agents. Myopic households observe the (last period) consumption of their neighbours (every household is its own neighbour, too), pick out the highest level, and attempt to consume as much as that, after accounting for their idiosyncratic "consumption drive" (d_{tk}), whose determination is explained again in the Adaptation-selection subsection. In formulas:

$$C_{tk}^P = d_{tk} \max(0, C_{tk}^{\max}),$$

where C_{tk}^{\max} is the maximum previous period consumption in the neighbourhood. A household with a higher d_{tk} has a stronger desire to emulate the consumption of the "most successful" among its acquaintances.

3.3 Consumption, capital accumulation and the credit market

After planning consumption each agent behaves identically, independently of its type. Essentially each household executes its consumption plan whenever own funds allow it. If not, the household tries to raise credit in order to fulfill the plan, though it may be restricted by a debt constraint.

In formulas: if

$$C_{tk}^P \leq TW_{tk}$$

then

$$C_{tk} = C_{tk}^P,$$

and the household's supply of funds ($W_{t+1,k}$) becomes

$$W_{t+1,k} = TW_{tk} - C_{tk} \geq 0,$$

while its borrowing demand is

$$D_{t+1,k} = 0.$$

Otherwise:

$$\begin{aligned} W_{t+1,k} &= 0 \\ C_{tk} &= \min(C_{tk}^P, \max(0, TW_{tk} + \overline{D}_{t+1})) , \end{aligned}$$

where \overline{D}_{t+1} is the debt constraint (see below), and r_{t+1} is the interest rate on loans maturing in period $t+1$.

In this case the borrowing demand of household k is:

$$D_{t+1,k} = \min(\overline{D}_{t+1}, C_{tk}^P - TW_{tk}) \geq 0.$$

The credit limit is set as a percentage of current labor income in the "bad" labor supply state ($L1$).

$$\overline{D}_{t+1} = \kappa w_t L1,$$

where κ is a parameter that determines the softness of the limit.

The interest rate is determined as

$$r_{t+1} = \omega_t \left(\frac{D_t}{Y_t} \right)^2 + r_t^K - \delta.$$

where $\omega_t \in (0, \bar{\omega})$ is an uniformly distributed random variable. Thus r_{t+1} is always at least as large as $r_t^K - \delta$, and the "premium" increases when the debt ratio $\left(\frac{D_t}{Y_t}\right)$ of the economy is higher, reflecting that lending is riskier than owning physical capital, and implying that to get credit people must pay a premium to those who have positive wealth, and may lend.

If the feasibility condition

$$\sum_j W_{t+1,j} \geq \sum_j D_{t+1,j}$$

is satisfied, the supply of funds is consistent with the demand for loans. In that case

$$B_{t+1,k} = W_{t+1,k} \frac{\sum_j D_{t+1,j}}{\sum_j W_{t+1,j}},$$

$$K_{t+1,k} = W_{t+1,k} - B_{t+1,k}.$$

Thus the portfolio weights with respect to credit and physical capital are the same across households, whenever a household's net supply of funds is positive. If the feasibility condition is not satisfied, the credit market collapses, and all debts are written off, but lending may recuperate in the next period.

3.4 Adaptation-selection

Agents' fitness is measured in terms of accumulated consumption as follows:

$$\begin{aligned} U_{tk} &= \lambda U_{t-1,k} + (1 - \lambda) C_{tk}, \\ 0 &< \lambda < 1. \end{aligned}$$

Thus past consumption experience is amortized at rate λ per period.

In each period there is a "small" chance (ρ_1) for any agent that a change in its type may occur. If this chance is realized, the agent examines his neighbors, and identifies the prudent, permanent-income and myopic type with the highest fitness value in each subgroup. Let these be $U_{t-1,pr(k)}^*$, $U_{t-1,pi(k)}^*$, and $U_{t-1,my(k)}^*$, respectively. Then the type of agent k becomes (τ, st) ($\tau \in \{pr, pi, my\}$), with probability

$$\Pr(\text{type}_t(k) = \tau, st) = \frac{\exp\left(\frac{U_{t-1,\tau(k)}^*}{\Gamma}\right)}{\exp\left(\frac{U_{t-1,pr(k)}^*}{\Gamma}\right) + \exp\left(\frac{U_{t-1,pi(k)}^*}{\Gamma}\right) + \exp\left(\frac{U_{t-1,my(k)}^*}{\Gamma}\right)},$$

where a very large $\Gamma > 0$ means that success is almost irrelevant, and Γ close to zero implies that there is a very high probability that the most successful type "wins". (We will call Γ sometimes "temperature" for brevity, alluding to its origin in thermodynamics. Alternatively $1/\Gamma$ can be called the strength of selection pressure.)

This two-tier formulation of the selection process can be interpreted as follows. When they have a chance to change behavior, people do not necessarily pick up the seemingly best behavioral style, as they might have other motives, independently of consumption. But when they are ready to make a choice between styles they imitate the best available within

each different style. With respect to a single-tier competition this formulation has two effects: first, inferior types have a better chance to succeed, and second, more successful subtypes are more likely to be adopted with respect to other subtypes. Thus it is made more difficult to eliminate main types, and at the same time, it is more difficult for inferior subtypes to survive.

