POLLACK PERIODICA
An International Journal for Engineering and Information Sciences
DOI: 10.1556/606.2017.12.3.3
Vol. 12, No. 3, pp. 23–32 (2017)
www.akademiai.com

RESISTANCE OF MEMBERS SUBJECTED TO COMBINED LOADS ACCORDING TO EC3 AND EC9

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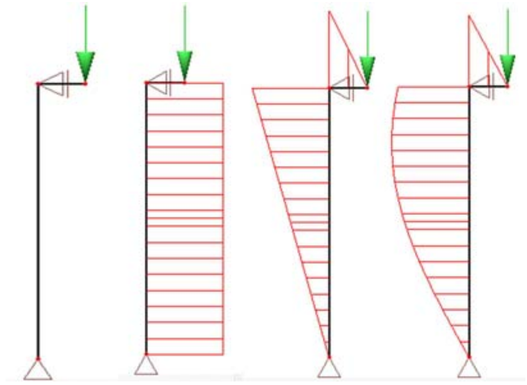
Received 31 December 2016; accepted 9 May 2017

Abstract: The aim of this article is to describe problems of members subjected to biaxial bending and axial compression force and the methods of their design. Members subjected to bending moments about two axes and axial compression force exhibit complex behavior. It is necessary to consider second order theory and imperfections when designing members like that, since they have noticeable influence on resistance in instability. There are examples shown in the article, using methods present in current European standards EN 1993-1-1 and EN 1999-1-1. Attention is drawn to differences in methods, both in applicability and practicality, in design of members with constant and linear bending moment. The purpose of presented paper is to show if the method, which is currently used in Eurocode for aluminum structures, can be also used for steel structures.

Keywords: Biaxial bending, Beam columns, Second order theory, Imperfections

1. Introduction

Members subjected to biaxial bending and axial compression force show complex behavior. Either transverse loads, or end moment loads result in first order bending moments on members $M_{y,Ed}$ and $M_{z,Ed}$. Furthermore, axial compression force will result in internal axial compression force N_{Ed}, but also a second order bending moment will be created (*Fig. 1*) possibly for both axis of bending. Biaxial bending by itself leads to complex deflection in both axes, which is not considered in approach presented in standards for practical reasons. Instead of one complicated expression, pairs of interactions formulae are used in standards. Simplification like that is rarely on the unsafe side.



a) b) c) d)

Fig. 1. Example of member a) subjected to eccentric load resulting in combined effect; b) with distribution of axial force; and c) bending moment*;* d) including second order effect

In general, distribution of internal forces is not constant, which presents another difficulty in determining the most loaded section on member. Beam columns are often present as parts of frame structures. In similar cases sway effects need to be taken into account. Unless a second order analysis with imperfections is carried out, stability of members subjected to combined loads must be checked using one of the methods present in current European standards. In [1] clause 6.3.3, two alternative methods for determining interaction factors are shown, another possibility is to use method presented in [2] clause 6.3.3. These methods are valid for doubly symmetrical cross sections resistant to warping. Clause 6.3.3 [1] requires two conditions (1), (2) to be satisfied when calculating the resistance of members subjected to axial compression and biaxial bending. It is also necessary to verify end section resistance. Determining resistance of beam columns requires analysis of behavior of both members in bending and in compression, while considering imperfections, both in steel and aluminum structures [3]-[9].

It may seem incorrect to compare methods, which are seemingly for different materials, it is however important to note that method, which is currently presented in EC 9 has been used in Swedish standards for steel structures [10] and is also proposed to be included in EC 3 [11].

2. Calculation methods

2.1. EN 1993

Method 1 presented in annex A of Eurocode 3 [1], [12] is based on transparency of the calculation by using formulas in which every factor represents single phenomenon.

Method 2 (annex B [1], [12]) is based on principle of global factors; therefore the calculation is less transparent compared to method 1. The aim of using lesser number of global factors is practicality.

The basis for interaction formulas is a single span member with hinged supports in both axes of bending and torsion, loaded by axial compression force and end bending moments or transverse load. Both methods presented in [1] are based on altering the formulas for flexural buckling to accommodate lateral torsional buckling. The result is calculation approach, which differentiates between members susceptible to torsion and those which are not. Members resistant to torsion sufficiently supported along the whole member, or members with closed sections will fail in flexural buckling.

