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COEFFICIENT ESTIMATES FOR A NEW SUBCLASS OF ANALYTIC AND BI-UNIVALENT FUNCTIONS DEFINED BY CONVOLUTION

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Abstract. In this paper, we introduce and investigate a new subclass $\mathcal{H}_{\Sigma}^{h,p}(\lambda;\Theta)$ of analytic and bi-univalent functions in the open unit disk $\mathbb{U}$. For functions belonging to this class, we obtain estimates on the first two Taylor-Maclaurin coefficients $|a_2|$ and $|a_3|$.

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1. INTRODUCTION

Let $\mathcal{A}$ denote the class of all functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1.1}$$

which are analytic in the open unit disk

$$\mathbb{U} = \{z : z \in \mathbb{C} \quad \text{and} \quad |z| < 1\}.$$

We also denote by $\mathcal{S}$ the class of all functions in the normalized analytic function class $\mathcal{A}$ which are univalent in $\mathbb{U}$.

Since univalent functions are one-to-one, they are invertible and the inverse functions need not be defined on the entire unit disk $\mathbb{U}$. In fact, the Koebe one-quarter theorem [7] ensures that the image of $\mathbb{U}$ under every univalent function $f \in \mathcal{S}$ contains a disk of radius $1/4$. Thus every function $f \in \mathcal{A}$ has an inverse f^{-1}, which is defined by

$$f^{-1}(f(z)) = z \qquad (z \in \mathbb{U})$$

and

$$f\left(f^{-1}(w)\right) = w \qquad \left(|w| < r_0(f);\; r_0(f) \geq \frac{1}{4}\right).$$

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In fact, the inverse function f^{-1} is given by

$$f^{-1}(w) = w - a_2 w^2 + \left(2a_2^2 - a_3\right) w^3 - \left(5a_2^3 - 5a_2 a_3 + a_4\right) w^4 + \cdots .$$

Denote by $f * \Theta$ the Hadamard product (or convolution) of the functions f and Θ, that is, if f is given by (1.1) and Θ is given by

$$\Theta(z) = z + \sum_{n=2}^{\infty} b_n z^n \qquad (b_n > 0), \tag{1.2}$$

then

$$(f * \Theta)(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n . \tag{1.3}$$

For two functions f and Θ, analytic in $\mathbb{U}$, we say that the function f is subordinate to Θ in $\mathbb{U}$, and write

$$f(z) \prec \Theta(z) \qquad (z \in \mathbb{U}),$$

if there exists a Schwarz function ω, which is analytic in $\mathbb{U}$ with

$$\omega(0) = 0 \qquad \text{and} \qquad |\omega(z)| < 1 \quad (z \in \mathbb{U})$$

such that

$$f(z) = \Theta(\omega(z)) \quad (z \in \mathbb{U}).$$

Indeed, it is known that

$$f(z) \prec \Theta(z) \quad (z \in \mathbb{U}) \Rightarrow f(0) = \Theta(0) \quad \text{and} \quad f(\mathbb{U}) \subset \Theta(\mathbb{U}).$$

Furthermore, if the function Θ is univalent in $\mathbb{U}$, then we have the following equivalence

$$f(z) \prec \Theta(z) \quad (z \in \mathbb{U}) \Leftrightarrow f(0) = \Theta(0) \quad \text{and} \quad f(\mathbb{U}) \subset \Theta(\mathbb{U}).$$

A function $f \in \mathcal{A}$ is said to be bi-univalent in $\mathbb{U}$ if both f and f^{-1} are univalent in $\mathbb{U}$. Let Σ denote the class of bi-univalent functions in $\mathbb{U}$ given by (1.1). For a brief history and interesting examples of functions in the class Σ, see [13] (see also [2]). In fact, the aforecited work of Srivastava *et al.* [13] essentially revived the investigation of various subclasses of the bi-univalent function class Σ in recent years; it was followed by such works as those by El-Ashwah [8], Frasin and Aouf [9], Aouf *et al.* [1], and others (see, for example, [3–6, 10–12, 14]).

