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STABLITY OF MAXIMUM PRESERVING QUADRATIC FUNCTIONAL EQUATION IN BANACH LATTICES

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Abstract. We have posed a version of the Hyers-Ulam stability problem by substituting addition in the quadratic functional equation with the maximum operation, to be called maximum preserving functional equations.

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1. INTRODUCTION

The stability problem was first posed by S. M. Ulam (see [32]) in the terms:

> "Let G_1 be a group and let G_2 be a metric group with a metric $d(.,.)$. Given $\epsilon > 0$, does there exist a $\delta > 0$ such that if a function $h : G_1 \to G_2$ satisfies the inequality $d(h(xy), h(x)h(y)) < \delta$ for all $x, y \in G_1$; then there is a homomorphism $H : G_1 \to G_2$ with $d(h(x), H(x)) < \epsilon$ for all $x \in G_1$? "

If the answer is affirmative, we say that the functional equation for homomorphisms is stable.

D. H. Hyers was the first mathematician to present the result concerning the stability of functional equations. He brilliantly answered the question of Ulam for the case where G_1 and G_2 were assumed to be Banach spaces (see [17]).

Since then various problems of stability on various spaces have arisen. For example: the stability of linear functional equation [5,6], quadratic and cubic functional equations [12,21,24,26,27,31], Jensen and Cauchy-Jensen functional equations [20], pexiderial functional equation [25, 29], non-Archimedean functional equation [19], functional differential equation [22], derivations and linear functions [7, 23], entropy equation [13–15], functional inequalities [11, 16, 18, 28]are some but not all.

N. K. Agbeko has studied the stability of maximum preserving functional equations motivated by the optimal average (see [1–4]). He has replaced addition operation with the maximum operation on a given Banach lattice.

The functional equation $f(x+y)+f(x-y)=2f(x)+2f(y)$ is called a *quadratic functional equation*. F. Skof [30] was the first person to prove the Hyers-Ulam stability of the quadratic functional equation for functions $f: E_1 \to E_2$ where E_1 and E_2 are a normed space and a Banach space, respectively. P. W. Cholewa [8] demonstrated that Skof's theorem is also valid if E_1 is replaced with an Abelian group, G. I. Fenyö [10] improved that result by replacing the bound $(1/2)\delta$ with the best possible one, $(1/3)(\delta+\|f(0)\|)$. Some other generalized results are obtained by S. Czerwik [9].

We used the technique of [3] and obtain following results about quadratic functional equation.

2. Main results

Let us recall some necessary definitions.

If B is a Banach lattice, then B^+ stands for its positive cone, i.e.

$$B^+ = \{x \in B : x \geq 0\} = \{|x| : x \in B\}.$$

Given two Banach lattices X and Y we say that a functional $F: X \to Y$ is cone-related if $F(X^+) = \{F(|x|) : x \in B\} \subset Y^+$(see [3]).

Let X and Y be two Banach lattices and $F: X \to Y$ be a cone-related functional, we may have following properties:

I) Maximum Preserving Functional Equation: $F(|x| \vee |y|)$ $= F(|x|) \vee F(|y|)$ for all members $x, y \in X$(see [3]).

II) Homogeneity of degree 2: $F(\alpha|x|) = \alpha^2 F(|x|)$ for all $x \in X$ and every number $\alpha \in [0,\infty)$.

III) Continuity From Below On The Positive Cone: The identity $\lim\limits_{n\to\infty} F(x_n)$ $= F(\lim_{n\to\infty} x_n)$ holds for every increasing sequence $(x_n) \subset X^+$ (see [3]).

IV): For any increasing sequence $(x_n) \subset X^+$ the inequality hereafter holds

$$\lim_{n\to\infty}\lim_{k\to\infty} 4^{-n}F(2^n x_k) \leq \lim_{k\to\infty}\lim_{n\to\infty} 4^{-n}F(2^n x_k)$$

provided that the limits exist.

V): For any increasing sequence $(x_n) \subset X^+$ the inequality hereafter holds

$$\lim_{n\to\infty}\lim_{k\to\infty} 4^{n}F(2^{-n} x_k) \leq \lim_{k\to\infty}\lim_{n\to\infty} 4^{n}F(2^{-n} x_k)$$

provided that the limits exist.

