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GENERALIZED DERIVATIONS ACTING AS HOMOMORPHISM OR ANTI-HOMOMORPHISM WITH CENTRAL VALUES IN SEMIPRIME RINGS

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Abstract. Let R be a semiprime ring with center $Z(R)$. A mapping $F : R \to R$ is called a generalized derivation if there exists a derivation $d : R \to R$ such that $F(xy) = F(x)y + xd(y)$ holds for all $x, y \in R$. In the present paper, our main object is to study the situations: (1) $F(xy) - F(x)F(y) \in Z(R)$, (2) $F(xy) + F(x)F(y) \in Z(R)$, (3) $F(xy) - F(y)F(x) \in Z(R)$, (4) $F(xy) + F(y)F(x) \in Z(R)$; for all x, y in some suitable subset of R.

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1. INTRODUCTION

Let R be an associative ring with center $Z(R)$. For $x, y \in R$, $[x, y]$ denotes the commutator element $xy - yx$. We use the notation to define the Engel type polynomial $[x, y]_{n+1} = [[x, y]_n, y]$ instead of $[x, y, y, \dots, y]$ for $n \geq 1$ and $[x, y]_1 = [x, y]$. Recall that a ring R is called prime if for any $a, b \in R$, $aRb = (0)$ implies that either $a = 0$ or $b = 0$ and is called semiprime if for any $a \in R$, $aRa = (0)$ implies $a = 0$. An additive mapping $F : R \to R$ is called a generalized derivation of R if there exists a derivation $d : R \to R$ such that $F(xy) = F(x)y + xd(y)$ holds for any $x, y \in R$. If $d = 0$, then F is said to be a left centralizer map of R. For any subset S of R, $r_R(S)$ denotes the right annihilator of S in R, that is, $r_R(S) = \{x \in R | Sx = 0\}$ and $l_R(S)$ denotes the left annihilator of S in R that is, $l_R(S) = \{x \in R | xS = 0\}$. If $r_R(S) = l_R(S)$, then $r_R(S)$ is called an annihilator ideal of R and is written as $ann_R(S)$.

Let S be a nonempty subset of a ring R. The mapping $F : R \to R$ is said to be a homomorphism (anti-homomorphism) acting on S if $F(xy) = F(x)F(y)$ holds for all $x, y \in S$ (respectively $F(xy) = F(y)F(x)$ holds for all $x, y \in S$).

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A series of papers in literature studied the homomorphism or anti-homomorphism of some specific type of additive mappings in prime and semiprime rings under certain conditions (see [1–4, 7, 10–12, 14, 15]).

In [4], Bell and Kappe showed that if a derivation d of a prime ring R can act as homomorphism or anti-homomorphism on a nonzero right ideal of R, then $d = 0$ on R. Then Ali, Rehman and Ali in [2] proved a similar result to Lie ideal case. They proved that if R is a 2-torsion free prime ring, L a nonzero Lie ideal of R such that $u^2 \in L$ for all $u \in L$ and d acts as a homomorphism or anti-homomorphism on L, then either $d = 0$ or $L \subseteq Z(R)$.

On the other hand, the authors developed above results, replacing the derivation d with a generalized derivation F of R. In this view, Rehman [14] proved the following:

Let R be a 2-torsion free prime ring and I be a nonzero ideal of R. Suppose $F : R \to R$ is a nonzero generalized derivation with d.

(i) If F acts as a homomorphism on I and if $d \neq 0$, then R is commutative.

(ii) If F acts as an anti-homomorphism on I and if $d \neq 0$, then R is commutative.

Recently, in [3] Ali and Huang studied the case when a generalized Jordan (α,β)-derivation F acts as homomorphism or anti-homomorphism on a square closed Lie ideal U in prime ring R.

It is natural to investigate the above situations in semiprime rings. Recently, in [7] the first author of this article has studied the situations, when a generalized derivation F of a semiprime ring R acts as homomorphism or anti-homomorphism in a nonzero left ideal of R.

From above results, it is natural to consider the situations, when the generalized derivations F satisfies the identities: (1) $F(xy) - F(x)F(y) \in Z(R)$, (2) $F(xy) + F(x)F(y) \in Z(R)$, (3) $F(xy) - F(y)F(x) \in Z(R)$, (4) $F(xy) + F(y)F(x) \in Z(R)$; for all x, y in some suitable subset of R.