Results of experiments and parametric studies on which the formulas are based on, represent mainly members with doubly symmetrical cross sections constant along the member. Because of that the usability of formulas is restricted to such members. It is possible however to adjust these formulas so they can be used for sections with one axis of symmetry [13]:

$$\frac{N_{Ed}}{\frac{x_y N_{Rk}}{\gamma_{M1}}} + k_{yy} \frac{M_{y,Ed} + \Delta M_{y,Ed}}{\chi_{LT} \frac{M_{y,Rk}}{\gamma_{M1}}} + k_{yz} \frac{M_{z,Ed} + \Delta M_{z,Ed}}{\chi_{LT} \frac{M_{y,Rk}}{\gamma_{M1}}} \leq 1\,, \tag{1}$$

$$\frac{N_{Ed}}{\frac{x_z N_{Rk}}{\gamma_{M1}}} + k_{zy} \frac{M_{y,Ed} + \Delta M_{y,Ed}}{\chi_{LT} \frac{M_{y,Rk}}{\gamma_{M1}}} + k_{zz} \frac{M_{z,Ed} + \Delta M_{z,Ed}}{\chi_{LT} \frac{M_{y,Rk}}{\gamma_{M1}}} \leq 1\,. \tag{2}$$

Note: Calculation of interaction factors k_{yy}, k_{yz}, k_{zy}, k_{zz}, depends on chosen alternative method. N_{Ed}, $M_{y,Ed}$, $M_{z,Ed}$ are the design values of the compression force and the maximum bending moments about the *y-y* and *z-z* axis along the member; $\Delta M_{y,Ed}$, $\Delta M_{z,Ed}$ respectively are the bending moments due to the shift of the centroidal axis for class 4 sections; χ_y, χ_z are the reduction factors due to flexural buckling; χ_{LT} is the reduction factor due to lateral torsional buckling; N_{Rk}, $M_{y,Rk}$, $M_{z,Rk}$ are characteristic values of compression force and bending moments cross section resistances; γ_{M1} is particular partial factor [1].

Both methods are based on similar principles like equivalent bending moment and buckling length. The basis for equivalent constant bending moment is to replace real distribution of bending moment with constant distribution, using interaction factors (*Fig. 2*).

2.2. EN 1999

Similarly to the approach in [1], first step is to verify cross section resistance [2] (cl. 6.2.9), and then the global member behavior. Cross section resistance in both elastic and plastic design depends on local buckling of parts of the section. Both Eurocodes take this into account by classification of cross sections. In case of combined loads, [2], [12] allows classification separately for stress from each type of load. The cross section is not classified for combined stress, which means that for each type of load the class can be different.

$$\left(\frac{N_{Ed}}{\chi_y \omega_x N_{Rd}}\right)^{\xi_{yc}} + \frac{M_{y,Ed}}{\omega_0 M_{y,Rd}} \leq 1.00\,, \tag{3}$$

$$\left(\frac{N_{Ed}}{\chi_z \omega_x N_{Rd}}\right)^{\eta_c} + \left(\frac{M_{z,Ed}}{\omega_0 M_{z,Rd}}\right)^{\xi_{zc}} \leq 1.00\,, \tag{4}$$

where ξ_{yc}, ξ_{zc}, η_c are interaction factors; ω_x is factor considering position of calculated section on member; ω_0 is factor considering softening due to heat affected zones [2].

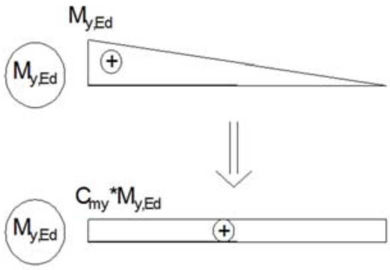


Fig. 2. Example of equivalent constant bending moment principle

Members resistant in torsion should satisfy conditions (3) a (4) according to [2] cl. 6.3.3. In case of members susceptible to torsion, criterion (5) must be satisfied as well. Values of exponents and their significance can be found in [2]. Conservatively the values can be taken as 0.8

$$\left(\frac{N_{Ed}}{\chi_z \omega_x N_{Rd}}\right)^{\eta_c} + \left(\frac{M_{y,Ed}}{\chi_{LT} \omega_{xLT} M_{y,Rd}}\right)^{\gamma_c} + \left(\frac{M_{z,Ed}}{\omega_0 M_{z,Rd}}\right)^{\gamma_c} \leq 1.00\,, \tag{5}$$

where γ_c is an interaction factor. Parameters ω_x a ω_{xLT} account for negative influence of local welds on aluminum alloys (heat affected zone), and influence of non-constant bending moment distribution

$$\omega_x = \frac{1}{\chi + (1-\chi)\sin\frac{\pi\, x_s}{l_c}}\,, \tag{6}$$

(Note: Based on the direction of deflection either χ_y or χ_z will be used.)

$$\omega_{xLT} = \frac{1}{\chi_{LT} + (1-\chi_{LT})\sin\dfrac{\pi\, x_s}{l_c}}\,, \tag{7}$$

where x_s is the smallest distance between calculated section and either a simple support, or point of contra flexure of the deflection curve for elastic buckling from axial force; l_c is the buckling length of elastic buckling [2].