We shall use the technics in [3] to prove the following two thearems.

Theorem 1. *Let X and Y be two Banach lattices and $F: X \to Y$ be a cone-related functional for which there are numbers $\theta > 0$, $\delta \geq 0$ and $p < 2$ such that*

$$\|F(\tau|x| \vee \eta|y|) - (\tau^2 F(|x|) \vee \eta^2 F(|y|))\| \leq \delta + \theta(\|x\|^p + \|y\|^p) \tag{2.1}$$

for all $x, y \in X$ and $\tau, \eta \in \mathbb{R}^+$; then there is a unique cone-related mapping $T : X \to Y$ which satisfies properties I, II and inequality.

$$\|T(|x|) - F(|x|)\| \le \frac{1}{3}\delta + \frac{2\theta}{4-2^p}\|x\|^p \tag{2.2}$$

Moreover, if F is continuous from below, then in order that T be continuous from below it is necessary and sufficient that F enjoy property (IV).

Proof. We first show by induction that for any fixed $x \in X$ and $n \in \mathbb{N}$,

$$\|F(2^n|x|) - 4^n F(|x|)\| \le \frac{1}{3}(4^n - 1)\delta + 2\cdot 4^{n-1}\theta\|x\|^p \sum_{j=0}^{n-1} 2^{j(p-2)}. \tag{2.3}$$

For $n = 1$, consider $\tau = \eta = 2$ and $x = y$ in inequality (2.1):

$$\|F(2|x|) - 4F(|x|)\| \le \delta + 2\theta\|x\|^p.$$

Suppose (2.3) is true for $n = k$, we must prove it for $n = k+1$. Let $2x$ be replaced by x then inequality (2.3) for $n = k$ becomes:

$$\|F(2^k|2x|) - 4^k F(|2x|)\| \le \frac{1}{3}(4^k - 1)\delta + 2\cdot 4^{k-1}\theta\|2x\|^p \sum_{j=0}^{k-1} 2^{j(p-2)}.$$

The triangle inequality yields

$$\begin{aligned}
&\|F(2^{k+1}|x|) - 4^{k+1}F(|x|)\| \\
&\qquad \le \|F(2^{k+1}|x|) - 4^k F(|2x|)\| + \|4^k F(|2x|) - 4^{k+1}F(|x|)\| \\
&\qquad \le \frac{1}{3}\delta(4^{k+1} - 1) + 2\cdot 4^k\theta\|x\|^p \sum_{j=1}^{k} 2^{j(p-2)} + 2\cdot 4^k\theta\|x\|^p \\
&\qquad = \frac{1}{3}\delta(4^{k+1} - 1) + 2\cdot 4^k\theta\|x\|^p \sum_{j=0}^{k} 2^{j(p-2)},
\end{aligned}$$

then inequality (2.3) is true for all $n \in \mathbb{N}$. Now, divide both side of inequality (2.3) by 4^n :

$$\|4^{-n}F(2^n|x|) - F(|x|)\| \le \frac{1}{3}(1 - 4^{-n})\delta + \frac{\theta}{2}\|x\|^p \sum_{j=0}^{n-1} 2^{j(p-2)}.$$

Since $\sum_{j=0}^{n-1} 2^{j(p-2)} \le \frac{1}{1-2^{p-2}} = \frac{4}{4-2^p}$, we obtain that

$$\|4^{-n}F(2^n|x|) - F(|x|)\| \le \frac{1}{3}\delta + \frac{2\theta}{4-2^p}\|x\|^p. \tag{2.4}$$