Each household type has a subtype in one dimension: o , the degree of optimism, in the case of permanent-income households, d , the drive to imitate, for myopic households, and ψ , the target puffer parameter, for prudent households. With probability ρ_2 these subtype parameters (st , generically) can also undergo change in each period. When this mutation occurs the new subtype parameter is determined as

$$st_t = st_{t-1} \exp(\eta),$$

where η is $N(0, \sigma_\eta^2)$.

Finally, there is also a probability (ρ_3) with which the Erdős-Rényi graph describing the neighborhood relationship is rewired in a given period.

3.5 The baseline simulation setup

Table 1: Baseline parameters

Parameters	Meaning of the parameters	Parameter values
N	number of agents	200
p	probability of staying in the low labor supply state	0.75
q	probability of staying in the high labor supply state	0.98
L_1	low labour supply	0.1
L_2	high labour supply	1
α	labour's share	0.67
δ	depreciation rate	0.005
σ	memory parameter for calculating permanent income	0.95
κ	debt limit parameter	30
ρ_1	the probability of type mutation	0.0033
ρ_2	the probability of subtype mutation	0.01
ρ_3	the probability of rewiring the neighbourhood graph	0.01
dg	degree of the graph	6
λ	memory parameter in the fitness function	0.95
σ_η^2	variance of subtype mutation	0.001
ω	interest rate determination parameter	0.001

We interpret one period as roughly one month, hence the depreciation rate is set at 0.005. Under the above parameters the (non-stochastic) golden rule capital stock equals 97.007, and the corresponding aggregate consumption is 984.8. The initial expected value of the target buffer parameter is set to be consistent with the golden rule capital stock (98 periods). In the tables that follow consumption and capital are expressed as ratios of golden rule values (multiplied by 100). Thus for instance, $K = 120$ would mean 1.2 times the golden rule value of the stock of capital. In each run the initial aggregate capital stock was set equal to the golden-rule

stock, and the initial period decision rule, where memory could not play a role, was consuming according to the golden-rule. The initial value of the target buffer stock was set consistently with golden rule accumulation, and the optimism and consumption-drive parameters were set neutrally, i.e. giving them the initial value of 1.

4 Simulation results¹

4.1 Stationarity and ergodicity

In stochastic macroeconomic models stationarity (usually after some appropriate transformation of variables) is usually taken for granted. Indeed macroeconomic models normally exhibit the stronger property of ergodicity, too. However, in our model with a definite evolutionary flavor none of these properties is natural, thus we have to test them.

Ergodicity rarely arises as a problem in economics. In practical terms stationarity implies that two distant observations on the same realization of a time series are almost independent. Thus tests for stationarity are most reliable when one has long observed time series, i.e. "large" samples. In addition, ergodicity means that all realizations are alike, while the lack of ergodicity implies that different realizations may behave qualitatively differently. Empirical economists can observe one realization of a time series, and though they have trust in the "largeness" of their samples, cannot distinguish between ergodicity and stationarity, as the distinction could be detected only if multiple realizations were available. Of course, with a simulation model one has the opportunity to test for both of these properties, as generating large samples and multiple runs with the same parameter values (and initial conditions) are feasible. As one can see in Figures 1–8, the trajectories of aggregate capital strongly suggest that our model is non-ergodic for the baseline parameterization, and stationarity is also questionable by visual inspection.

¹The model's code was written in MATLAB (2007), and we made use of the routine "erdosRenyi" for generating the Erdős–Rényi graphs (Blinder 2005).