3. Worked examples

This chapter shows verification examples of members loaded by axial compression force and biaxial bending moments with various distributions. Verification is carried out according to [1], using both method 1 and 2, and also according to [2]. The results are compared. Three examples are calculated, first two are members with length of 7 m, and third one is a member of 5 m. All members are made of steel S235 with hot rolled doubly symmetrical *I* profile HEA200. Cross section properties were taken from literature. Elastic modulus in tension was taken with value of 210 GPa. All members are verified using both elastic and plastic cross-section considerations. Even though [2] allows separate classification for load types, either only elastic or only plastic properties are used in examples, to match [1] more closely.

Example 1 (*Fig. 3*) is a member with hinged supports in both axes of bending and forked in torsion, loaded by end bending moments with same magnitudes $M_{y.Ed}$=10 kNm, $M_{z.Ed}$=4 kNm, (ψ=1) and axial compression force N_{Ed}=250 kN. Distribution of (5) is shown in (*Fig. 4*).

Example 2 (*Fig. 5*) is a member with same dimensions as in example 1; the difference is in bending moment distribution, which is linear in this case, since the member is loaded by single end moment. This applies in both planes *x-z* and *x-y* (ψ=0). Distribution of (5) is shown (*Fig. 6*).

Example 3 (*Fig. 7*) was chosen to show an example where end section verification will be deciding. The member is 5 m long, with hinged supports and loaded by single end moment in both planes, with values $M_{y.Ed}$=20 kNm, $M_{z.Ed}$=20 kNm and axial force N_{Ed}=60 kN. Distribution of (5) is shown in (*Fig. 8*).

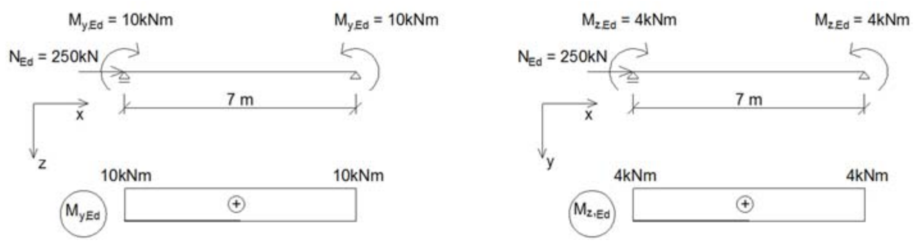


Fig. 3. Example 1 with boundary conditions and first order bending moment distribution in planes *x-z* and *x-y*