Recently, Albas [1] studied the above mentioned identities in prime rings. Albas proved the following theorems:

Theorem A. *Let R be a prime ring with center $Z(R)$ and I be a nonzero ideal of R. If R admits a nonzero generalized derivation F of R, with associated derivation d such that $F(xy) - F(x)F(y) \in Z(R)$ or $F(xy) + F(x)F(y) \in Z(R)$ for all $x, y \in I$, then either R is commutative or $F = I_{id}$ or $F = -I_{id}$, where I_{id} denotes the identity map of the ring R.*

Theorem B. *Let R be a prime ring with center $Z(R)$ and I be a nonzero ideal of R. If R admits a nonzero generalized derivation F of R, with associated derivation d such that $F(xy) - F(y)F(x) \in Z(R)$ or $F(xy) + F(y)F(x) \in Z(R)$ for all $x, y \in I$, then R is commutative.*

In the present paper our main object is to investigate the situations in semiprime rings.

2. Preliminaries

We shall use following basic identities which will be used frequently: for $x, y, z \in R$,

$$[xy, z] = x[y, z] + [x, z]y \quad \text{and} \quad [x, yz] = y[x, z] + [x, y]z.$$

We need the following facts which will be used to prove our theorems:

Fact-1. [5, Theorem 3] *Let R be a semiprime ring and U a nonzero left ideal of R. If R admits a derivation d which is nonzero on U and $[d(x), x] \in Z(R)$ for all $x \in U$, then R contains a nonzero central ideal.*

Fact-2. [8, Fact-4]*Let R be a semiprime ring, d a nonzero derivation of R such that $x[[d(x), x], x] = 0$ for all $x \in R$. Then d maps R into its center.*

Fact-3. [13, Corollary 2] *If R is a semiprime ring and I is an ideal of R, then $I \cap ann_R(I) = 0$.*

Fact-4. [6, Lemma 2] *(a) If R is a semiprime ring, the center of a nonzero one-sided ideal is contained in the center of R; in particular, any commutative one-sided ideal is contained in the center of R.*
(b) If R is a prime ring with a nonzero central ideal, then R must be commutative.

Fact-5. [9, Corollary 2.6] *Let R be a prime ring, I a nonzero ideal of R and $F : R \to R$ a nonzero left centralizer map. (1) If $F(x)F(y) - F(xy) \in Z(R)$ for all $x, y \in I$, then either R is commutative or $F(r) = r$ for all $r \in R$. (2) If $F(x)F(y) + F(xy) \in Z(R)$ for all $x, y \in I$, then either R is commutative or $F(r) = -r$ for all $r \in R$.*

3. Main results

Theorem 1. *Let R be a semiprime ring with center $Z(R)$ and I a nonzero ideal of R. Let $F : R \to R$ be a generalized derivation associated with the derivation $d : R \to R$. If $F(xy) - F(x)F(y) \in Z(R)$ for all $x, y \in I$, then one of the following holds:*
(1) R contains a nonzero central ideal;
(2) $d(I) = (0)$ and F is a left centralizer map on I such that $[F(x), x] = 0$ for all $x \in I$.