For every $n \in \mathbb{N}$, define the mapping $T_n : X \to Y$ by $T_n(x) = 4^{-n}F(2^n|x|)$. If $x = 0$ it is trivial to see that $\{T_n(0)\}$ is a Cauchy sequence. If $x \in X\backslash\{0\}$ for $n > m$, inequality (2.3) implies

$$\begin{aligned}\|T_n(|x|) - T_m(|x|)\| &= \|4^{-n}F(2^n|x|) - 4^{-m}F(2^m|x|)\| \\ &= 4^{-n}\|F(2^{n-m}\cdot 2^m|x|) - 4^{n-m}F(2^m|x|)\| \\ &\le 4^{-n}3^{-1}(4^{n-m}-1)\delta + 2\cdot 4^{-m-1}\theta\|2^m x\|^p \sum_{j=0}^{n-m-1} 2^{j(p-2)}.\end{aligned}$$

Since $\sum_{j=0}^{n-m-1} 2^{j(p-2)} \le \dfrac{4}{4-2^p}$ we have

$$\|T_n(|x|) - T_m(|x|)\| \le 3^{-1}4^{-m}\delta + 2^{m(p-2)-1}\ \frac{4}{4-2^p}\theta\|x\|^p.$$

The assumption $p < 2$ implies that $T_n(x)$ is a Cauchy sequence. Since Y is complete we can define

$$T(|x|) = \lim_{n\to\infty} T_n(|x|) \tag{2.5}$$

for any $x \in X$. Clearly, T is a cone-related operator. Let us show that T is maximum preserving. Let $\tau = \eta = 2^n$ in (2.1) we have

$$\|F(2^n(|x| \vee |y|)) - 2^{2n}(F(|x|) \vee F(|y|))\| \le \delta + \theta(\|x\|^p + \|y\|^p).$$

Substituting x with $2^n x$ and y with $2^n y$ in the last inequality:

$$\|F(4^n(|x| \vee |y|) - 4^n(F(2^n|x|) \vee F(2^n|y|)\| \le \delta + 2^{np}\theta(\|x\|^p + \|y\|^p).$$

Thus

$$\begin{aligned}&\|F(4^{-2n}(4^n(|x| \vee |y|)) - 4^{-n}(F(2^n|x|) \vee F(2^n|y|))\| \\ &\le 4^{-2n}\delta + 2^{n(p-4)}\theta(\|x\|^p + \|y\|^p)\end{aligned}$$

By letting $n \to \infty$ we get for all $x, y \in X$ the equality

$$\|T(|x| \vee |y|) - T(|x|) \vee T(|y|)\| = 0$$

or equivalently

$$T(|x| \vee |y|) = T(|x|) \vee T(|y|),$$

because,

$$\lim_{n\to\infty} 4^{-2n}F(2^{2n}|z|) = \lim_{m\to\infty} 4^{-m}F(2^m|z|), \quad z \in X$$

Now, we must show $T(r|x|) = r^2T(|x|)$ for all $x \in X$ and $r \in [0,\infty)$. Use inequality (2.1) with $\eta = \tau$, $y = 0$ and substituting τ with $2^n\tau$:

$$\|F(2^n\tau|x|) - (2^n\tau)^2F(|x|)\| \le \delta + \theta\|x\|^p.$$

If we replace x with $2^n x$:

$$\|F(4^n \tau|x|) - 4^n \tau^2 F(2^n|x|)\| \leq \delta + \theta\|x\|^p 2^{np}.$$

Divide by 4^{2n} both side of above inequality:

$$\|4^{-2n} F(4^n \tau|x|) - 4^{-n} \tau^2 F(2^n|x|)\| \leq 4^{-2n}\delta + \theta\|x\|^p 2^{n(p-4)}.$$

Since $p < 2$ and

$$\lim_{n\to\infty} 4^{-2n} F(4^n \tau|x|) = \tau^2 \lim_{n\to\infty} 4^{-n} F(2^n|x|) = \tau^2 T(|x|),$$

by taking $z = \tau|x|$, we have

$$r^2 T(|x|) = \lim_{n\to\infty} 4^{-2n} F(4^n \tau|x|) = \lim_{n\to\infty} 4^{-2n} F(4^n|z|) = T(|z|) = T(\tau|x|).$$