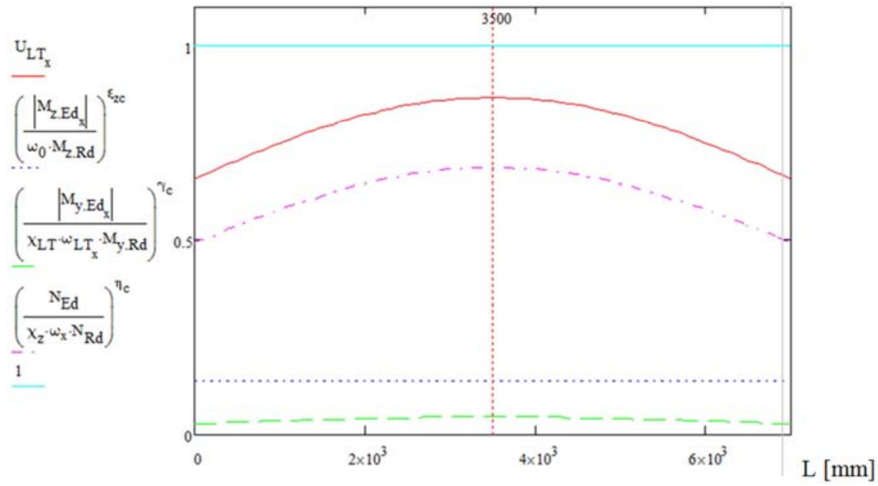


Fig. 4. Interaction formula (5) on member in example 1

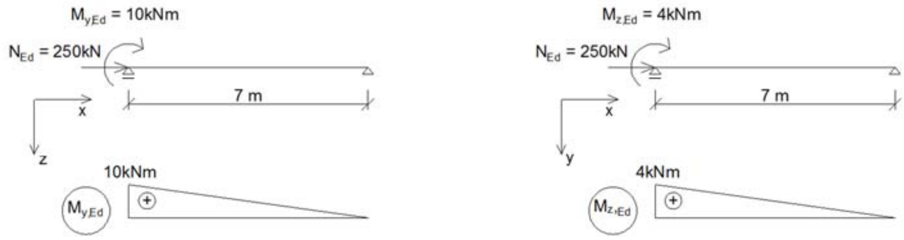


Fig. 5. Example 2 with boundary conditions and first order bending moment distribution in planes *x-z* and *x-y*

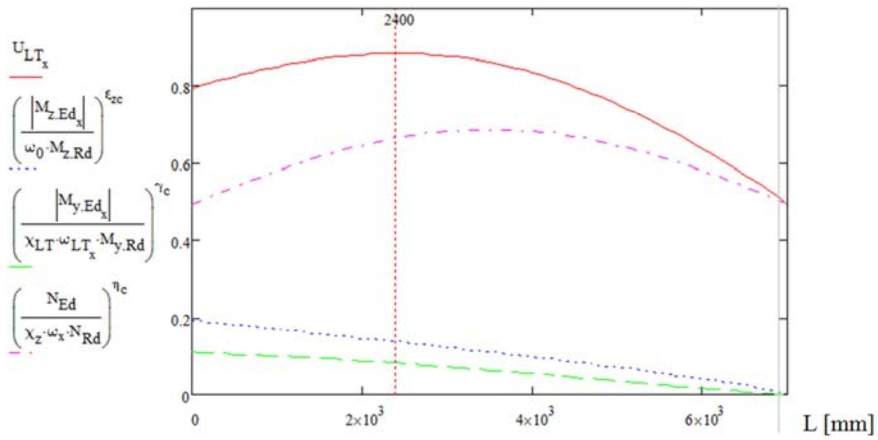


Fig. 6. Interaction formula (5) on member in example 2

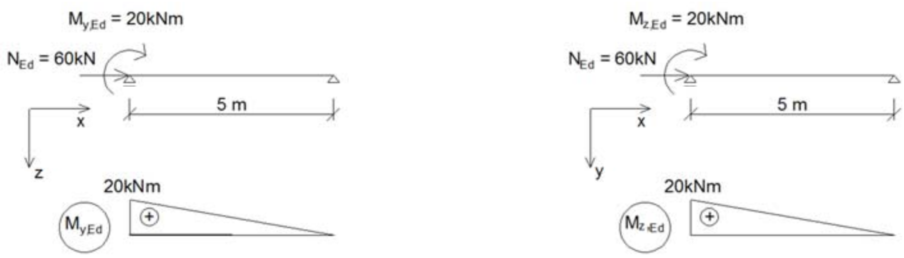


Fig. 7. Example 3 with boundary conditions and first order bending moment distribution in planes *x-z* and *x-y*

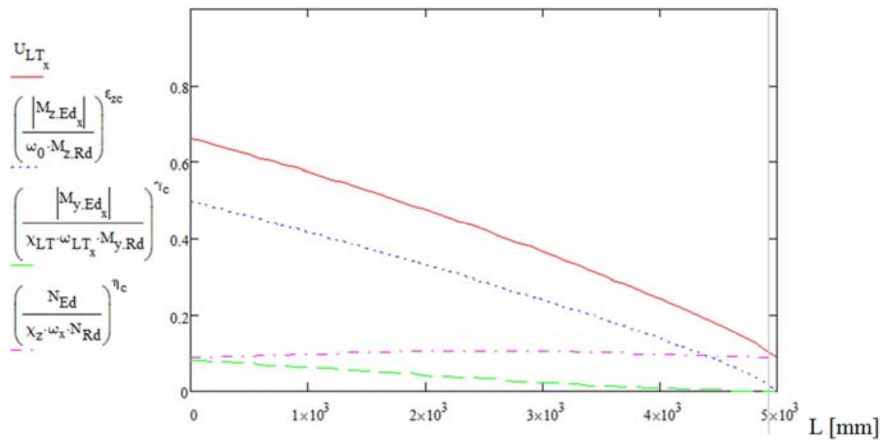


Fig. 8. Interaction formula (5) on member in example 3

Fig. 4, *Fig. 6* and *Fig. 8* show the distribution of utilization (5) along the member, to illustrate the significance of each part of the formula, as well as point out the necessity to check multiple sections.