Proof. By our assumption, we have

$$F(xy) - F(x)F(y) \in Z(R) \tag{3.1}$$

for all $x, y \in I$. Replacing y with yz, where $z \in I$, we get

$$F(xyz) - F(x)F(yz) \in Z(R) \tag{3.2}$$

which implies

$$F(xy)z + xyd(z) - F(x)\{F(y)z + yd(z)\} \in Z(R) \tag{3.3}$$

that is

$$(F(xy) - F(x)F(y))z + (x - F(x))yd(z) \in Z(R). \tag{3.4}$$

Commuting both sides with z, we get

$$[(F(xy) - F(x)F(y))z + (x - F(x))yd(z), z] = 0 \tag{3.5}$$

for all $x, y, z \in I$. By using (3.1), above relation yields

$$[(x - F(x))yd(z), z] = 0 \tag{3.6}$$

for all $x, y, z \in I$. Now we put $x = xz$, and then obtain that

$$[(xz - F(x)z - xd(z))yd(z), z] = 0 \tag{3.7}$$

which is

$$[(x - F(x))zyd(z), z] - [xd(z)yd(z), z] = 0 \tag{3.8}$$

for all $x, y, z \in I$. In (3.6), replacing y with zy, we get

$$[(x - F(x))zyd(z), z] = 0 \tag{3.9}$$

for all $x, y, z \in I$. Using (3.9), (3.8) implies

$$[xd(z)yd(z), z] = 0 \tag{3.10}$$

for all $x, y, z \in I$. Now we put $x = d(z)x$ in (3.10), and then we see that

$$0 = [d(z)xd(z)yd(z), z] = d(z)[xd(z)yd(z), z] + [d(z), z]xd(z)yd(z) \tag{3.11}$$

for all $x, y, z \in I$. As an application of (3.10), (3.11) reduces to

$$[d(z), z]xd(z)yd(z) = 0 \tag{3.12}$$

for all $x, y, z \in I$. Replacing x with xz and y with zy respectively in (3.12), we get

$$[d(z), z]xzd(z)yd(z) = 0 \tag{3.13}$$

and

$$[d(z), z]xd(z)zyd(z) = 0 \tag{3.14}$$

for all $x, y, z \in I$. Subtracting one from another yields

$$[d(z), z]x[d(z), z]yd(z) = 0 \tag{3.15}$$

for all $x, y, z \in I$. Replacing y with yz in (3.15) and right multiplying (3.15) by z respectively and then subtracting one from another yields

$$[d(z), z]x[d(z), z]y[d(z), z] = 0 \tag{3.16}$$

for all $x,y,z \in I$, which implies $(I[d(z),z])^3 = (0)$ for all $z \in I$. Since R is semiprime, it contains no nonzero nilpotent left ideal, implying $I[d(z),z] = (0)$ for all $z \in I$. Thus, $[d(z),z] \in Ann_R(I)$ for all $z \in I$. Since I is an ideal, we conclude that $[d(z),z] \in I$ for all $z \in I$. This implies that $[d(z),z] \in I \cap Ann_R(I)$ for all $z \in I$. In view of Fact-3, $[d(z),z] = 0$ for all $z \in I$. Further, if d is derivation such that $d(I) \neq (0)$, then by Fact-1, R contains a nonzero central ideal.

Let $d(I) = (0)$. Then $F(xy) = F(x)y + xd(y) = F(x)y$ for all $x,y \in I$, i.e., F is a left centralizer map on I. Then by our hypothesis, we have

$$F(x)(y - F(y)) \in Z(R) \tag{3.17}$$

for all $x,y \in I$. Replacing y with yu, where $u \in I$, we get

$$F(x)(y - F(y))u \in Z(R) \tag{3.18}$$

for all $x,y,u \in I$. Commuting both sides with v, where $v \in I$, we get

$$F(x)(y - F(y))uv - vF(x)(y - F(y))u = 0. \tag{3.19}$$

By using (3.18), it reduces to

$$vF(x)(y - F(y))u \in Z(R) \tag{3.20}$$

for all $u,v,x,y \in I$. We choose $x,y \in I$ such that $a = F(x)(y - F(y)) \neq 0$. Then from above, we have $IaI \subseteq Z(R)$, that is, R contains a central ideal. If this ideal is zero ideal, then

$$I(F(x)(y - F(y))) = (0)$$

for all $x,y \in I$. Replacing x with xz, $z \in I$, this gives

$$I(F(x)z(y - F(y))) = (0)$$

for all $x,y,z \in I$. Thus $F(x)z(y - F(y)) \in I \cap ann_R(I) = (0)$ for all $x,y,z \in I$. This gives

$$[F(x),x]z(y - F(y)) = 0$$

for all $x,y,z \in I$. Putting $y = y^2$ and $z = zy$ respectively and then subtracting one from another, we get

$$[F(x),x]z[F(y),y] = 0$$

for all $x,y,z \in I$. Since I is an ideal of R, it follows that $([F(x),x]I)^2 = (0)$ for all $x \in I$. Since semiprime ring contains no nonzero nilpotent ideal, we have $[F(x),x]I = (0)$ for all $x \in I$. Thus by Fact-3, $[F(x),x] \in I \cap ann_R(I) = (0)$ for all $x \in I$. Thereby, the proof is completed. □

