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Cost-optimal model predictive scheduling of home appliances ⋆

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Abstract: A novel heuristic model-based optimal scheduling algorithm is proposed in this paper to operate heating and cooling type home appliances connected to smart grids where the price of the electrical energy is known in advance and temperature constraints are present.
The properties and the use of the proposed algorithm are shown using a simple refrigerator model. The accuracy and the computational properties of the proposed method are compared to the schedule generated by the MPT toolbox. The algorithm works well with a relatively short prediction horizon using a fraction of the computing time needed for the MPT-based method. The robustness of the algorithm is also investigated with respect to the load of the refrigerator. It is shown that the designed schedule with an empty refrigerator is always suitable, yet not optimal, for any loaded case.

Keywords: Smart grids; Demand side management; Model predictive control; Heuristics; Scheduling algorithms;

1. INTRODUCTION

Nowadays, electrical energy providers and line operators, and also the electrical appliances themselves are providing more and more smart solutions with economical, technical and environmental goals, that facilitates the development of smart grid technologies and solutions both on the demand and on the supplier sides. An important influencing factor of this development is the day-ahead electricity market, that is continuously expanding, and the amount of energy being traded through them is increasing. Therefore, the cost-optimal operation of the composite system consisting of suppliers, consumers and the electrical grid presents a wide variety of operating, scheduling and control problems.

From the suppliers side, the approaches of optimized pricing (Joe-Wong et al. (2012)) are of great interest that aim at balancing the electrical grid subject to variations in the supply (e.g. caused by the changing availability of renewable energy sources), and also in the demand. As a result of optimized pricing, hourly changing electrical energy prices are available for the day-ahead electricity market (see e.g. Spot (2010)).

From the side of demand management, one may optimally operate certain electrical appliances with controllable on/off switching taking into account the dynamically changing electrical energy prices and the operating constraints. In the simplest case this problem leads to an optimal scheduling one, for which nice solutions have been proposed in the literature. An optimal day ahead microgrid scheduling method for an office building considering weather scenarios is developed by Shimomachi et al. (2014), while an optimal residential load control method with price prediction is reported in the paper by Mohsenian-Rad and Leon-Garcia (2010). Household appliances can also be a subject of optimal operation or scheduling, see e.g. the paper of Du and Lu (2011).

An important, yet relatively simple class of household appliances are the heating/cooling devices, such as refrigerators, boilers, etc. Their optimal operation is also widely investigated under various circumstances, see the recent PhD thesis of Vinther (2014) and the references therein. The subject of this paper is the cost-optimal operation of a cooling household appliance, a refrigerator. Although food safety is a primary concern, it is aways possible to operate the refrigerator in a cost-optimal way which is in compliance with the safety temperature limits. Considering its simple dynamic model with the input of the electrical switch taking into account the known but hourly changing prices of electrical energy and temperature constraints, one can formulate a model-predictive control problem with linear piecewise affine model for designing an optimal schedule. In our earlier work (Bálint and Magyar (2016)) we used the MPT toolbox (see in the paper of Herceg et al. (2013)) to solve this optimal scheduling problem, that turned out to be computationally too demanding for this simple task. Therefore, the aim of this work is to propose an improved version of the optimal scheduling algorithm using heuristics based on the physics of the system.

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2. PROBLEM STATEMENT

An important possible service of a smart electrical grid is to operate our household appliances in a cost-optimal way. This can be achieved not only by improving the energy efficiency of the appliances, but also to optimally schedule their time of operation taking into account the price of electrical energy.

2.1 Day-ahead market

In the modern power grid the day-ahead market serves as the marketplace for trading power. The service provider gives the electricity price, i.e. the price for electrical energy, for the next 24 hours. Fig. 1 shows the hourly electricity prices for a week, where each line corresponds to the prices of a day. As it is apparent in Fig. 1, the energy price of

