Developing teacher knowledge by engaging with a mathematical task and reflecting on the experience

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Abstract
In this workshop participants get a first-hand experience of a method that the Budapest Semesters in Mathematics Education program uses to develop the knowledge of prospective teachers. They are exposed to a mathematical task as students, then they reflect on the experience from the perspective of a teacher. Workshop participants will experience doing a mathematical task involving matchbox constructions and reflecting on it the same way as BSME's prospective teachers do, and will also have the opportunity to reflect on the experience as a whole.

Introduction

Mathematical problem solving has had a long tradition in Hungarian classrooms, where a strong and explicit focus is placed on problem solving, creativity, and communication. Students learn concepts by working on mathematically meaningful problems that emphasize procedural fluency, conceptual understanding, logical thinking, and connections between various topics. For every lesson, an overarching goal is to learn what it means to engage in mathematics and to feel the excitement of discovery (Stockton, 2010).

This *Hungarian approach* aligns closely with recommendations in the *Principles and Standards for School Mathematics* (NCTM, 2000). Thus, American and Hungarian mathematicians and mathematics educators collaborated to start a semester-long program for American preservice teachers (PSTs) in Hungary, called *Budapest Semesters in Mathematics Education* (BSME). The program aims to develop PSTs' teacher knowledge by guiding them to shape their own vision of mathematics and by providing them tools to impact their own students similarly. The fundamental principles of BSME are based on the problem-solving heuristics delineated by George Pólya (1962), which were introduced into Hungarian primary and secondary education by Tamás Varga (Szendrei, 2007).

We analyze the ways in which BSME develops teacher knowledge using Shulman's (1986) framework of Content Knowledge (CK), Pedagogical

Content Knowledge (PCK) and Pedagogical Knowledge (PK). We also consulted the mathematical knowledge for teaching framework studied by many researchers—e.g., Ball, Thames, and Phelps (2008) and Davis and Simmt (2006).

We believe that a powerful way to develop preservice teacher knowledge is exposing them to new modes of learning and teaching that challenge their own classroom experiences as students (Watson & Mason, 2007). We also believe in the power or reflection in teacher training (Cooney, 1999). Hence at BSME we (1) provide productive struggle to PSTs by posing them mathematical tasks from secondary school setting that are challenging at their own level; and (2) have them reflect on this problem-solving experience from a teacher's point of view.

Benefits of grappling with challenging mathematical tasks include:

- PSTs learn how to think like mathematicians. They improve in problem solving, experimenting, problem posing, definition making, and communication. (CK)
- PSTs' views of mathematics as a discipline are developed and/or refined. (CK, PCK)
- PSTs experience a pedagogical approach that likely differs from what they have seen in their own education; thus their view of teaching is further developed and/or refined (Liljedahl et al., 2009). (PCK, PK)

The mathematical tasks we pose PSTs possess the following properties. First, they have a "low threshold, high ceiling" nature, i.e., accessible without much prerequisite knowledge, but offering possibilities for rich exploration; this is important, since we want PSTs to understand that all students can have authentic mathematical experiences. Second, the tasks have multiple entry points, or different ways in which they can be approached. Third, the tasks have complexity and structure that require students to persevere in solving them and to reflect on their strategies.

Reflection is an essential component of any learning experience (Mason & Johnston-Wilder, 2006). It is particularly important for PSTs, who are learning about the learning process itself (Cooney, 1999). Typical reflectional prompts include:

- PSTs reflect on mathematical content and experience: big underlying ideas, different solution approaches, difficulties faced. (CK, PCK)
- PSTs analyze the pedagogical context of the task: target student age, prerequisite knowledge, common errors, follow-up activities, curricular connections. (PCK)
- PSTs reflect on pedagogical approaches used. (PCK, PK)

- PSTs consider possible adaptations for different groups of students by modifying the task in content, difficulty levels, and instructional methods. (PCK)

(Excerpted from: (Juhász, Kiss, Matsuura & Szász, 2016)).

A mathematical task

Workshop participants are given a mathematical task with which the BSME students engage in the role of students. The task was created by Lajos Pósa as a modified version of a task by Edward De Bono (1967), and it is originally used in a secondary school setting.

> *There are five matchboxes. Build a construction where*
>
> *(a) each box touches two others*
>
> *(b) each box touches three others*
>
> *(c) each box touches four others*
>
> *By "touching" we mean that two sides touch with a positive area.*

Reflection

Workshop participants are given questions that BSME students would use to reflect on the experience of engaging with the mathematical task.

They work in four groups, where each group has a different focus for reflection.

Group 1: This task is a modified version of a task by Edward De Bono (1967), where there are 6 boxes, and 2 / 3 / 4 / 5 touches are required (the modified version was created by Lajos Pósa). Compare the two tasks.

Group 2: In what ways could you use this problem in your future classroom? What grade level, curricular area and purpose?

Group 3: What hints can you give to students who struggle with the task, and what additional challenges can you give to those who solve it faster?

Group 4: Design a problem that could be assigned on a previous lesson to scaffold the matchbox task, or a problem assigned on a later lesson that builds on it.

Conclusion

The aim of this phase is for participants to share and discuss the BSME experience.

They form groups of four, one person from each group of the reflection phase. They share what they group established, and reflect on their experience with the BSME method.

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