Figure 1: The time series of the aggregate capital in case of $\Gamma = 0.0001$

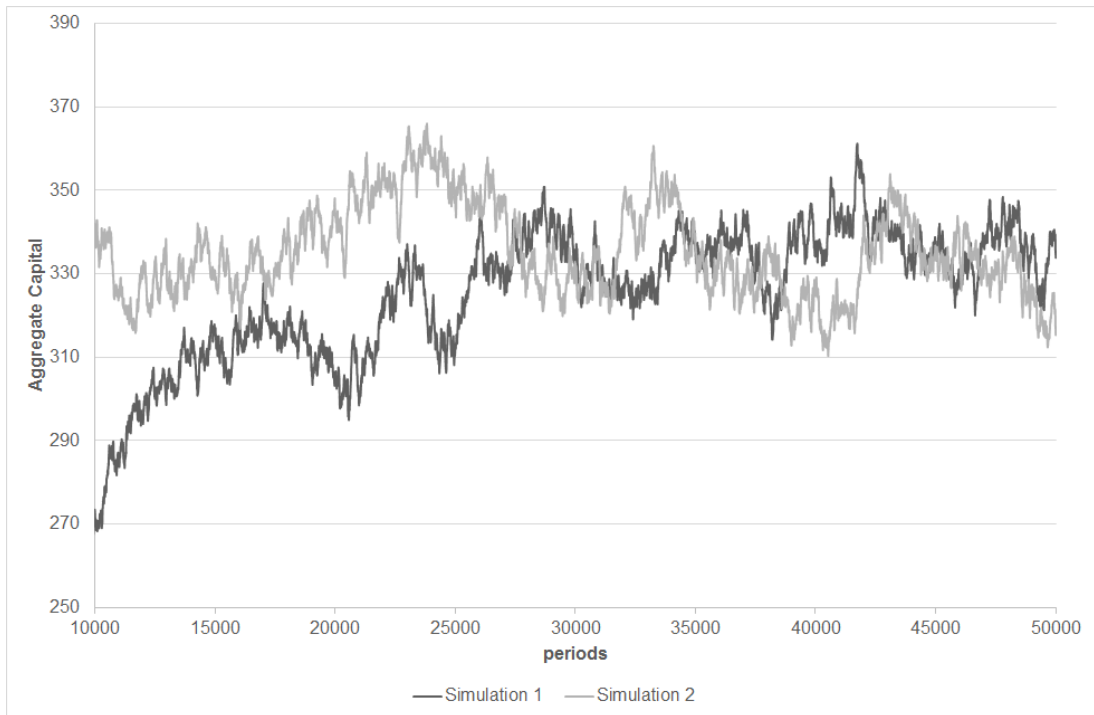


Figure 2: The time series of the aggregate capital in case of $\Gamma = 0.001$

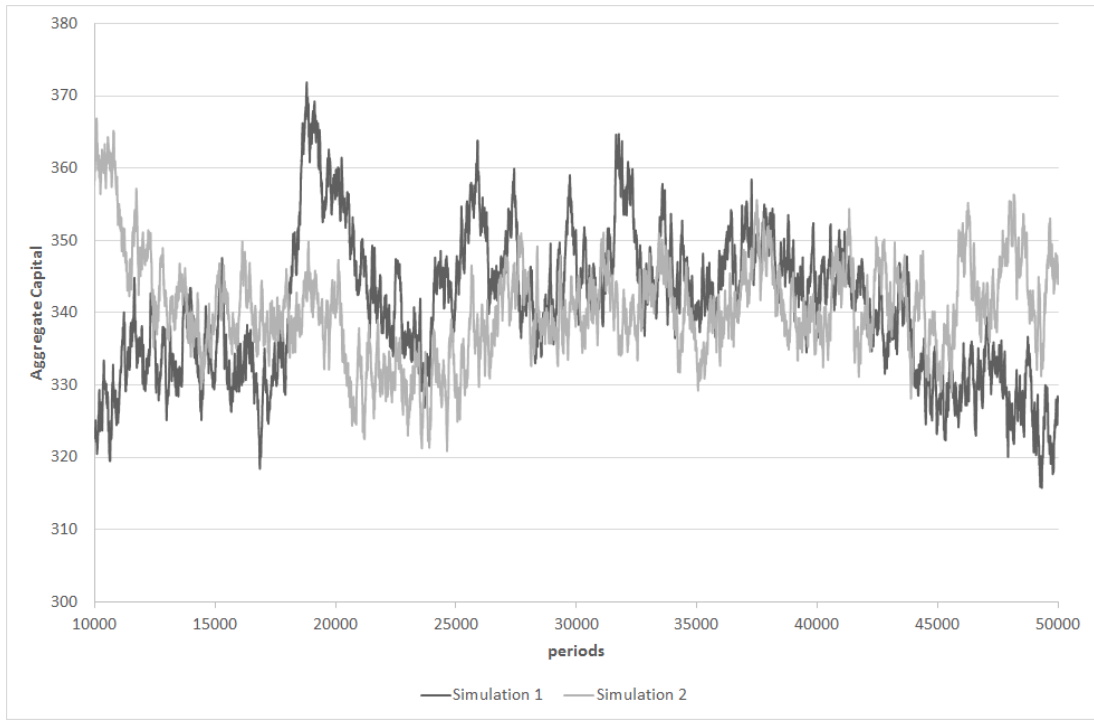


Figure 3: The time series of the aggregate capital in case of $\Gamma = 0.01$

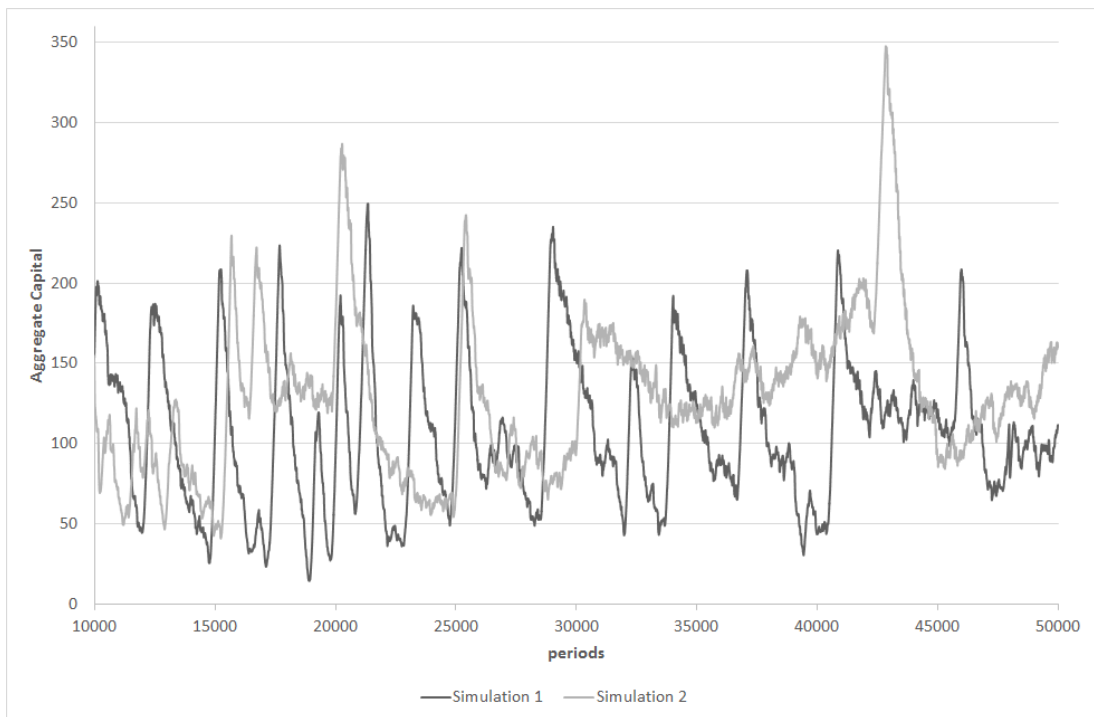


Figure 4: The time series of the aggregate capital in case of $\Gamma = 0.1$

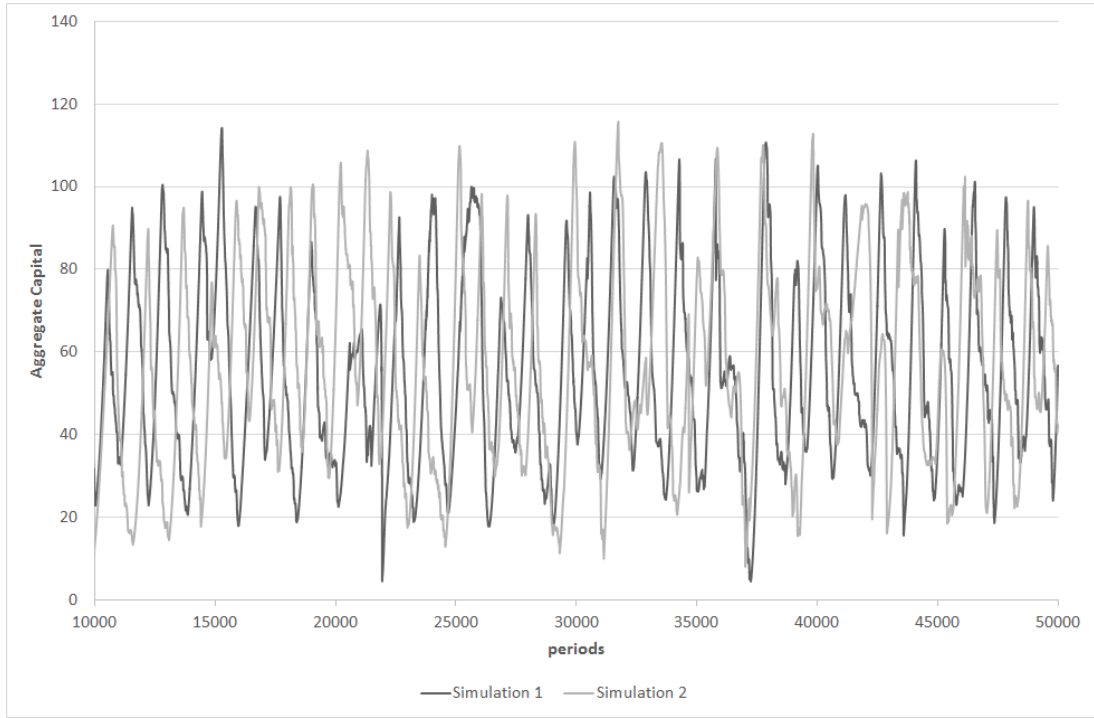


Figure 5: The time series of the aggregate capital in case of $\Gamma = 1$

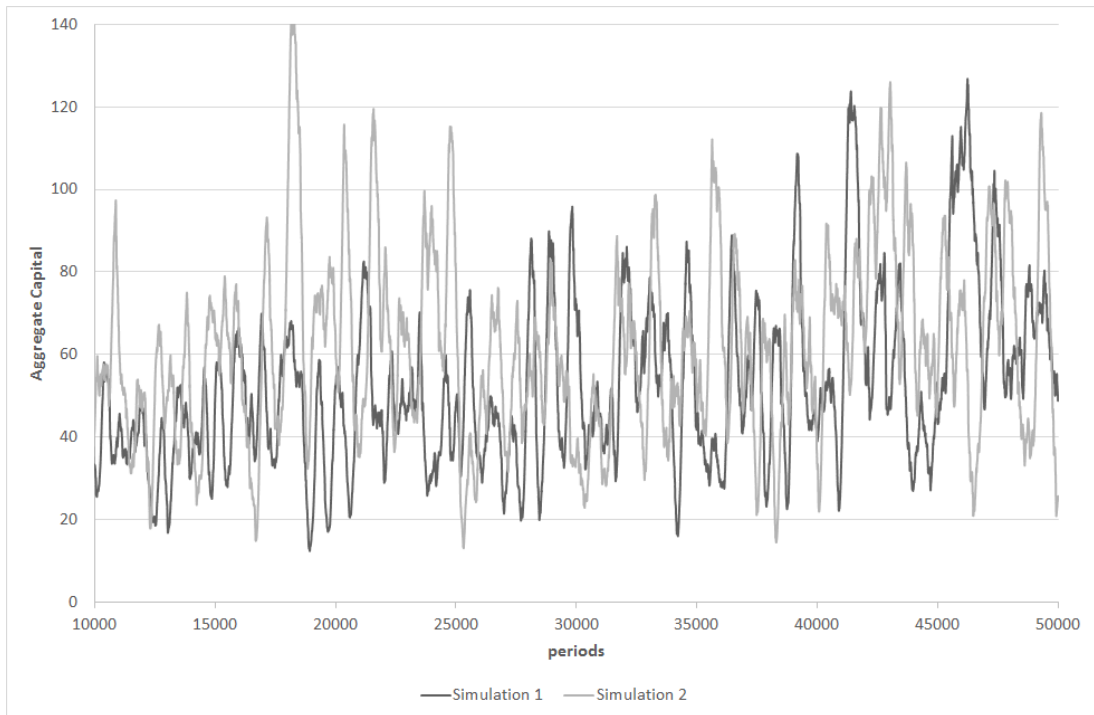


Figure 6: The time series of the aggregate capital in case of $\Gamma = 10$

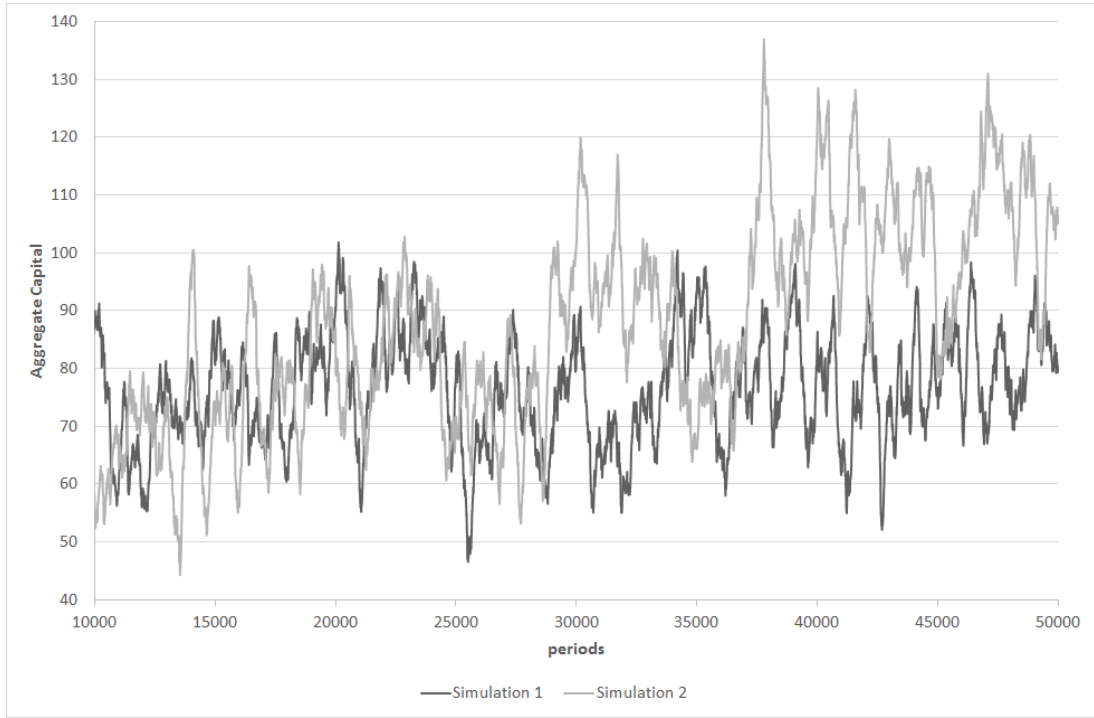


Figure 7: The time series of the aggregate capital in case of $\Gamma = 100$

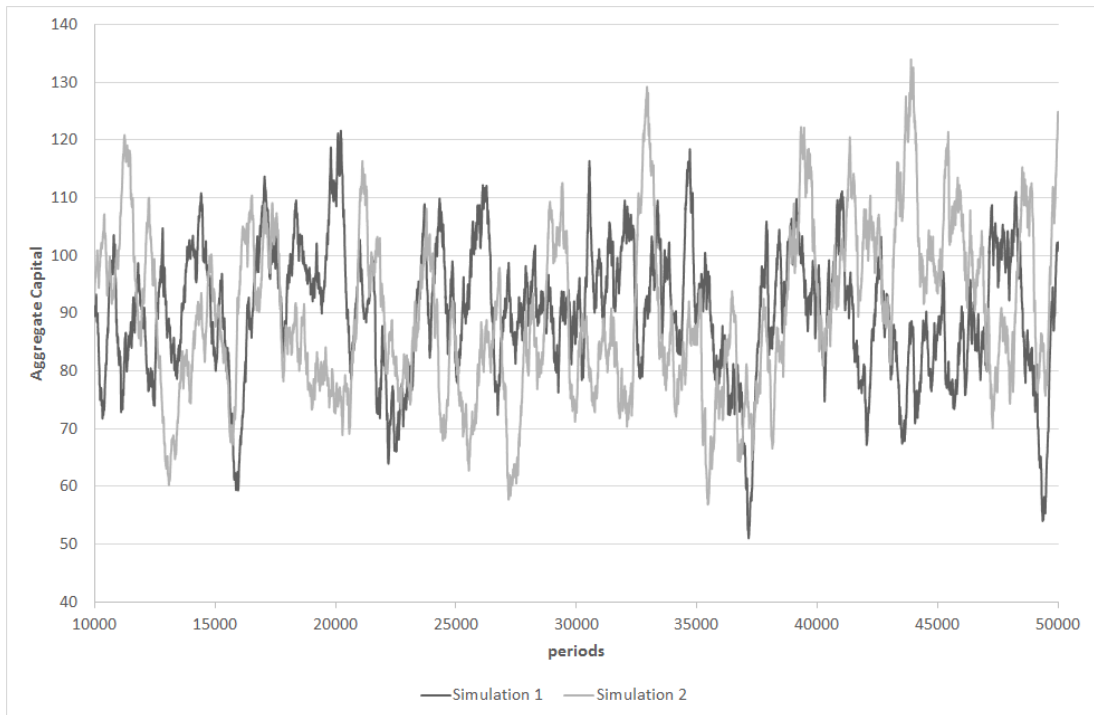


Figure 8: The time series of the aggregate capital in case of $\Gamma = 1000000$

4.1.1 Testing for stationarity and ergodicity

Parametric tests Firstly, we performed the augmented Dickey-Fuller test (the most commonly used parametric test, when the null is nonstationarity (unit-root)), on the capital stock time series with different Γ ("temperature") values (see Table 2). The optimal lag structure of the model was determined according to the Akaike information criterion. As Table 2 shows for each Γ the null hypothesis of an unit root is rejected at conventional significance levels.

Table 2: Results of the Augmented Dickey Fuller test in case of different Γ values

Γ values	0.0001	0.001	0.01	0.1	1	10	100	100000
MacKinnon p-values	0.0006	0.0000	0.0113	0.0005	0.0000	0.0000	0.0000	0.0000