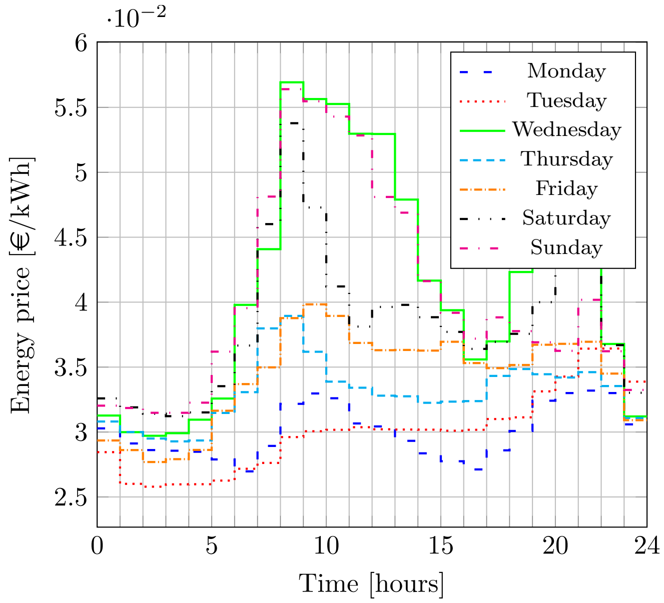

Fig. 1. Electricity price of a day-ahead market for a week. Source: Spot (2010)

the same period for different days is highly fluctuating, so the price-optimal operation of home appliances in a day-ahead market can be formulated as an optimal scheduling problem (see in Joe-Wong et al. (2012)). The real data used for the simulations are obtained from the report of Spot (2010).

2.2 Piecewise affine modelling of heating and cooling appliances

As an example of a cooling appliance, consider a refrigerator that is cooled by a cooling liquid circuit driven by an electrical motor. The schematic picture of the main elements of the refrigerator is shown in Fig. 2.

The containment is characterized by its air temperature T_a. It is heated by the outer environment (its temperature is T_o) through the door of the fridge, and cooled by the wall with temperature T_w . A liquid cooling system with liquid temperature T_c provides cooling when the cooling binary switch S is on, i.e. $S = 1$, while there is no cooling of the wall when $S = 0$. The wall is also heated by the outer environment.

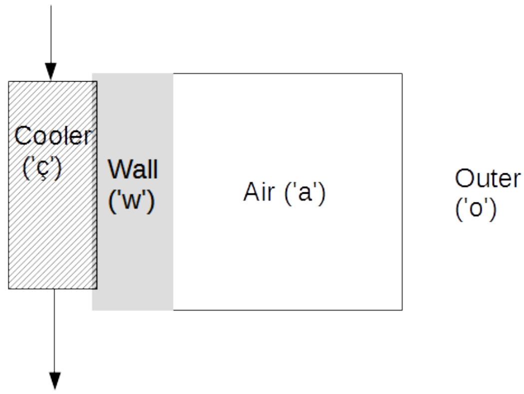

Fig. 2. The schematic picture of the refrigerator

The engineering model The simplest possible dynamic model that describes the dynamics of the refrigerator can be constructed from the energy balances for the containment air and that of the wall in the following form (see Hangos and Cameron (2001))

$$C_a \frac{dT_a}{dt} = K_w(T_w - T_a) + K_o(T_o - T_a) \quad (1)$$

$$C_w \frac{dT_w}{dt} = K_w(T_a - T_w) + K_x(T_o - T_w) + \\ + S \cdot K_c(T_c - T_w) \quad (2)$$

where T_a is the containment air temperature, and T_w is the wall temperature. The first terms in the right-hand sides of the equations correspond to the heat transfer between the air and wall, the second transfer terms correspond to the transfer between the outer environment and the air or wall, respectively, and the last term in the second equation describes the effect of the cooling liquid. The constant positive parameters of the model are C_w and C_a being the heat capacities of the containment air and the wall, respectively, and K_w, K_o K_c and K_x are the heat transfer coefficients for the air-wall, air-environment, wall-cooling liquid, and wall-environment transfers, respectively. The outer environment temperature T_o and the cooling liquid temperature T_c are assumed to be constant.

The state and input variables Now we can identify the state and input variables of the dynamic model as follows.

$$\boldsymbol{x} = \begin{bmatrix} T_a \\ T_w \end{bmatrix} \quad , \quad \boldsymbol{u} = S \quad (3)$$

where S is the position of the switch.

Piecewise affine model Let us define two operating modes of the refrigerator: the cooling and the reheating modes. In both cases the state space model is in the standard affine model form

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\,\boldsymbol{x} + \boldsymbol{B}\,\boldsymbol{u} + \boldsymbol{f} \quad (4)$$

$$\boldsymbol{y} = \boldsymbol{C}\,\boldsymbol{x} \quad (5)$$

but the value of the coefficient matrices $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}$ and the constant vector $\boldsymbol{f}$ differ.

