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Further Characterizations for the Jacobson Radical of a Ring

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180. *Further Characterizations for the Jacobson Radical of a Ring*[*)]

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Ring will mean in this note always an associative ring. For the fundamental notations, used here, we refer the reader e.g. to N. Divinsky [2] and N. Jacobson [3]. Various characterizations for the Jacobson radical F of a ring A are given (cf. N. Jacobson [3], manuscript of author's book [4] or survey [5]). Among others, author's paper [8] asserts, that any quasiprimitive ideal coincides with a primitive ideal of the ring, the Jacobson radical being the intersection of all quasiprimitive ideals (cf. author's papers [6] and [7]). Four new characterizations for the Brown-McCoy radical, for the other well used concrete radical of rings, can be found in author's earlier paper [11].

Let A be a ring. Then the equalities $(a)_r=(b)_r$ for the principal right ideals $(x)_r=Tx+xA$ $(a, b, x \in A$ and T is the ring of the rational integers), of A, define an equivalence relation $a\equiv b$ in the set of the elements of the ring, such that $\equiv$ is suitable for yielding of some characterizations of the Jacobson radical. Obviously the relation $\equiv$ is a left congruence of the multiplicative semigroup of the ring.

Here we mention only without proof our results, obtained spring 1968, whose details [10] will be published with their proofs later:

Theorem 1. *The Jacobson radical F of a ring A coincides with the subset B of those elements b of A, such the equivalence relation $a\equiv a+ac$ for any $a\in A$ and for any element c of the principal right ideal $(b)_r$, generated by $b\in B$, of A, holds.*

Theorem 2. *F coincides with the subset D of those elements d of A, such the equivalence relation $a\equiv a+adb$ for any $a, b\in A$ holds.*

Remark. On the basis of these statements, F can be considered, as a "right-sided antisimple" (twosided) radical (cf. V. A. Andrunakievitch [1]).

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