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The solution of a problem on upper radicals of rings

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The aim of this little article is to give a solution of R. WIEGANDT's "Problem 1" from his paper [6].

The rings, considered in this note, are assumed to be associative. All notions, used here, can be found in [1], [2], [3], [4] and [5]. In particular, UH denotes the upper radical class, determined by the class of all JACOBSON semisimple linearly compact rings. Here I denotes the JACOBSON radical class.

By H. LEPTIN [3], the cardinality of the set of elements of a primitive linearly compact ring A is either finite, or at least $2^{\aleph_0}$, i.e. $|A| < \aleph_0$ or $|A| \geqq 2^{\aleph_0}$. Thus we have the following result:

Theorem. *$I \underset{+}{\leq} UH$ holds for the* JACOBSON *radical I and the upper radical UH determined by the class of all (I-semisimple) primitive linearly compact rings.*

Proof. Let A be the algebra of all infinite matrices of only finite number of nonzero entries, over the racional number field K_0. Then the algebra A has a basis e_{ij} $(i, j = 1, 2, 3, \ldots)$ over K_0, where

$$e_{ij}e_{kl} = \delta_{jk}e_{il}$$

and δ_{jk} denotes the KRONECKER delta. Obviously A is a simple and primitive MHR-ring [5] which is not linearly compact [3], being $|A| = \aleph_0$. It is sufficient to show that A is not a metaideal, in the sense of R. BAER (i.e. an accessible subring) of a JACOBSON semisimple linearly compact ring. But this follows from the elementary fact that A is a proper subring of the *simple* ideal of all infinite matrices of only finite number of nonzero columnes of a primitive linearly compact ring.

This completes the proof.

References

[1] N. DIVINSKY, Rings and Radicals, London 1965.

[2] N. JACOBSON, Structure of Rings, Providence 1964.

[3] H. LEPTIN, Linear kompakte Moduln und Ringe, I., Math. Z. **62**, 241—267 (1955), II. ibidem **66**, 289—327 (1957).

[4] F. SZÁSZ, Radikale der Ringe, Budapest (to appear).

[5] —, Über Ringe mit Minimalbedingung für Hauptrechtsideale, I., Publ. Math. Debrecen **7**, 54—64 (1960); II. Acta Math. Acad. Sci. Hung. **12**, 417–439 (1961); III. ibidem **14**, 447–461 (1963).

[6] R. WIEGANDT, Radicals coinciding with the JACOBSON radical on linearly compact rings, Beiträge zur Algebra und Geometrie **1**, 195–199 (1971).