Cooling dynamics The first case is when the switch is closed ($S = 1$), i.e. the refrigerator is cooling. Then the parameter matrices and vector are

$$\boldsymbol{A_{on}} = \begin{bmatrix} -\dfrac{K_w + K_o}{C_a} & \dfrac{K_w}{C_a} \\ \dfrac{K_w}{C_w} & -(\dfrac{K_w}{C_w} + \dfrac{K_c}{C_w} + \dfrac{K_x}{C_w}) \end{bmatrix} \quad (6)$$

$$\boldsymbol{B_{on}} = \begin{bmatrix} 0 \\ \dfrac{T_c K_c}{C_w} \end{bmatrix} \ , \ \boldsymbol{f_{on}} = \begin{bmatrix} \dfrac{K_o T_o}{C_a} \\ \dfrac{K_x T_o}{C_w} \end{bmatrix} \ , \ \boldsymbol{C} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \ . \quad (7)$$

Reheating dynamics The second case is when the switch is open ($S = 0$), i.e. the refrigerator is reheated to the environmental temperature. Then the parameter matrices and vector are as follows:

$$\boldsymbol{A_{off}} = \begin{bmatrix} -\dfrac{K_w + K_o}{C_a} & \dfrac{K_w}{C_a} \\ \dfrac{K_w}{C_w} & -(\dfrac{K_w}{C_w} + \dfrac{K_x}{C_w}) \end{bmatrix} \quad (8)$$

$$\boldsymbol{B_{off}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \ , \ \boldsymbol{f_{off}} = \begin{bmatrix} \dfrac{K_o T_o}{C_a} \\ \dfrac{K_x T_o}{C_w} \end{bmatrix} \ , \ \boldsymbol{C} \ = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \ . \quad (9)$$

2.3 Control aim

Given the dynamically changing but known-ahead price for electrical energy, the aim of optimally operating the refrigerator described above in subsection 2.2 can be formulated as a constrained optimization problem using the following assumptions:

- The operating cost is the cost of the electrical energy consumption of the refrigerator during the day.
- The price of the electrical energy $p(t)$ changes hourly in a piece-wise constant way.
- The energy price function is known for 24 hours in advance.
- The temperatures in the refrigerator must be between the following operating constraints
  - · the inner air temperature T_a should be between $T_{a,min}$ and $T_{a,max}$,
  - · the cooled back wall temperature T_w should be between $T_{w,min}$ and $T_{w,max}$
- The outer air temperature T_o is constant.
- The input variable is then the value of the switch S that is binary (on: $S = 1$, off: $S = 0$).
- The simple piecewise affine model with its cooling and reheating models described in subsection 2.2 is considered.

The aim of the control is to minimize the operating cost, that is in the following general form

$$\int_{\tau=0}^{24h} (p(\tau) \cdot S(\tau)) d\tau \ . \quad (10)$$

3. MODEL PREDICTIVE SCHEDULING

Although the system dynamics and the control aim is given in continuous-time, any implementation of the control system will work in discrete-time, so the piecewise affine model (6-9) will be discretized with a suitable sampling time.

3.1 MPC problem formulation

In order to be able to apply the tools of model predictive control theory, the model and the control aim of sections 2.2 and 2.3 is to be reformulated in the frame of a model predictive control (scheduling) problem as follows.

System model As a first step the continuous-time model (6-9) is discretized with sampling time h in order to get the discrete-time piecewise affine (PWA) system model used in the sequel. Based on the preliminary experiments and knowledge about the system dynamics to be controlled, $h = 5\,\text{min}$ will be used.

$$\Sigma_i : \begin{cases} \boldsymbol{x}_{k+1} = \boldsymbol{\Phi_i}\, \boldsymbol{x}_k + \boldsymbol{\Gamma_i}\, u_k + h \boldsymbol{f}_i \\ \boldsymbol{y}_k = \boldsymbol{C}\, \boldsymbol{x}_k \end{cases} \ , \ i \in \{on; off\} \quad (11)$$

where $\boldsymbol{x}_k$ stands for the value of the vector valued signal $\boldsymbol{x}$ at the discrete time instant k, matrices $\boldsymbol{\Phi} = \boldsymbol{e^{A\,T}}$ and $\boldsymbol{\Gamma} = \boldsymbol{A}^{-1}(\boldsymbol{e^{A\,T}} - \boldsymbol{I})\boldsymbol{B}$ are the state- and input matrices of the state equation discretized by sampling time h, and $\boldsymbol{f}$ is the constant vector in the continuous time model.