Definition 1. Let the functions $h, p : \mathbb{U} \to \mathbb{C}$ be so constrained that

$$\min\{\Re(h(z)), \Re(p(z))\} > 0 \quad (z \in \mathbb{U}) \quad \text{and} \quad h(0) = p(0) = 1.$$

Also let the function f, defined by (1.1), be in the analytic function class $\mathcal{A}$ and let $\Theta \in \Sigma$. We say that

$$f \in \mathcal{H}_{\Sigma}^{h,p}(\lambda; \Theta) \qquad (\lambda \geq 1)$$

if the following conditions are satisfied:

$$f \in \Sigma \quad \text{and} \quad (1-\lambda)\frac{(f*\Theta)(z)}{z} + \lambda (f*\Theta)'(z) \in h(\mathbb{U}) \quad (z \in \mathbb{U}) \tag{1.4}$$

and

$$(1-\lambda)\frac{(f*\Theta)^{-1}(w)}{w} + \lambda\left((f*\Theta)^{-1}\right)'(w) \in p(\mathbb{U}) \quad (w \in \mathbb{U}), \tag{1.5}$$

where the function $(f*\Theta)^{-1}$ is defined by

$$\begin{aligned}(f*\Theta)^{-1}(w) = w - a_2b_2w^2 + \left(2a_2^2b_2^2 - a_3b_3\right)w^3 \\ - \left(5a_2^3b_2^3 - 5a_2b_2a_3b_3 + a_4b_4\right)w^4 + \cdots.\end{aligned} \tag{1.6}$$

Remark 1. If we let

$$\Theta(z) = \frac{z}{1-z} = z + \sum_{n=2}^{\infty} z^n,$$

then the class $\mathcal{H}_{\Sigma}^{h,p}(\lambda;\Theta)$ reduces to the class denoted by $\mathcal{B}_{\Sigma}^{h,p}(\lambda)$ which is the subclass of the functions $f \in \Sigma$ satisfying

$$(1-\lambda)\frac{f(z)}{z} + \lambda f'(z) \in h(\mathbb{U})$$

and

$$(1-\lambda)\frac{g(w)}{w} + \lambda g'(w) \in p(\mathbb{U}),$$

where the function g is defined by

$$g(w) = w - a_2w^2 + \left(2a_2^2 - a_3\right)w^3 - \left(5a_2^3 - 5a_2a_3 + a_4\right)w^4 + \cdots. \tag{1.7}$$

This class is introduced and studied by Xu *et al.* [15]. Also we get the function class

$$\mathcal{H}_{\Sigma}^{h,p}\left(1;\frac{z}{1-z}\right) = \mathcal{H}_{\Sigma}^{h,p}$$

introduced and studied by Xu *et al.* [14].

Remark 2. If we let

$$h(z) = \left(\frac{1+z}{1-z}\right)^{\alpha} \quad \text{and} \quad p(z) = \left(\frac{1-z}{1+z}\right)^{\alpha} \qquad (0 < \alpha \le 1),$$

then the class $\mathcal{H}_{\Sigma}^{h,p}(\lambda;\Theta)$ reduces to the class denoted by $\mathcal{B}_{\Sigma}(\Theta,\alpha,\lambda)$ which is the subclass of the functions $f \in \Sigma$ satisfying

$$\left|\arg\left\{(1-\lambda)\frac{(f*\Theta)(z)}{z} + \lambda (f*\Theta)'(z)\right\}\right| < \frac{\alpha\pi}{2}$$

and

$$\left|\arg\left\{(1-\lambda)\frac{(f*\Theta)^{-1}(w)}{w} + \lambda\left((f*\Theta)^{-1}\right)'(w)\right\}\right| < \frac{\alpha\pi}{2},$$

where the function $(f * \Theta)^{-1}$ is defined by (1.6). This class is introduced and studied by El-Ashwah [8]. In this class,

(i) if we set

$$\Theta(z) = \frac{z}{1-z} = z + \sum_{n=2}^{\infty} z^n,$$

then the class $\mathcal{B}_\Sigma(\Theta, \alpha, \lambda)$ reduces to the class denoted by $\mathcal{B}_\Sigma(\alpha, \lambda)$ introduced and studied by Frasin and Aouf [9]. Also we have the class

$$\mathcal{B}_\Sigma(\alpha, 1) = \mathcal{H}_\Sigma^\alpha$$

introduced and studied by Srivastava *et al.* [13].

