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A second almost subidempotent radical for rings

To the memory of Professor OTTÓ VARGA (1909–1969)

By FERENC A. SZÁSZ of Budapest

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The purpose of this paper, which can be considered, as a continuation of the author's papers [23] and [24], is to show, with the help of some elementfree methods of C. ROOS [19], that the class S of all E_6 -rings (see Definition 1 of our paper) forms a radical class in the sense of AMITSUR and KUROSH (Theorem 8). This radical is almost subidempotent, but it is not hereditary (Remarks 15(3) and (4)). We give examples for E_6 -rings, and assert equivalent conditions, each of which characterizes the E_6 -rings (Propositions 3 and 4). Every locally nilpotent ring is strongly S -semisimple for this radical S (Theorem 9). On the other hand, every strongly S -semisimple ring is antisimple (Theorem 11). We mention some particular strongly semisimple rings (Corollary 12(1) and (2)), furthermore, we characterize the nilpotent rings among all right artinian rings (Theorem 14) with the help of the S -semisimplicity. The relationship $a \in S(a) = aA + Aa + AaA$ defines a so-called F -regularity for $F(a) = S(a)$. In this way an F -radical $\text{rad}_F A$ in the sense of B. BROWN and N. H. MCCOY [7] can be defined for every ring A such that $\text{rad}_S A \supseteq S(A)$, and $\text{rad}_S A$ is a more general radical in the sense of J. M. MARANDA [16] and G. MICHLER [17]. The rings A with $\text{rad}_S A = 0$ are discussed (Proposition 16). Finally six open problems are raised on E_6 -rings.

The fundamental notions, used in this paper, can be found in N. DIVINSKY [11], N. JACOBSON [13] and S. LANG [15]. All rings, considered here, are associative. The symbol \mathbb{Z} always denotes the ring of rational integers. The principal left, right and twosided ideal, generated by the element a of a ring A , will be denoted by $(a)_l$, $(a)_r$, and (a) , respectively. The symbol $B \triangleleft A$ denotes, that B is a twosided ideal of the ring A . If $B \triangleleft A$, then A is an EVERETT ring extension [12] of B . For the arbitrary additive subgroups B_1 and B_2 of the additive group A^+ of a ring A , by the product $B_1 \cdot B_2$ we mean the additive subgroup, generated by all products $b_1 \cdot b_2$ with $b_1 \in B_1$ and $b_2 \in B_2$, of A^+ . We point out again the notation $S(a)$ for the twosided ideal $aA + Aa + AaA$ of the ring A , for every element a of A . Furthermore, A^n denotes the intersection of all powers A^n of A , for $n = 1, 2, 3, 4, 5, \dots$ etc. The rings A , satisfying $A^2 = A$, are called idempotent. Following V. A. ANDRUNAKIEVICH [3], a ring is said to be strongly idempotent, if every its twosided ideal is idempotent. Important instances for strongly idempotent rings are the

regular rings in the sense of J. VON NEUMANN [18], the class of which forms a radical class by B. BROWN and N. H. MCCOY [8], and the biregular rings of R. F. ARENS and I. KAPLANSKY [5] and of V. A. ANDRUNAKIEVICH [2]. Let us remember, that a ring A is called NEUMANN regular, and biregular, if for every element a of A the inclusion $a \in aAa$ holds; and the principal ideal (a) contains for every $a \in A$ a twosided unity element, respectively. Furthermore, a ring is said to be antisimple (see V. A. ANDRUNAKIEVICH [3]), if it cannot be homomorphically mapped onto a subdirectly irreducible ring with an idempotent heart. Examples for antisimple rings are the locally nilpotent rings, in which every finitely generated subring is nilpotent.