Cost function The cost function (10) is approximated with the discrete sum (12). Although the sampling time h is kept considerably smaller than one hour (i.e. the sampling time of the price) the values of (10) and (12) may be (and usually are) different so the discrete-time implementation is suboptimal with respect to the cost function (10)

$$cost = \sum_{j=1}^{N} p_j\, u_j\, h \ , \quad (12)$$

where N is supposed to be the prediction horizon size. It is supposed that the price levels of the next day are known at least $H = N\,h$ time (prediction time) before midnight.

Constraints The state constraints given in section 2.3 can be used directly in the MPC framework.

$$\underline{\boldsymbol{x}} \le \boldsymbol{x}_k \le \overline{\boldsymbol{x}} \quad (13)$$

where the lower- and upper bounds of the state variables are evaluated component-wise. In notation (13), the bounds are

$$\underline{\boldsymbol{x}} = \begin{bmatrix} T_{a,min} \\ T_{w,min} \end{bmatrix}, \quad \overline{\boldsymbol{x}} = \begin{bmatrix} T_{a,max} \\ T_{w,max} \end{bmatrix} \quad (14)$$

Summarized, the MPC problem is to minimize (12) in u_k with respect to (11) and the constraints (13). In each iteration the optimization of the cost (12) is performed from the actual time to a fixed size prediction horizon $H = N\,h$, and the first element of the optimizing input sequence is applied to the real system.

3.2 Heuristic optimal scheduling algorithm

From the modelling and the problem formulation given in the previous sections it is clear, that the optimization problem is a model predictive optimal scheduling problem, where the cost function depends on the time varying energy price, $p(t)$, or its discrete time counterpart p_k.

The proposed algorithm is a version of branch and bound type optimization where the branch step introduces possible switching sequences and the bound step decreases the size of the solution space based on the following three heuristic rules:

Rule 1: Any scheduling sequence that yields an $\boldsymbol{x}$ breaking the bounds (13) is not allowed.
Rule 2: Any scheduling sequence that yields a higher $\boldsymbol{x}$ at a higher cost is not optimal.
Rule 3: Any scheduling sequence containing a cooling step that could have been performed later for a lower energy price is not optimal.

The first two rules are easy to check, the third one can be checked by the following analysis step. As the hourly energy prices are known 24 hours ahead, it is possible to calculate a price-equivalent cooling time t_i^p for all subsequent constant-price periods of the day. For the i^{th} hour, it is calculated as

$$t_i^p = \left\lceil \frac{p_{i+1}}{p_i} h \right\rceil, \quad i = 1, \ldots, 24 \tag{15}$$

where p_i is the price for the i^{th} hour and $\lceil . \rceil$ represents the ceiling function.

Using these price-equivalent cooling times, off-line dynamical simulations are made for all price periods using the model (11) and $\overline{\boldsymbol{x}}$ as the initial state with the simulation time t_i^p for all periods, respectively. The final states $\hat{\boldsymbol{x}}_{\boldsymbol{i}}$ can be used as the reference values of the comparison step **Rule 3'**: If the actual state $\boldsymbol{x} < \hat{\boldsymbol{x}}_i$ during the i^{th} price period then switching the cooling on yields a suboptimal sequence.

It is important to note, that the calculation of t_i^p and $\hat{\boldsymbol{x}}_i$ can be calculated off-line, once a day, preferably when the service provider gives the prices of the next day.

The pseudocode of the proposed optimal scheduling algorithm is given in Algorithm 1. A simulation based analysis of the algorithm working as the optimizer of a model predictive scheduling control system is given in section 4.