(ii) if we set

$$\Theta(z) = z + \sum_{n=2}^{\infty} \frac{(a_1)_n \cdots (a_q)_n}{(b_1)_n \cdots (b_s)_n} \frac{1}{n!} z^n,$$

then the class $\mathcal{B}_\Sigma(\Theta, \alpha, \lambda)$ reduces to the class denoted by $\mathcal{T}_{q,s}^{\Sigma}[a_1; b_1, \alpha, \lambda]$ introduced and studied Aouf *et al.* [1].

Remark 3. If we let

$$h(z) = \frac{1+(1-2\beta)z}{1-z} \quad \text{and} \quad p(z) = \frac{1-(1-2\beta)z}{1+z} \qquad (0 \leq \beta < 1),$$

then the class $\mathcal{H}_\Sigma^{h,p}(\lambda; \Theta)$ reduces to the class denoted by $\mathcal{B}_\Sigma(\Theta, \beta, \lambda)$ which is the subclass of the functions $f \in \Sigma$ satisfying

$$\Re\left\{(1-\lambda)\frac{(f*\Theta)(z)}{z} + \lambda (f*\Theta)'(z)\right\} > \beta$$

and

$$\Re\left\{(1-\lambda)\frac{(f*\Theta)^{-1}(w)}{w} + \lambda\left((f*\Theta)^{-1}\right)'(w)\right\} > \beta,$$

where the function $(f * \Theta)^{-1}$ is defined by (1.6). This class is introduced and studied by El-Ashwah [8]. In this class,

(i) if we set

$$\Theta(z) = \frac{z}{1-z} = z + \sum_{n=2}^{\infty} z^n,$$

then the class $\mathcal{B}_\Sigma(\Theta, \beta, \lambda)$ reduces to the class denoted by $\mathcal{B}_\Sigma(\beta, \lambda)$ introduced and studied by Frasin and Aouf [9]. Also we have the class

$$\mathcal{B}_\Sigma(\beta, 1) = \mathcal{H}_\Sigma(\beta)$$

introduced and studied by Srivastava *et al.* [13].

(ii) if we set

$$\Theta(z) = z + \sum_{n=2}^{\infty} \frac{(a_1)_n \cdots (a_q)_n}{(b_1)_n \cdots (b_s)_n} \frac{1}{n!} z^n,$$

then the class $\mathcal{B}_\Sigma(\Theta, \beta, \lambda)$ reduces to the class denoted by $\mathcal{T}_{q,s}^{\Sigma}[a_1; b_1, \beta, \lambda]$ introduced and studied by Aouf *et al.* [1].

2. A SET OF GENERAL COEFFICIENT ESTIMATES

In this section, we state and prove our general results involving the bi-univalent function class $\mathcal{H}_{\Sigma}^{h,p}(\lambda; \Theta)$ given by Definition 1.

Theorem 1. *Let the function $f(z)$ given by the Taylor-Maclaurin series expansion (1.1) be in the function class*

$$\mathcal{H}_{\Sigma}^{h,p}(\lambda; \Theta) \qquad (\lambda \geq 1)$$

with

$$\Theta \in \Sigma \qquad \text{and} \qquad \Theta(z) = z + \sum_{n=2}^{\infty} b_n z^n \qquad (b_n > 0).$$

Then

$$|a_2| \leq \frac{1}{b_2} \min\left\{ \sqrt{\frac{|h'(0)|^2 + |p'(0)|^2}{2(1+\lambda)^2}}, \sqrt{\frac{|h''(0)| + |p''(0)|}{4(1+2\lambda)}} \right\} \tag{2.1}$$

and

$$|a_3| \leq \frac{1}{b_3} \min\left\{ \frac{|h'(0)|^2 + |p'(0)|^2}{2(1+\lambda)^2} + \frac{|h''(0)| + |p''(0)|}{4(1+2\lambda)}, \frac{|h''(0)|}{2(1+2\lambda)} \right\}. \tag{2.2}$$