Radical and semisimplicity are used here in the sense of S. A. AMITSUR [1] and of A. G. KUROSH [14]. But at the Remark 15(2) also the more general radicals $\text{rad}_F A$ of a ring A in the sense of J. M. MARANDA [16] and G. MICHLER [17] are mentioned. For a detailed exposition on general radicals in the sense of AMITSUR and KUROSH, until the results of 1964, see DIVINSKY [11]. For a radical T [1], [14], the T -radical of a ring A will be denoted by $T(A)$. The symbol \bar{C} , for an arbitrary class C of rings, always denotes the lower radical class, defined by C . Therefore, C is a radical class if and only if $\bar{C} = C$ holds. Following S. A. AMITSUR [1] and V. A. ANDRUNAKIEVICH [4], for a radical T , a ring A is said to be strongly T -semisimple, if every homomorphic image of A is again T -semisimple. For example, every NEUMANN regular ring is strongly JACOBSON semisimple [13], and every simple ring, which is also T -semisimple, obviously is strongly T -semisimple. Furthermore, a radical T is called hereditary, if every twosided ideal of a T -radical ring is a T -radical ring. A radical T is said to be almost subidempotent, if every T -radical ring is idempotent. According to V. A. ANDRUNAKIEVICH [4], the hereditary and almost subidempotent radicals are called subidempotent. Thus, the radical studied in BROWN and MCCOY [8] is subidempotent, but our radical S , which contains the radical R of the author's papers [23] and [24], is not subidempotent. Also this R from [24] and [24] is not subidempotent. R and S are only almost subidempotent.

In the author's paper [21], which gives some criteria for the existence of the twosided unity element of a ring, the E_i -rings were discussed for $i = 0, 1, 2, 3, 4$ and 5. According to this paper, a ring was called an E_5 -ring, if every homomorphic image of it has no nonzero left annihilators. See yet the author's papers [22], [23] and [24] and C. ROOS' paper [19]. Following the notation of the author [21], we give the:

Definition 1. A ring is called an E_6 -ring, if every of its homomorphic images has no nonzero twosided annihilators.

Let R and S denote the class of all E_5 -rings and that of all E_6 -rings, respectively. One trivially has $R \subseteq S$. Now $R \neq S$ is shown by the following:

Example 2. Let A be the algebra, generated by the elements a and b , over the field of two elements, with the table of multiplication:

		a
a		a
b		0

Then a is a left unity element $A \in R$ and $A \in S$, which to the ring in Example 1

One obtains further important:

Proposition 3. (1) Every (2) Every ring with one-sided $A^2 = A$, every VON [2], every strongly idempotent VINSKY [10], every de la I every ring A such that I E_6 -ring.

Proof is obvious, thus It is clear, that the ri

Proposition 4. For an equivalent:

- (i) A is an E_6 -ring;
- (ii) $I = I \cdot A + AI$ hold.
- (iii) $(a) = (a) \cdot A + A \cdot (a)$
- (iv) $(a) = (a)_l \cdot A + A \cdot (a)$
- (v) $(a) = S(a)$ holds for a
- (vi) $a \in S(a)$ holds for every

Proof is almost trivial G. SZÁSZ [25] has discussed I of A .

Proposition 5. A ring of every EVERETT ring ext

Proof (cf. C. ROOS [(A \cap C) \triangleleft B, consequent $A \cap C = (A \cap C) \cdot A +$ yields, at once, the desire

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	a	b
a	a	b
b	0	0

Then a is a left unity element and b is a left annihilator of A , whence we have $A \in R$ and $A \in S$, which imply $R \neq S$, indeed. This example is antiisomorphic to the ring in Example 13 of [23].

One obtains further instances for E_6 -rings by the following trivial but important:

Proposition 3. (1) Every homomorphic image of an E_6 -ring is again an E_6 -ring. (2) Every ring with one-sided unity elements, furthermore every simple ring A satisfying $A^2 = A$, every VON NEUMANN regular ring [8], [18], every biregular ring [5], [2], every strongly idempotent ring [3], every D -regular ring in the sense of N. DIVINSKY [10], every de la ROSA's λ -radical ring [9], every E_2 -ring, every E_5 -ring, and every ring A such that $I = I \cdot A + AI$ holds for every twosided ideal I of A , is an E_6 -ring.

Proof is obvious, thus we omit it.

It is clear, that the rings A , satisfying $A^2 = 0$, are not E_6 -rings.

Proposition 4. For an arbitrary ring A the following conditions mutually are equivalent:

- (i) A is an E_6 -ring;
- (ii) $I = I \cdot A + AI$ holds for every twosided ideal I of the ring A ;
- (iii) $(a) = (a) \cdot A + A \cdot (a)$ holds for every element $a \in A$;
- (iv) $(a) = (a)_l \cdot A + A \cdot (a)_r$ holds for every element $a \in A$;
- (v) $(a) = S(a)$ holds for every element $a \in A$;
- (vi) $a \in S(a)$ holds for every element $a \in A$.