Non-parametric tests As the null hypothesis of the Dickey-Fuller test is rather specific we looked for nonparametric tests, too. In any case it was necessary to conduct a test for ergodicity, and there exist fundamentally similar testing procedures applicable to both problems (see Gibbons 1985). In this paper we opted for a methodology that has been applied to agent-based models, which is an application of the extension of the Wald-Wolfowitz test (Wald and Wolfowitz 1940) by Grazzini (Grazzini 2012).

The Wald-Wolfowitz ("Runs") test checks whether two samples come from the same distribution. Its extension tests whether a function fits well some set of observations, by noticing that the observations should be randomly distributed above and below the function, regardless of the distribution of errors. Given the estimated function, 1 is assigned to the observations above the fitted line, and 0 to the observations below the fitted line. The statistics used to test the null hypothesis is the number of runs, where a run is defined as "a succession of one or more identical symbols which are followed and preceded by a different symbol or no symbol at all" (Gibbons 1985). The number of runs, too many or too few runs, may reflect the existence of non-randomness in the sequence. The null-hypothesis is that a given set of observations is randomly distributed around a given function.

In order to check whether the mean is stationary, a model was simulated for 50,500 periods, and this long series was divided into 100 windows, after dropping the first 500 observations. For each window the mean was computed, and it was checked whether the means of the subsamples were above or below the mean of the whole time series (the function whose "fit" we tested) by performing the Runs Test in its form with a two-sided alternative (see Grazzini 2012). Had the sub-sample means been randomly distributed around the overall mean, we