Algorithm 1 Heuristic scheduling algorithm

procedure HEURISTIC B&B
Input:
  $\Sigma \leftarrow \Sigma_{on}, \Sigma_{off}$
  $\boldsymbol{x}$ actual state
  $\underline{\boldsymbol{x}}, \overline{\boldsymbol{x}}$ bound
  $\hat{\boldsymbol{x}}$ bound (Rule 3)
  p electricity prices
  $N \leftarrow$ horizon size
Initialization:
  cost empty column vector
  U, X empty matrices
  for $i = 0 : 1 : N$ **do**
branch:

$$U = \left[\begin{array}{c|c} U & \begin{array}{c}1\\ \vdots\\ 1\end{array} \\ \hline U & \begin{array}{c}0\\ \vdots\\ 0\end{array}\end{array}\right], \quad X = \left[\frac{X}{X}\right], \quad cost = \left[\frac{cost}{cost}\right]$$

    for $k = 1 : \mathbf{rows}(U)$ **do**
      $X_{k,i+1} = \Sigma(X_{k,i}, U_{k,i})$
      update $cost_k$
bound (Rule 1):
      if $X_{k,i+1} \notin [\underline{\boldsymbol{x}}, \overline{\boldsymbol{x}}]$ **then**
        delete row $X_{k,.}$, $U_{k,.}$ and $cost_k$
      end if
bound (Rule 3):
      if $X_{k,i+1} < \hat{\boldsymbol{x}}_k$ and $U_{k,i+1} = 1$ **then**
        delete row $X_{k,.}$, $U_{k,.}$ and $cost_k$
      end if
    end for
bound (Rule 2):
    for $k, l = 1 : \mathbf{rows}(U)$, $k \neq l$ **do**
      if $X_{k,i+1} > X_{l,i+1}$ **and** $cost_k > cost_l$ **then**
        delete row $X_{k,.}$, $U_{k,.}$, $cost_k$
      else
        if $X_{l,i+1} > X_{k,i+1}$ **and** $cost_l > cost_k$ **then**
          delete row $X_{l,.}$, $U_{l,.}$, $cost_l$
        end if
      end if
    end for
  end for
optimal solution:
  minimal value of $cost = cost_{k_{opt}}$
  Minimizing sequence $U_{k_{opt},.}$
end procedure

4. CASE STUDY

In order to verify the proposed optimization based scheduling algorithm, different simulation experiments are performed. The parameter values of the model used in the experiments were obtained from Schné et al. (2014) and are given in Table A.1.

In order to be able to make comparisons with respect to optimization time, the following notation is used. t_{opt} denotes the time spent for calculating the actual input, t_{opt}^{day} denotes the sum of the t_{opt} values for a whole day. All the experiments are simulated using a modern PC with the following parameters

- Intel Pentium B970 CPU (2x2.30 GHz)
- 6 GB RAM
- Windows 10 Home 64bit
- Matlab r2014

4.1 Model predictive scheduling of a refrigerator

In the first experiment setup the refrigerator model used by the model predictive scheduling algorithms was identical to the controlled plant model of Schné et al. (2014) i.e. the controller had full information about the refrigerator dynamics. Fig. 3 shows the temperatures of the refrigerator and the corresponding price level for a period of time when the refrigerator has been controlled by the proposed heuristic model predictive scheduling algorithm. It is apparent that before price jumps the controller cools down the refrigerator while before price falls it keeps the temperature around its upper bound.

The results of the simulation have been compared against the classical results obtained by formalizing the problem in MPT Toolbox (see in Herceg et al. (2013)), while the details of the MPT implementation are given in the paper of Bálint and Magyar (2016).

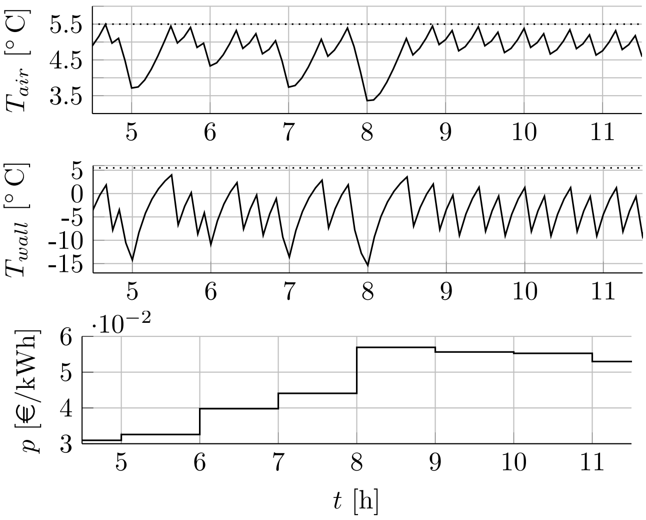

Fig. 3. Temperatures T_{air} and T_{wall} the price levels for a period of time for the proposed algorithm. The temperature bounds are denoted by dotted line.