Theorem 2. *Let R be a semiprime ring with center $Z(R)$ and I a nonzero ideal of R. Let $F : R \to R$ be a generalized derivation associated with the derivation $d : R \to R$. If $F(xy) + F(x)F(y) \in Z(R)$ for all $x,y \in I$, then one of the following holds:*

(1) R contains a nonzero central ideal;

(2) $d(I) = (0)$ and F is a left centralizer map on I such that $[F(x), x] = 0$ for all $x \in I$.

Proof. If we replace F with $-F$ and d with $-d$ in Theorem 1, we conclude that $(-F)(xy) - (-F)(x)(-F)(y) \in Z(R)$ for all $x, y \in I$, implies $[(-d)(x), x] = 0$ for all $x \in I$, that is, $F(xy) + F(x)F(y) \in Z(R)$ for all $x, y \in I$, implies $[d(x), x] = 0$ for all $x \in I$. Hence conclusion follows by Theorem 1. □

The following corollary is immediate consequences of the Theorem 1 and Theorem 2 by using Fact-4 and Fact-5.

Corollary 1. *Let R be a prime ring with center $Z(R)$ and $F : R \to R$ be a generalized derivation associated with the derivation $d : R \to R$.*
(1) If R satisfies $F(xy) + F(x)F(y) \in Z(R)$, then either R is commutative or $F(x) = -x$ for all $x \in R$.
(2) If R satisfies $F(xy) - F(x)F(y) \in Z(R)$, then either R is commutative or $F(x) = x$ for all $x \in R$.

Theorem 3. *Let R be a semiprime ring with center $Z(R)$. Let $F : R \to R$ be a generalized derivation associated with the derivation $d : R \to R$. If $F(xy) - F(y)F(x) \in Z(R)$ for all $x, y \in R$, then one of the following holds: (1) R contains a nonzero central ideal; (2) F is a left centralizer map of R such that $F : R \to Z(R)$.*

Proof. By hypothesis, we have

$$F(xy) - F(y)F(x) \in Z(R) \tag{3.21}$$

for all $x, y \in R$. Putting $x = xz$, we have

$$F(xzy) - F(y)(F(x)z + xd(z)) \in Z(R) \tag{3.22}$$

which gives

$$F(x)zy + xd(zy) - F(y)F(x)z - F(y)xd(z) \in Z(R). \tag{3.23}$$

Commuting both sides with z, we have

$$[F(x)zy - F(y)F(x)z - F(y)xd(z) + xd(zy), z] = 0 \tag{3.24}$$

that is

$$[F(x)zy, z] - [F(y)F(x), z]z - [F(y)xd(z) - xd(zy), z] = 0 \tag{3.25}$$

for all $x, y, z \in R$. From (3.21), we can write that $[F(xy) - F(y)F(x), z] = 0$ for all $x, y, z \in R$, that is, $[F(xy), z] = [F(y)F(x), z]$ for all $x, y, z \in R$. Thus (3.25) reduces to

$$[F(x)zy, z] - [F(xy), z]z - [F(y)xd(z) - xd(zy), z] = 0 \tag{3.26}$$

for all $x, y, z \in R$. Putting $y = z^2$ in (3.26), we have

$$[F(x)z^3, z] - [F(x)z^2 + xd(z^2), z]z - [(F(z)z + zd(z))xd(z) - xd(z^3), z] = 0 \tag{3.27}$$

that is,

$$[F(z)zxd(z)+zd(z)xd(z)-xd(z^3)+xd(z^2)z,z]=0 \tag{3.28}$$

for all $x,z\in R$. Putting $x=zx$ in (3.26), we have

$$[(F(z)x+zd(x))zy,z]-[F(z)xy+zd(xy),z]z-[F(y)zxd(z)-zxd(zy),z]=0 \tag{3.29}$$

that is,

$$\begin{aligned}&[F(z)xzy,z]-[F(z)xy,z]z-\\&\qquad[F(y)zxd(z)-zxd(zy)-zd(x)zy+zd(xy)z,z]=0\end{aligned} \tag{3.30}$$