would have concluded that the hypothesis of stationarity for the first moment cannot be rejected. As for all Γ s the p-values were practically zeros, the null hypothesis of stationarity was flatly rejected. Thus the parametric test (with nonstationarity as the null), and the non-parametric tests (with stationarity as the null) provided contradictory answers.

To test the ergodic property, the Runs Test was used again, but in its original form, proposed by Wald and Wolfowitz (1940), as suggested by Grazzini. The first steps of the ergodicity test are similar to the stationarity test: again a 50,500 periods long time series was simulated, then divided into 100 windows, and the mean of each window was computed. The first sample of the test is formed by the means of the 100 sub-samples. As a second step, 100 time series of 1500-period length were generated with different random seeds, and the means were computed for

each one, after dropping the first 500 values. The second sample for the ergodicity test consisted of these 100 sample means. We merged them, and created a set which sorts the elements of these two sets into one of ascending order. Then the Runs Test was applied, where under the null hypothesis (ergodicity) the two samples had the same mean (see Grazzini 2012). For every value of Γ the p-values were again very close to zero, so the null hypothesis of ergodicity was in each case definitely rejected.

4.1.2 Interpretation

What is the content of non-ergodicity? Ergodic processes are such that different realizations of the process are essentially the same, thus one can discover the properties of a process from a single (long enough) realization as well as from an infinity (in practice a very large number) of different realizations. Stationarity but