Proof. First of all, we write the argument inequalities in (1.4) and (1.5) in their equivalent forms as follows:

$$(1-\lambda)\frac{(f*\Theta)(z)}{z} + \lambda (f*\Theta)'(z) = h(z) \quad (z \in \mathbb{U})$$

and

$$(1-\lambda)\frac{(f*\Theta)^{-1}(w)}{w} + \lambda \left((f*\Theta)^{-1}\right)'(w) = p(w) \quad (w \in \mathbb{U}),$$

respectively, where $h(z)$ and $p(w)$ satisfy the conditions of Definition 1. Furthermore, the functions $h(z)$ and $p(w)$ have the following Taylor-Maclaurin series expansions:

$$h(z) = 1 + h_1 z + h_2 z^2 + \cdots$$

and

$$p(w) = 1 + p_1 w + p_2 w^2 + \cdots,$$

respectively. Now, upon equating the coefficients of

$$(1-\lambda)\frac{(f*\Theta)(z)}{z}+\lambda(f*\Theta)'(z)$$

with those of $h(z)$ and the coefficients of

$$(1-\lambda)\frac{(f*\Theta)^{-1}(w)}{w}+\lambda\left((f*\Theta)^{-1}\right)'(w)$$

with those of $p(w)$, we get

$$(1+\lambda)b_2a_2 = h_1, \tag{2.3}$$

$$(1+2\lambda)b_3a_3 = h_2, \tag{2.4}$$

$$-(1+\lambda)b_2a_2 = p_1 \tag{2.5}$$

and

$$-(1+2\lambda)b_3a_3+2(1+2\lambda)b_2^2a_2^2 = p_2. \tag{2.6}$$

From (2.3) and (2.5), it follows that

$$h_1 = -p_1 \tag{2.7}$$

and

$$2(1+\lambda)^2b_2^2a_2^2 = h_1^2+p_1^2. \tag{2.8}$$

Also from (2.4) and (2.6), we find that

$$2(1+2\lambda)b_2^2a_2^2 = h_2+p_2. \tag{2.9}$$

Therefore, we find from the equations (2.8) and (2.9) that

$$|a_2|^2 \leq \frac{|h'(0)|^2+|p'(0)|^2}{2(1+\lambda)^2b_2^2}$$

and

$$|a_2|^2 \leq \frac{|h''(0)|+|p''(0)|}{4(1+2\lambda)b_2^2},$$

respectively. So we get the desired estimate on the coefficient $|a_2|$ as asserted in (2.1).

Next, in order to find the bound on the coefficient $|a_3|$, by subtracting (2.6) from (2.4), we obtain

$$2(1+2\lambda)b_3a_3-2(1+2\lambda)b_2^2a_2^2 = h_2-p_2. \tag{2.10}$$

Upon substituting the value of a_2^2 from (2.8) into (2.10), it follows that

$$a_3 = \frac{h_1^2+p_1^2}{2(1+\lambda)^2b_3}+\frac{h_2-p_2}{2(1+2\lambda)b_3}.$$

We thus find that

$$|a_3| \le \frac{|h'(0)|^2 + |p'(0)|^2}{2(1+\lambda)^2 b_3} + \frac{|h''(0)| + |p''(0)|}{4(1+2\lambda) b_3}.$$

On the other hand, upon substituting the value of a_2^2 from (2.9) into (2.10), it follows that

$$a_3 = \frac{h_2}{(1+2\lambda) b_3}.$$

Consequently, we have

$$|a_3| \le \frac{|h''(0)|}{2(1+2\lambda) b_3}.$$

This evidently completes the proof of Theorem 1. □

Remark 4. If we take

$$\Theta(z) = \frac{z}{1-z} = z + \sum_{n=2}^{\infty} z^n$$

in Theorem 1, then we have the estimates obtained by Srivastava *et al.* [12, Corollary 1] which is an improvement of the estimates given by Xu *et al.* [15, Theorem 3]. In addition the above condition on Θ, if we set $\lambda = 1$, then we get [14, Theorem 3].

If we let

$$h(z) = \left(\frac{1+z}{1-z}\right)^{\alpha} \quad \text{and} \quad p(z) = \left(\frac{1-z}{1+z}\right)^{\alpha} \qquad (0 < \alpha \le 1)$$

in Theorem 1, then we have Corollary 1 below.