for all $x,y,z\in R$. Assuming $y=z$, we have

$$[F(z)zxd(z)-zxd(z^2)-zd(x)z^2+zd(xz)z,z]=0 \tag{3.31}$$

for all $x,z\in R$. Subtracting (3.31) from (3.28), we get

$$[zd(z)xd(z)-xd(z^3)+xd(z^2)z+zxd(z^2)+zd(x)z^2-zd(xz)z,z]=0 \tag{3.32}$$

for all $x,z\in R$. This reduces to

$$[zd(z)xd(z),z]+[-xd(z^3)+xd(z^2)z+zxd(z^2)-zxd(z)z,z]=0 \tag{3.33}$$

for all $x,z\in R$. Now putting $x=zx$ in (3.33), we get

$$[zd(z)zxd(z),z]+z[-xd(z^3)+xd(z^2)z+zxd(z^2)-zxd(z)z,z]=0 \tag{3.34}$$

for all $x,z\in R$. Left multiplying (3.33) by z and then subtracting from (3.34), we get

$$[z[d(z),z]xd(z),z]=0 \tag{3.35}$$

for all $x,z\in R$. Again putting $x=xz$ in above relation, we get

$$[z[d(z),z]xzd(z),z]=0 \tag{3.36}$$

for all $x,z\in R$. Now right multiplying (3.35) by z and then subtracting from (3.36), we obtain

$$[z[d(z),z]x[d(z),z],z]=0 \tag{3.37}$$

and hence

$$[z[d(z),z]xz[d(z),z],z]=0 \tag{3.38}$$

for all $x,z\in R$. This implies

$$z[d(z),z]xz[d(z),z]z-z^2[d(z),z]xz[d(z),z]=0 \tag{3.39}$$

for all $x,z\in R$. In (3.39), replacing x with $xz[d(z),z]u$, we obtain

$$z[d(z),z]xz[d(z),z]uz[d(z),z]z-z^2[d(z),z]xz[d(z),z]uz[d(z),z]=0 \tag{3.40}$$

for all $x,u,z\in R$. Using (3.39), (3.40) gives

$$z[d(z),z]xz^2[d(z),z]uz[d(z),z]-z[d(z),z]xz[d(z),z]zuz[d(z),z]=0 \tag{3.41}$$

that is

$$z[d(z),z]x[z[d(z),z],z]uz[d(z),z]=0 \tag{3.42}$$

for all $x,u,z \in R$. This implies $[z[d(z),z],z]x[z[d(z),z],z]u[z[d(z),z],z]=0$ for all $x,u,z \in R$, which is $(R[z[d(z),z],z])^3=(0)$ for all $z \in R$. Since R is semiprime, we conclude that $R[z[d(z),z],z]=(0)$ for all $z \in R$. Hence, $z[[d(z),z],z]=0$ for all $z \in R$. Then by Fact-2, either $d(R)=(0)$ or $d(R) \subseteq Z(R)$. If $d(R) \neq (0)$, then the second case implies $[d(x),x]=0$ for all $x \in R$. Hence in view of Fact-1, R contains a nonzero central ideal.

Let $d(R)=(0)$. Then $F(xy)=F(x)y+xd(y)=F(x)y$ for all $x,y \in R$, i.e., F is a left centralizer map of R. Then by our hypothesis, we have

$$F(x)y-F(y)F(x) \in Z(R) \tag{3.43}$$

for all $x,y \in R$. Replacing y with yu, where $u \in R$, we get

$$F(x)yu-F(y)uF(x) \in Z(R) \tag{3.44}$$

that is

$$(F(x)y-F(y)F(x))u+F(y)[F(x),u] \in Z(R) \tag{3.45}$$

for all $x,y,u \in R$. Commuting both sides with u, we get

$$[(F(x)y-F(y)F(x))u,u]+[F(y)[F(x),u],u]=0 \tag{3.46}$$

for all $x,y,u \in R$. Since $F(x)y-F(y)F(x) \in Z(R)$ for all $x,y \in R$, we have from (3.46) that