non-ergodicity means that we have to carry out many simulations with the same parameters, and computing statistics therefrom (ensemble averaging) to estimate the characteristics of the time series. In principle this averaging is appropriate only when stationarity prevails, but inappropriate when even stationarity is refuted. As we have seen we can have serious doubts even with respect to stationarity. Does it mean that calculating ensemble averages would be meaningless? We believe that it is not necessarily the case, and Table 3, summarizing our results for the baseline parameterization, seems to justify this claim. In this table we report the mean, the minimum and maximum of the average capital stock for different values of the "temperature" parameter. It can be seen that the figures are not totally arbitrary, there are clear differences attributable to varying selection pressure. This statement is true not only for the mean but also for the ranges. Therefore we feel vindicated that in the following tables like Table 3 are reported, and conclusions are drawn from them.

Table 3: Mean, minimum and maximum of the average aggregate capital in simulations with different Γ values (as a percentage of the golden-rule capital)

Γ	mean	min	max
0.0001	241.80	117.57	392.04
0.001	263.71	142.99	349.52
0.01	239.00	68.32	411.28
0.1	87.93	51.55	346.24
1	58.85	0.04	101.75
10	48.99	30.63	64.31
100	111.04	84.02	146.22
1000000	106.37	60.66	149.24

4.2 Main findings

For any given parameter setting we ran 20 simulations, in each case for 5000 periods. For every run we dropped the first 3000 periods to eliminate any effects stemming from the assumptions that we used for starting the simulation, and calculated all statistics on the basis of the last 2000 simulated "observations" only.