Corollary 1. *Let the function $f(z)$ given by the Taylor-Maclaurin series expansion (1.1) be in the function class*

$$\mathcal{B}_{\Sigma}(\Theta, \alpha, \lambda) \qquad (\lambda \ge 1, 0 < \alpha \le 1)$$

with

$$\Theta \in \Sigma \qquad \textit{and} \qquad \Theta(z) = z + \sum_{n=2}^{\infty} b_n z^n \qquad (b_n > 0).$$

Then

$$|a_2| \le \frac{1}{b_2} \begin{cases} \frac{2\alpha}{1+\lambda} & , \quad \lambda \ge 1+\sqrt{2} \\ \sqrt{\frac{2}{1+2\lambda}}\alpha & , \quad 1 \le \lambda < 1+\sqrt{2} \end{cases}$$

and

$$|a_3| \le \frac{2\alpha^2}{(1+2\lambda) b_3}.$$

Remark 5. It is easy to see, for the coefficient $|a_2|$, that

$$\frac{2\alpha}{1+\lambda} \leq \frac{2\alpha}{\sqrt{(1+\lambda)^2+\alpha\left(1+2\lambda-\lambda^2\right)}} \qquad \left(0<\alpha\leq 1\,,\,\lambda\geq 1+\sqrt{2}\right)$$

and

$$\sqrt{\frac{2}{1+2\lambda}}\alpha \leq \frac{2\alpha}{\sqrt{(1+\lambda)^2+\alpha\left(1+2\lambda-\lambda^2\right)}} \qquad \left(0<\alpha\leq 1\,,\,1\leq\lambda<1+\sqrt{2}\right).$$

On the other hand, for the coefficient $|a_3|$, we have

$$\begin{aligned}\frac{2\alpha^2}{1+2\lambda} &\leq \frac{2\alpha}{1+2\lambda}\\ &\leq \frac{4\alpha^2}{(1+\lambda)^2}+\frac{2\alpha}{1+2\lambda} \qquad (0<\alpha\leq 1\,,\,\lambda\geq 1)\,.\end{aligned}$$

Thus, clearly, Corollary 1 is an improvement of the estimates obtained by El-Ashwah [8, Theorem 1].

Remark 6. **(i)** If we take

$$\Theta(z)=\frac{z}{1-z}=z+\sum_{n=2}^{\infty}z^n$$

in Corollary 1, then we have an improvement of the estimates obtained by Frasin and Aouf [9, Theorem 2.2]. In addition the above condition on Θ, if we set $\lambda=1$, then we get an improvement of the estimates obtained by Srivastava *et al.* [13, Theorem 1].

(ii) If we take

$$\Theta(z)=z+\sum_{n=2}^{\infty}\frac{(a_1)_n\cdots\left(a_q\right)_n}{(b_1)_n\cdots(b_s)_n}\frac{1}{n!}z^n$$

in Corollary 1, then we have an improvement of the estimates obtained by Aouf *et al.* [1, Theorem 4].

If we let

$$h(z)=\frac{1+(1-2\beta)z}{1-z} \quad \text{and} \quad p(z)=\frac{1-(1-2\beta)z}{1+z} \qquad (0\leq\beta<1)$$

in Theorem 1, then we have Corollary 2 below.

Corollary 2. *Let the function $f(z)$ given by the Taylor-Maclaurin series expansion (1.1) be in the function class*

$$\mathcal{B}_{\Sigma}(\Theta,\beta,\lambda) \qquad (\lambda\geq 1\,,\,0\leq\beta<1)$$

with

$$\Theta \in \Sigma \qquad \textit{and} \qquad \Theta(z) = z + \sum_{n=2}^{\infty} b_n z^n \qquad (b_n > 0).$$

Then

$$|a_2| \leq \frac{1}{b_2} \min\left\{ \frac{2(1-\beta)}{1+\lambda}, \sqrt{\frac{2(1-\beta)}{1+2\lambda}} \right\}$$

and

$$|a_3| \leq \frac{2(1-\beta)}{(1+2\lambda)\,b_3}.$$

Remark 7. Corollary 2 is an improvement of the estimates obtained by El-Ashwah [8, Theorem 2].