4. Results

The results were acquired using program written with MathCAD software. In first two examples according to [1] criterion 2 was deciding, meaning the member buckled perpendicular to axis *z*. Verification according to [2] distinguishes between buckling perpendicular to axis y (3), perpendicular to axis *z* (4) and lateral torsional buckling (5). In all cases criterion (5) was deciding. The place of a critical section varied (see *Fig. 4*, *Fig. 6* and *Fig. 8*). According to [2] each section needs to be checked, therefore no further end section check is required, like it is with [1] (column titled U.CS in *Table I, Table II*).

Table I

Results of interaction formulas according to [1]

			C.my	C.mz	U.1 (1)	U.2 (2)	U.CS
Example 1	EC 3 A	Elastic	1	1	0.765	0.947	0.434
		Plastic			0.676	0.875	0.38
	EC 3 B	Elastic	1	1	0.648	0.943	0.434
		Plastic			0.538	0.874	0.38
Example 2	EC 3 A	Elastic	0.668	0.738	0.588	0.824	0.434
		Plastic			0.5	0.754	0.38
	EC 3 B	Elastic	0.6	0.6	0.493	0.845	0.434
		Plastic			0.428	0.794	0.38
Example 3	EC 3 A	Elastic	0.668	0.738	0.721	0.813	0.901
		Plastic			0.528	0.536	0.663
	EC 3 B	Elastic	0.6	0.6	0.608	0.734	0.901
		Plastic			0.354	0.575	0.663

Table II

Results of interaction formulas according to [2]

			U.F1 (3)	U.F2 (4)	U.LT (5)
Example 1	EC 9	Elastic	0.49	0.878	1.028
		Plastic	0.476	0.823	0.868
Example 2	EC 9	Elastic	0.441	0.804	0.883
		Plastic	0.431	0.769	0.784
Example 3	EC 9	Elastic	0.302	0.824	1.043
		Plastic	0.246	0.583	0.662

Table I also contains values of equivalent constant moment factors $C_{m.y}$ and C_{mz}, which had been calculated using both method 1 and 2. Comparison between elastic and plastic analysis in examples 1 and 2 is showing difference, which is caused by the fact that the prevailing load is result of axial force N_{Ed}=250 kN. In third example the plastic reserve is greater which corresponds with ratio of bending to axial loading.

Another set of examples is presented in *Table III*, where multiple steel rolled ‘I’ sections were checked. These sections were loaded by biaxial constant bending moment and compression force, with values of each load corresponding to one third of maximum load of individual effect, resulting in utilization close to 1.00. For comparison also finite element model was used, using RFEM software. Only elastic design was used. The table shows values of highest utilization in all checks in each method.

5. Conclusion

While the approach in Eurocode 9 is more conservative, in the chosen examples it is shown that it is indeed possible to use it to determine resistance of steel beam-columns. The difference in utilization between three methods is relatively small. For calculation of chosen examples all three methods are applicable. From perspective of practicality, method in Eurocode 9 is the most desirable, since it contains only a small number of formulas needed for verification. It is however necessary to verify several sections using those formulas, which can be time consuming in certain cases. Method A in Eurocode 3 is less practical compared to other two, because it contains many formulas and factors, which increases the possibility of making a mistake.

Table III

Comparison of various 'I' rolled sections with FEM model

		RFEM	EC 3 A	EC 3 B	EC 9
IPE 300	Utilization	0.99	1.03	1.06	1.17
	Difference [%]	0.00	4.75	7.48	18.12
HEA 300	Utilization	1.05	1.09	1.04	1.16
	Difference [%]	0.00	4.47	-0.40	10.68
HEB 300	Utilization	1.04	1.09	1.05	1.16
	Difference [%]	0.00	4.45	0.23	11.34
HEM 300	Utilization	1.03	1.08	1.05	1.17
	Difference [%]	0.00	4.76	2.04	13.42

Acknowledgements

Project No. 1/0819/15 was supported by Slovak Scientific grant agency VEGA.

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