The total optimization times and the one-day cost of the MPT based approach and that of the proposed one are collected in Table 1. Regarding optimization time the heuristic optimization based MPC performs far better than the MPT based solution (two orders of magnitude). On the other hand, the proposed scheduler mananaged to keep the cost at a lower level as opposed to the MPT based solution.

Table 1. Comparison of total optimization time t_{opt} and operating cost of the MPT based approach and the proposed scheduler for one day with $H = 1$ hour.

	MPT	heuristic B&B	
t_{opt}^{day} [min]	139.22	0.10	-99.93%
cost [€/kW]	0.3003	0.2814	-6.29%

4.2 *Effect of prediction horizon*

The next set of experiments were aimed towards investigating performance of the proposed heuristic branch and bound scheduler algorithm with respect to the prediction horizon size. Fig. 4 shows a comparative simulation result of the heuristic branch and bound optimization algorithm for different prediction horizon sizes. The results are in line with the engineering expectations i.e. a larger prediction horizon tends to be computationally more demanding while (according to Table 2) the accuracy of the optimal solution does not depend on the prediction horizon size.

Table 2. Operating cost and total optimization time vs horizon size H.

H	24 h	1 h	2 h	3 h	4 h
cost [€/kW]	0.278	0.281	0.281	0.281	0.281
t_{opt}^{day} [min]	234	0.10	0.45	1.59	3.71

4.3 *Effect of parameter uncertainty*

The third simulation experiment was a robustness analysis of the method with respect to the parameter uncertainty

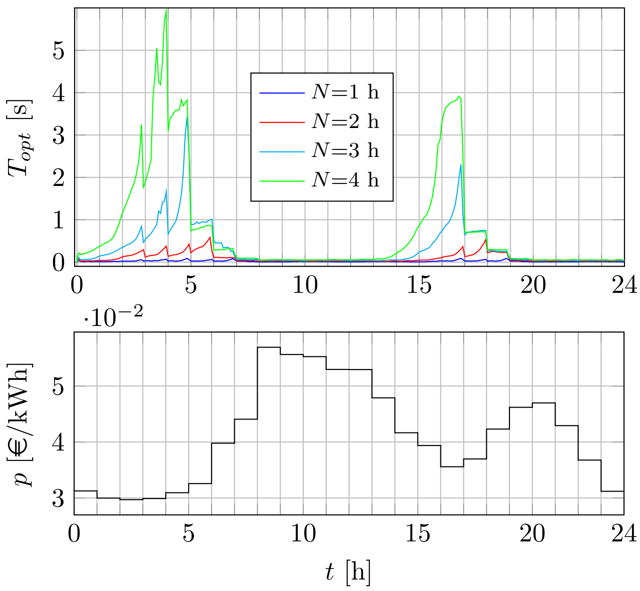

Fig. 4. The effect of prediction time H on t_{opt}. When the energy price is increasing the optimization problem gets computationally more demanding.

between the model used by the MPC algorithm and the actual refrigerator. Since the refrigerator is used for cooling meals and other goods, its natural that from time to time different goods appear in the containment changing (increasing) the overall heat capacity of the containment. This is described through an increase of value of the containment heat capacity $\tilde{C}_a$ which then differs from C_a. Simulations have been performed with $\tilde{C}_a = 2\,C_a$ and prediction horizon $H = 2$ hours. Two different cases have been examined: when the actual value of the uncertain parameter $\tilde{C}_a$ is known for the optimal scheduler and when it is unknown. The information about $\tilde{C}_a$ in the former case can be obtained e.g. by an online parameter estimation performed in parallel with the scheduling.

The simulation results are shown in Fig. 5. Using the information about $\tilde{C}_a$ the scheduler (solid line) was able to keep the T_{air} at the neighbourhood of the upper bound (dotted line). On the other hand, without this extra knowledge the scheduler was conservative. Table 3 shows the daily cost values for the different $\tilde{C}_a$ valued, compared to the optimal case ($H = 24$ h and $C_a = \tilde{C}_a$ in the model used by the scheduling algorithm). It can be seen that the reached daily cost does not really depend on the actual value of $\tilde{C}_a$ but the fact that the scheduler knows its actual value yield lower daily costs.

Table 3. The effect of $\tilde{C}_a$ on the daily cost. All values are given in [€/kW]. Second row: the scheduler has information about the parameter change. Third row: the value of $\tilde{C}_a$ is unknown for the algorithm.