Remark 8. **(i)** If we take

$$\Theta(z) = \frac{z}{1-z} = z + \sum_{n=2}^{\infty} z^n$$

in Corollary 2, then we have an improvement of the estimates obtained by Frasin and Aouf [9, Theorem 3.2]. In addition the above condition on Θ, if we set $\lambda = 1$, then we get an improvement of the estimates obtained by Srivastava *et al.* [13, Theorem 2].

(ii) If we take

$$\Theta(z) = z + \sum_{n=2}^{\infty} \frac{(a_1)_n \cdots (a_q)_n}{(b_1)_n \cdots (b_s)_n} \frac{1}{n!} z^n$$

in Corollary 2, then we have an improvement of the estimates obtained by Aouf *et al.* [1, Theorem 8].

References

[1] M. Aouf, R. El-Ashwah, and A. Abd-Eltawab, "New subclasses of biunivalent functions involving Dziok-Srivastava operator," *ISRN Math. Anal.*, vol. 2013, no. Art. ID 387178, p. 5 pp., 2013, doi: 10.1155/2013/387178.

[2] D. Brannan and T. Taha, "On some classes of bi-univalent functions," *Studia Univ. Babeş -Bolyai Math.*, vol. 31, no. 2, pp. 70–77, 1986.

[3] S. Bulut, "Coefficient estimates for a class of analytic and bi-univalent functions," *Novi Sad J. Math.*, vol. 43, no. 2, pp. 59–65, 2013.

[4] S. Bulut, "Coefficient estimates for new subclasses of analytic and bi-univalent functions defined by Al-Oboudi differential operator," *J. Funct. Spaces Appl.*, vol. 2013, no. Art. ID 181932, p. 7 pp., 2013, doi: 10.1155/2013/181932.

[5] S. Bulut, "Coefficient estimates for a new subclass of analytic and bi-univalent functions," *Analele Stiint. Univ. Al. I. Cuza Iasi Math.*, in press.

[6] M. Çağlar, H. Orhan, and N. Yağmur, "Coefficient bounds for new subclasses of bi-univalent functions," *Filomat*, vol. 27, no. 7, pp. 1165–1171, 2013.

[7] P. Duren, *Univalent Functions*, ser. Grundlehren der Mathematischen Wissenschaften. New York: Springer, 1983, vol. 259.
[8] R. El-Ashwah, "Subclasses of bi-univalent functions defined by convolution," *J. Egyptian Math. Soc.*, vol. 22, no. 3, pp. 348–351, 2014, doi: 10.1016/j.joems.2013.06.017.
[9] B. Frasin and M. Aouf, "New subclasses of bi-univalent functions," *Appl. Math. Lett.*, vol. 24, pp. 1569–1573, 2011, doi: 10.1016/j.aml.2011.03.048.
[10] T. Hayami and S. Owa, "Coefficient bounds for bi-univalent functions," *Pan Amer. Math. J.*, vol. 22, no. 4, pp. 15–26, 2012.
[11] S. Porwal and M. Darus, "On a new subclass of bi-univalent functions," *J. Egyptian Math. Soc.*, vol. 21, no. 3, pp. 190–193, 2013, doi: 10.1016/j.joems.2013.02.007.
[12] H. Srivastava, S. Bulut, M. Çağlar, and N. Yağmur, "Coefficient estimates for a general subclass of analytic and bi-univalent functions," *Filomat*, vol. 27, no. 5, pp. 831–842, 2013, doi: 10.2298/FIL1305831S.
[13] H. Srivastava, A. Mishra, and P. Gochhayat, "Certain subclasses of analytic and bi-univalent functions," *Appl. Math. Lett.*, vol. 23, pp. 1188–1192, 2010, doi: 10.1016/j.aml.2010.05.009.
[14] Q.-H. Xu, Y.-C. Gui, and H. Srivastava, "Coefficient estimates for a certain subclass of analytic and bi-univalent functions," *Appl. Math. Lett.*, vol. 25, pp. 990–994, 2012, doi: 10.1016/j.aml.2011.11.013.
[15] Q.-H. Xu, H.-G. Xiao, and H. Srivastava, "A certain general subclass of analytic and bi-univalent functions and associated coefficient estimate problems," *Appl. Math. Comput.*, vol. 218, pp. 11 461–11 465, 2012, doi: 10.1016/j.amc.2012.05.034.

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