Tables 3 and 4 present statistics for the most important macrovariables: aggregate capital stock and aggregate consumption. The crucial observations concern the dependence of the

capital stock on Γ (Reminder: low gamma amounts to high selection pressure.) One can see that very strong selection pressure tends to result in substantial overaccumulation, where the capital stock is, on average, much higher than its golden rule value. Though there are significant differences across runs (the symptom of non-ergodicity or non-stationarity) the ensemble average of capital is close to the golden rule value, when selection pressure across types is very low (high Γ). There seems to be an interesting non-linearity: at medium levels of Γ there tends to be underaccumulation of capital, and the transition from under- to over-accumulation is not smooth, wildly different average capital stocks are realized for Γ s in that parameter range.

Table 4: Mean, minimum and maximum of the average aggregate consumption in simulations with different Γ values (as a percentage of the golden-rule capital)

Γ	mean	min	max
0.0001	77.38	40.57	98.80
0.001	73.56	53.32	96.90
0.01	77.62	34.51	97.77
0.1	90.82	53.44	104.57
1	85.47	5.16	98.09
10	92.85	85.59	98.15
100	99.34	97.02	101.03
1000000	99.13	95.75	101.24

The aggregate consumption figures in Table 4, reflect, of course, the evolution of the capital stock. The lesser degree of volatility is due to the concavity and non-monotonicity of the relationship between capital and consumption. One additional thing that can be observed here is that in terms of consumption, over-accumulation is more harmful than under-accumulation.

These findings may constitute a puzzle. Naive evolutionists would have believed that letting evolutionary forces establish "preferences" of individuals would conduce to a socially "optimal" state, as the golden rule consumption is certainly very close to our intuition about social optimality. (Environmental concerns cannot play a role here, as there are no non-reproducible resources in the model.) To understand why increasing selection pressure may cause harm to society we have to look at the fate of the different types. In Tables 5–7 statistics for buffer-stock savers, myopics and permanent-income households are reported, respectively.

Table 5: Average proportion, average share in aggregate consumption and average buffer stock of the *prudent* type: mean, minimum and maximum of the mean in simulations with different Γ values

Γ	proportion			share in consumption			optimism		
	mean	min.	max.	mean	min.	max.	mean	min.	max.
0.0001	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	200.10	110.97	293.40
0.001	1.0000	0.9999	1.0000	1.0000	0.9996	1.0000	203.17	127.08	260.74
0.01	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	201.93	76.47	311.33
0.1	0.3052	0.1344	0.9980	0.2233	0.1078	0.9927	374.76	259.48	474.91
1	0.2164	0.0000	0.3931	0.1500	0.0000	0.3772	405.81	13.98	602.99
10	0.2304	0.1720	0.3313	0.2277	0.1510	0.3241	300.96	175.79	360.30
100	0.3364	0.3033	0.3672	0.1642	0.1044	0.2403	356.84	242.95	446.27
1000000	0.3315	0.3037	0.3640	0.1796	0.0846	0.2811	326.11	212.96	450.18

Table 6: Average proportion, average share in aggregate consumption and average optimism of the *permanent-income* type: mean, minimum and maximum of the mean in simulations with different Γ values

Γ	proportion			share in consumption			optimism		
	mean	min.	max.	mean	min.	max.	mean	min.	max.
0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-	-	-
0.001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-	-	-
0.01	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-	-	-
0.1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-	-	-
1	0.2614	0.0000	1.0000	0.2620	0.0000	1.0000	1.0440	0.9277	1.1265
10	0.0906	0.0451	0.1792	0.0925	0.0478	0.1678	1.0271	0.8541	1.1219
100	0.2889	0.2586	0.3268	0.3349	0.2991	0.3745	1.3335	1.1848	1.5115
1000000	0.3350	0.3103	0.3614	0.3786	0.3494	0.4062	1.2778	1.1193	1.4753

Table 7: Average proportion, average share in aggregate consumption and average consumption drive of the *myopic* type: mean, minimum and maximum of the mean in simulations with different Γ values

Γ	proportion			share in consumption			drive		
	mean	min.	max.	mean	min.	max.	mean	min.	max.
0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-	-	-
0.001	0.0000	0.0000	0.0001	0.0000	0.0000	0.0003	-	-	0.9288
0.01	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-	-	-
0.1	0.6948	0.0020	0.8656	0.7767	0.0073	0.8922	1.0105	0.8930	1.1044
1	0.5222	0.0000	0.8835	0.5880	0.0000	0.8974	1.0242	0.9516	1.1220
10	0.6789	0.5274	0.7429	0.6798	0.5081	0.7442	1.0243	0.9198	1.0913
100	0.3746	0.3349	0.4282	0.5009	0.4123	0.5917	1.0093	0.9166	1.2068
1000000	0.3335	0.3136	0.3627	0.4418	0.3582	0.5128	1.0325	0.8944	1.1550

Columns (1) of these tables provide a proximate explanation. At very low Γ s buffer stock savers (prudent agents) dominate, but as the evolutionary pressure is reduced, myopics survive in more and more substantial numbers, and finally, at low selection pressure, the distribution of the three types, as expected, becomes roughly equal in the long term. These observations suggest a rather clear viability ranking: 1. prudent, 2. myopic, 3. permanent-income. The proximate "explanation" of the puzzle seems to work like this: when selection pressure is high prudent households drive out myopic and more risk tolerant agents, but they behave with excessive caution, making harm to each other eventually.