$\tilde{C}_a$	C_a	$1.5\,C_a$	$2\,C_a$	$3\,C_a$	$5\,C_a$
optimal daily cost	0.2787	0.2731	0.2716	0.2680	0.2649
known $\tilde{C}_a$	0.2817	0.2748	0.2722	0.2683	0.2654
unknown $\tilde{C}_a$	0.2817	0.2817	0.2817	0.2837	0.2833

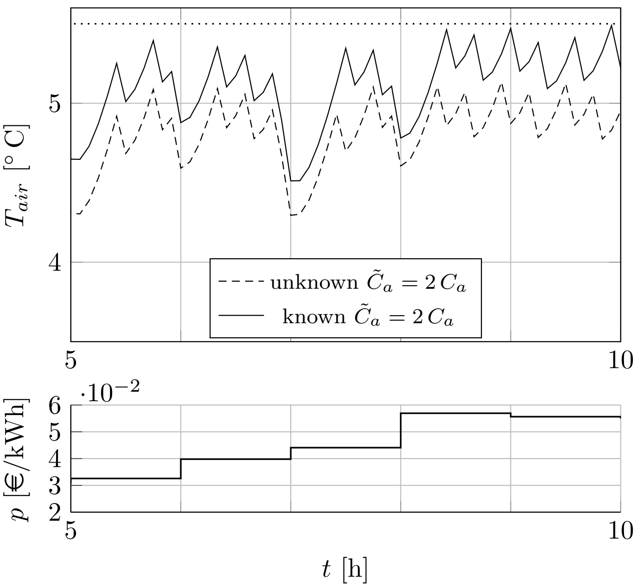

Fig. 5. The effect of the knowledge of $\tilde{C}_a$ on the scheduling algorithm. With the exact model (known $\tilde{C}_a$), the scheduler keeps the inner air temperature near the upper bound (dotted line) resulting in a lower cost.

5. CONCLUSIONS

A novel heuristic model-based optimal scheduling algorithm is proposed in this paper to operate heating and cooling type home appliances connected to smart grids where the price of the electrical energy is known in advance for 24 hours and upper and lower constraints are given for the temperature variables in the system. The algorithm is of branch-and-bound type where the bounding is driven by rules describing the qualitative properties of the step response function of the temperature to be controlled with respect to the electricity supply switch position.

The properties and the use of the proposed algorithm are shown using a refrigerator for which a simple second order model is developed. The effect of the prediction horizon and that of the electricity price function on the computing time and on the accuracy are investigated comparing the results with the schedule computed by the MPT toolbox. The algorithm works well with a relatively short prediction horizon using a fraction of the computing time (less than 1 %) needed for the MPT-based method.

The robustness of the algorithm is also investigated with respect to the load of the refrigerator that is described through the containment air heat capacity value. It is shown that the designed schedule with an empty refrigerator always respects the given temperature constraints. As a future step, an online parameter estimation of the inner air heat capacity will be performed in order to ensure the optimality of the scheduling for any loaded case.

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Appendix A. PARAMETERS OF THE MODEL

Table A.1. Parameter values of the case study

parameter	symbol	value	unit
sampling time	h	300	s
outer air temperature	T_o	20	°C
cooling liquid temperature	T_c	-36.71	°C
minimal inner air temperature	$T_{a,min}$	0.1	°C
maximal inner air temperature	$T_{a,max}$	5.5	°C
minimal back wall temperature	$T_{w,min}$	-19	°C
maximal back wall temperature	$T_{a,max}$	5.8	°C
air-wall heat transfer coeff.	K_w	$3.78 \cdot 10^3$	$\frac{\text{kW}}{°\text{C}}$
air-env. heat transfer coeff.	K_o	$2.04 \cdot 10^3$	$\frac{\text{kW}}{°\text{C}}$
wall-env. heat transfer coeff.	K_x	$0.52 \cdot 10^3$	$\frac{\text{kW}}{°\text{C}}$
wall-cool. liq. heat transfer coeff.	K_c	$5.01 \cdot 10^3$	$\frac{\text{kW}}{°\text{C}}$
heat capacity of containment air	C_a	$1.21 \cdot 10^7$	$\frac{\text{kJ}}{°\text{C}}$
heat capacity of wall	C_w	$3.41 \cdot 10^6$	$\frac{\text{kJ}}{°\text{C}}$