Proof is almost trivial, therefore it will be omitted.

G. SZÁSZ [25] has discussed semigroups S such that $I = I \cdot S$ for every ideal I of A .

Proposition 5. A ring A is an E_6 -ring if and only if for every twosided ideal C of every EVERETT ring extension [12] B of A holds: $A \cap C = A \cdot C + C \cdot A$.

Proof (cf. C. ROOS [19]). Let us assume $A \in S$, $A \triangleleft B$ and $C \triangleleft B$. Then $(A \cap C) \triangleleft B$, consequently $(A \cap C) \triangleleft A$. Therefore, by Proposition 4, we obtain $A \cap C = (A \cap C) \cdot A + A(A \cap C) \subseteq C \cdot A + AC \subseteq C \cap A = A \cap C$, which yields, at once, the desired condition $A \cap C = AC + CA$.

Conversely, if the condition of the criterion, formulated in Proposition 5, holds, then for the particular case $B = A$ one immediately has $C = AC + CA$ for every $C \triangleleft A$, which by Proposition 4 implies $A \in S$, indeed.

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Then every homomorphic
 arbitrary ideal of B . If
 $+ C \cdot c \cdot C = S(c)$ in C ,

for every $c \in C$. Therefore there exist elements c_1, c_2, c_i, c_j such that holds:

$$c = c \cdot c_1 + c_2 \cdot c + \sum c_i \cdot c \cdot c_j \quad (i, j \geq 2; \quad i, j \in \mathbb{Z}).$$

Let D be the finitely generated subring $\{c_1, c_2, \dots, c_i, \dots, c_j, \dots\}$ (i and j run over a finite set of indices). Then, by our assumption, there exists an exponent m such that $D^m = 0$. Now $c \in c \cdot D + Dc + D \cdot c \cdot D$, by induction on n , obviously implies for every $n = 1, 2, 3, 4, \dots$:

$$c \in c \cdot D^n + D^n \cdot c + \sum_{k=1}^{n-1} D^k \cdot c \cdot D^{n-k}.$$

If, in particular, $n = 2m - 1$, then we obtain by $D^m = 0$ also $c = 0$ and $C = 0$, thus A is strongly S -semisimple, indeed.

Corollary 10. (1) Every BAER [6] lower nil radical ring, and every nilpotent ring is strongly S -semisimple. (2) Every JACOBSON radical ring with minimum condition on principal right ideals, every right noetherian nilring and every right artinian nilring is strongly S -semisimple (cf. author [20]).

Theorem 11. Every strongly S -semisimple ring A is antisimple.

Proof. Let C be the heart of an arbitrary subdirectly irreducible homomorphic image B of a strongly S -semisimple ring A . Since B is S -semisimple, one has $S(B) = S(C) = 0$ and thus $C \notin S$. Then, by Proposition 4, there exists an element $c \in C$ such that holds: $c \notin C^* = c \cdot C + C \cdot c + C \cdot c \cdot C \subseteq C$. Let $C^{**} = C^* + C^*B + B \cdot C^* + B \cdot C^*B$, which is the ideal of B , generated by the ideal C^* of C . Then, by the famous lemma of V. A. ANDRUNAKIEVICH [2], we have $(C^{**})^3 \subseteq C^*$ (since obviously $(C^{**})^3 \subseteq C \cdot C^{**} \cdot C = C \cdot (C^* + C^*B + BC^* + B \cdot C^*B) \subseteq C^* + C^*C + CC^* + C \cdot C^*C$). If $C^{**} = 0$, then also $C^* = 0$; thus $c \cdot C = C \cdot c = 0$ and $c \notin C^*$, but $c \in C$, imply $C^2 = 0$. If $C^{**} \neq 0$, then by $c \notin C^*$, $(C^{**})^3 \subseteq C^* \subseteq C$ and by the minimality of C , one obtains $(C^{**})^3 = 0$. Let m be a minimal exponent, such that $(C^{**})^m = 0$. Then $(C^{**})^{m-1} \neq 0$ and by $C^{**} \neq 0$, evidently $2 \leq m \leq 3$. Now $C \subseteq (C^{**})^{m-1}$ and $(C^{**})^m = 0$ again imply $C^2 = 0$, even in the case $C^{**} \neq 0$. This completes the proof of the antisimplicity of A .