We can gain further insight if we look at Column (3) in Table 5. It appears that the average buffer is lower the more buffer-stock savers dominate. Thus they are more cautious when have "rivals" among other types than when they compete only among themselves at high selection pressure. With low selection pressure not only target capital buffers get higher, but they become rather diffuse (see the large range for high Γ in Column (3) Table 5). Inspecting column (2) in Table 6-7 shows a curious situation: whenever other types survive they consume (per capita) more than prudent savers, making the latter *de facto* benefactors of others, at their own expense.

From the point of view of evaluating our approach to "endogenous preferences" it is important to notice that the auxiliary parameters of the permanent-income and myopic types behave reasonably. Column (4) in Tables 6-7 shows the outcomes with respect to these. Concerning permanent-income households their optimism is definitely higher when the selection pressure across types is low. Quite "rationally" they tend to exploit prudent households by forming more optimistic pictures of the future, and therefore increasing consumption. On the other hand the consumption-drive of myopics is almost non-biased, irrespective of selection pressure.

4.2.1 Robustness

Our principal concern was to provide an evolutionary alternative to "utility function" modelling in one of the basic areas of macroeconomics, and see what this alternative approach offers for the analysis of long-run capital accumulation. It is clearly imperative to check whether variations in the proxy utility parameters affect the qualitative results or not. See below the list of

these parameters:

Table 8: List of parameters in the robustness check

σ	memory parameter for calculating permanent income
ρ_1	the probability of type mutation
ρ_2	the probability of subtype mutation
ρ_3	the probability of rewiring the neighbourhood graph
dg	degree of the graph
λ	memory parameter in the fitness function
σ_η^2	variance of subtype mutation

We conducted simulations with small deviations from the baseline values of these parameters, and practically no change in the qualitative behaviour of the model could be detected. Thus we can say that the model's results are not singular.

Also, we wanted to ensure that changes in credit market parameters do not unduly or unreasonably affect the results, since the modelling of the credit market is rather arbitrary. For these parameters we experimented with larger changes as well. It turned out that only unreasonably high credit limits and interest rate premia have significant impact on the model, mostly by making the trajectories more "chaotic".

5 Conclusions

In this study we addressed two interrelated problems: 1. can we set up a macromodel focusing on saving without the assumption of utility maximization, but preserving certain "rational" aspects of human decision making?, 2. in this framework can we derive something interesting about long-run capital accumulation? Into a quite traditional production-primary income distribution framework we introduced consumption-types as embodiments of saving-functions akin to saving-functions that have been popular in the traditional literature, and whose characteristics have been given some confirmatory evidence. These saving-functions evolve endogenously in our model due to evolutionary forces (selection, mutation). We have argued throughout this paper that this model is a possible solution to the first problem, though in no case the only one. We think that the stochastic properties of our model (non-ergodicity) are in its favour, despite the fact that these properties traditionally disqualify a macroeconomic model. It is obvious also that without a positive answer to the second problem this may turn out to be a rather shallow solution.

We believe that the model has provided us with interesting insights about capital accumulation. It tells us that "ecologically rational" risk-aversion may turn out to be exaggerated from the social point of view, and it happens exactly when the fight for survival is intense. It means that lowering evolutionary pressure, resulting in more diversity in saver types, may be socially beneficial. Crickets may be, unintentionally, useful for society as a whole, including for prudent and cautious ants.

Upon receiving this message many may raise their eyebrows. Especially when it is coupled with another corollary of our model: the wealthy may let themselves exploited in a sense, the fruits of their thrift are reaped by overoptimistic and myopic persons. This does not seem to

be the real world at all, and it may be a formidable argument against the model. However one must notice two or three things. First, in this model there is no lazy agent, everybody works and wants to work. Capital accumulation helps the worker, indeed excessive accumulation helps only the worker, and this is partly behind the perverse exploitation phenomenon. Second, this model is clearly an abstract one, it does not aim to draw a map of any existing economy. (Not unlike many very useful models in the neoclassical tradition whose sole goal was generating insights.) Third, if we extend our imagination back in history and today globally, we can find cases where one part of humankind behave cricket-like, while another part seem to emulate ants. Sixteenth-seventeenth century Spain versus the Low Countries and England may come to one's mind as an obvious example, but early twenty-first century United States and China can pass for another example. In the expression of Kindleberger (1990) there have always been low and high "absorbers" (of savings), and is it such an outrageous assumption that one cannot exist without the other? The fact that in the past low absorbers tended to impoverish does not mean that low absorbers as a group do not consume more than the high absorbers in the long run.

Coming down to earth we have another—more practical—corollary. One motive for undertaking this study was that agent-based macromodels have failed to reserve a place for endogenously selected saving-rules, to allow for "ecological rationality" the role it plays in our decisions. For those who are interested in this type of research our results suggest that at least two types (buffer stock and myopic) of savers are needed in a model. Heterogeneity must be looked for within those types, and there are promising ways to model this heterogeneity endogenously, making use of the idea of ecological rationality. Though our specific manner of doing this will and should not be the final solution, we feel that perhaps we have made a step in the right direction.

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