$$[F(y)[F(x),u],u]=0 \tag{3.47}$$

for all $x,y,u \in R$. We put $y=yr$ in above and get

$$[F(y)r[F(x),u],u]=0 \tag{3.48}$$

for all $x,y,u,r \in R$. Now putting $y=yu$ in above, we have

$$[F(y)ur[F(x),u],u]=0 \tag{3.49}$$

for all $x,y,u,r \in R$. Left multiplying (3.48) by u, we get

$$[uF(y)r[F(x),u],u]=0 \tag{3.50}$$

for all $x,y,u,r \in R$. Subtracting (3.50) from (3.49), we obtain that

$$[[F(y),u]r[F(x),u],u]=0 \tag{3.51}$$

for all $x,y,u,r \in R$. In particular, above relation yields

$$[[F(x),u]r[F(x),u],u]=0 \tag{3.52}$$

that is

$$[F(x),u]r[F(x),u]u-u[F(x),u]r[F(x),u]=0 \tag{3.53}$$

for all $x,u,r \in R$. In (3.53), replacing r with $r[F(x),u]v$ we get

$$[F(x),u]r[F(x),u]v[F(x),u]u-u[F(x),u]r[F(x),u]v[F(x),u]=0. \tag{3.54}$$

By using (3.53), (3.54) becomes

$$[F(x),u]ru[F(x),u]v[F(x),u]-[F(x),u]r[F(x),u]uv[F(x),u]=0, \tag{3.55}$$

which is

$$[F(x),u]r[[F(x),u],u]v[F(x),u]=0 \tag{3.56}$$

for all $x,r,u,v \in R$. Replacing v with vu in (3.56) and right multiplying (3.56) by u respectively and then subtracting one from another, we have

$$[F(x),u]r[[F(x),u],u]v[[F(x),u],u]=0. \tag{3.57}$$

Similarly, just from above relation, we can write

$$[[F(x),u],u]r[[F(x),u],u]v[[F(x),u],u]=0. \tag{3.58}$$

Thus $([[F(x),u],u]R)^3=(0)$ for all $x,u \in R$. Since R is semiprime, R contains no nonzero nilpotent ideals. Hence $[[F(x),u],u]R=(0)$ and so $[[F(x),u],u]=0$ for all $x,u \in R$. Then by Fact-1, we conclude that either $[F(x),u]=0$ for all $x,u \in R$ or R contains a nonzero central ideal. If $[F(x),u]=0$ for all $x,u \in R$, then F maps R into its center. Thus we obtain our all conclusions. $\square$

Theorem 4. *Let R be a semiprime ring with center $Z(R)$. Let $F:R \to R$ be a generalized derivation associated with the derivation $d:R \to R$. If $F(xy)+F(y)F(x) \in Z(R)$ for all $x,y \in R$, then one of the following holds: (1) R contains a nonzero central ideal; (2) F is a left centralizer map of R such that $F:R \to Z(R)$.*

Proof. If we replace F with $-F$ and d with $-d$ in Theorem 3, we conclude that $(-F)(xy)-(-F)(y)(-F)(x) \in Z(R)$ for all $x,y \in R$ implies $x[(-d)(x),x]_2=0$ for all $x \in I$, that is, $F(xy)+F(y)F(x) \in Z(R)$ for all $x,y \in R$, implies $x[d(x),x]_2=0$ for all $x \in R$. Hence the conclusion follows by Theorem 3. $\square$

We conclude our paper with the following example which shows that the above theorems do not hold for arbitrary rings.

Example: Consider the ring $R=\left\{\begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} | a,b \in \mathbb{Z}\right\}$. Obviously, R is not semiprime, because $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} R \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}=(0)$.

We define maps $F,d:R \to R$ by $F\begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix}=\begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix}$ and $d\begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix}=\begin{pmatrix} 0 & -b \\ 0 & 0 \end{pmatrix}$. Then F is a generalized derivation of R associated with the derivation d of R. For $I=R$, we have that $F(xy)-F(x)F(y) \in Z(R)$ for all $x,y \in I$ and $F(xy)-F(y)F(x) \in Z(R)$ for all $x,y \in I$. Since $d(R) \neq (0)$ and R contains no nonzero central ideal for $Z(R)=\left\{\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}\right\}$, the semiprimeness hypothesis in Theorem 1 and Theorem 3 is not superfluous.

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