Corollary 12. (1) Every strongly S -semisimple ring with minimum condition on principal twosided ideals is a nil ring. (2) Every strongly S -semisimple ring with minimum condition on all twosided ideals is nilpotent.

Proof follows from Theorem 11 and from V. A. ANDRUNAKIEVICH's [3]. Theorem 7 and 6.

Theorem 13. For every right artinian ring A the following two conditions are equivalent:

- (i) A is nilpotent;
- (ii) $\text{rad}_S A = 0$ (see remark 15.2), and A has no nonzero left annihilators, contained in the intersection A^∞ of all A^n , for $n = 1, 2, 3, 4, \dots$ etc.

Proof follows from Theorem 5 of N. DIVINSKY [10] and from $D \subseteq S$, where D means the class of all D -regular rings [10].

Theorem 14. For every ring A , which is right artinian and also left artinian, the following two conditions are equivalent:

- (i) A is nilpotent;
- (ii) $\text{rad}_S A = 0$ (see remark 15.2), and A has no nonzero twosided annihilators, contained in A^0 .

Proof follows from Theorem 6 of N. DIVINSKY [10], from $D \subseteq S$ and the left-right selfduality of S .

Remarks 15. (1) In Corollary 12 of the author's paper [23] "ring A with minimum condition on right ideals and minimum condition on left ideals" must be read instead of "ring A with minimum condition on right ideals".

(2) The relationship $a \in S(a) = aA + Aa + AaA$ for the elements a of a not necessarily E_0 -ring A , obviously defines a so-called F -regularity in the sense of B. BROWN and N. H. MCCOY [7]. Thus, by [7] in every ring A there exists the F -radical $\text{rad}_F A$, which contains every F -regular ideal of A ; $\text{rad}_F A$ is F -regular in A , for $F(a) = S(a)$, and $\text{rad}_F(A/\text{rad}_F A) = 0$ holds. Then $\text{rad}_S A$ is a radical in the sense of J. M. MARANDA [16] and of G. MICHLER [17].

(3) Every S -radical ring A must be, by Proposition 4, idempotent, that is, $A = S(A)$ implies $A^2 = A$.

(4) However, the radical S is not hereditary, since $Z \in S$, $2. Z \notin S$. Therefore the radical is not subidempotent, only almost subidempotent.

Proposition 16. (1) In every ring A holds $\text{rad}_S A \subseteq S(A)$ (see remark 15 (2)).
 (2) $A/\text{rad}_S A$ is, for every ring A , a subdirect sum of some subdirectly irreducible rings S_x with $\text{rad}_S S_x = 0$ such that $S_x H_x = H_x S_x = 0$ holds for the hearts H_x of S_x .

Proof of (1) is trivial. The proof of (2) immediately follows from B. BROWN and N. H. MCCOY [7].

Now we would like to mention some open questions on our radical S :

Problem 1. Does there exist a ring A such that $\text{rad}_S A \neq S(A)$?

Problem 2. Does there exist a nil ring, which is not strongly S -semisimple?

Remark 17. Problem 2 is equivalent to each of the following two questions: "Does there exist a nil ring which is not S -semisimple?" or "Does there exist a nonzero nil ring which is an E_0 -ring?" In fact, any two of the following three assertions are equivalent:

- (1) Every nil ring is strongly S -semisimple.
- (2) Every nil ring is S -semisimple.
- (3) A nonzero nil ring is not an E_0 -ring.

Proof. (1) \Rightarrow (2) trivially.

(2) \Rightarrow (3). Let A be a nil ring. By (2) $S(A) = 0$. If, in addition, A is an E_0 -ring, then also $A = S(A)$. Hence $A = 0$.

(3) \Rightarrow (1). If A is a nil ring which is not strongly S -semisimple, then A has a homomorphic image A/I such that $S(A/I) \neq 0$. Since $S(A/I) = K/I$ with a

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Problem 3. Does there exist an antisimple ring, which is not strongly S -semisimple?

Problem 4. Does there exist a ring A such that $S(A)$ does not contain every S -radical right ideal of A ?

Problem 5. Is $S(A_n) = (S(A))_n$ true for every $n \times n$ full matrix ring over the ring A ?

Problem 6. For every right artinian ring A , does follow condition (i) of Theorem 14 from the condition (ii)?

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