For the validity of inequality (2.2); it is enough to take limit in (2.4) when $n \to \infty$. For the uniqueness of T: assume there exist cone-related maps S and T from X to Y such that

$$\|T(|x|) - F(|x|)\| \leq a_1 + b_1\|x\|^p,$$
$$\|S(|x|) - F(|x|)\| \leq a_2 + b_2\|x\|^p,$$

for all $x \in X\backslash\{0\}$; Since $T(2^n|x|) = 4^n T(|x|)$ and $S(2^n|x|) = 4^n S(|x|)$ we have

$$\begin{aligned}\|T(|x|) - S(|x|)\| &= 4^{-n}\|T(2^n|x|) - S(2^n|x|)\|\\ &\leq 4^{-n}(\|T(2^n|x|) - F(2^n|x|)\| + \|S(2^n|x|) - F(2^n|x|)\|)\\ &= 4^{-n}(a_1 + a_2) + 2^{n(p-2)}(b_1 + b_2)\|x\|^p,\end{aligned}$$

since $p < 2$,when $n \to \infty$ we have

$$T(|x|) = S(|x|).$$

To end the proof, we simply mention that the moreover part can be carried out exactly the same way its counterpart in Theorem 1 was proved in [3]. □

Theorem 2. *Let X and Y be two Banach lattices and $F : X \to Y$ be a cone-related functional for which there are numbers $\theta > 0$ and $p > 2$ such that*

$$\|F(\tau|x| \vee \eta|y|) - \tau^2 F(|x|) \vee \eta^2 F(|y|)\| \leq \theta(\|x\|^p + \|y\|^p) \tag{2.6}$$

for all $x, y \in X$ and $\tau, \eta \in \mathbb{R}^+$; then there is a cone-related mapping $T : X \to Y$ such that

$$\|T(|x|) - F(|x|)\| \leq \frac{2}{2^p - 4}\theta\|x\|^p \tag{2.7}$$

, if $p > 4$ then T is unique and satisfies properties I, II .

Moreover, if F is continuous from below, then in order that T be continuous from below it is necessary and sufficient that F enjoy property (V).

Proof. We first show by induction that for any fixed $x \in X$ and $n \in \mathbb{N}$,

$$\|F(|x|) - 4^n F(2^{-n}|x|)\| \leq 2^{1-p}\theta\|x\|^p \sum_{j=0}^{n-1} 2^{j(2-p)} \tag{2.8}$$

For $n = 1$, consider $\tau = \eta = 2$ and $x = y = t/2$ in inequality (2.6):

$$\|F(|t|) - 4F(2^{-1}|t|)\| \leq 2^{1-p}\theta\|t\|^p$$

Suppose (2.8) is true for $n = k$, we must prove it for $n = k+1$. The triangle inequality yields

$$\begin{aligned}
&\|F(|x|) - 4^{k+1}F(2^{-k-1}|x|)\| \\
&\quad \leq \|F(|x|) - 4^k F(2^{-k}|x|)\| + \|4^k F(2^{-k}|x|) - 4^{k+1}F(2^{-k-1}|x|)\| \\
&\quad = \|F(|x|) - 4^k F(2^{-k}|x|)\| + 4^k\|F(2^{-k}|x|) - 4F(2^{-1}(2^{-k}|x|))\| \\
&\leq 2^{1-p}\theta\|x\|^p \sum_{j=0}^{k-1} 2^{j(2-p)} + 2^{1-p}\theta\|x\|^p 2^{k(2-p)} = 2^{1-p}\theta\|x\|^p \sum_{j=0}^{k} 2^{j(2-p)}
\end{aligned}$$

Then inequality (2.8) is true for all $n \in \mathbb{N}$. Next, define $T_n(|x|) = 4^n F(2^{-n}|x|)$, for $x = 0$ it is trivial to see $\{T_n(0)\}$ is a Cauchy sequence; If $x \in X\backslash\{0\}$ for $n > m$, use (2.8) then we have:

$$\begin{aligned}
\|T_n(|x|) - T_m(|x|)\| &= 4^m\|4^{n-m}F(2^{-(n-m)}|2^{-m}x|) - F(|2^{-m}x|)\| \\
&\leq 4^m(2^{1-p}\theta\|2^{-m}x\|^p) \sum_{j=0}^{n-m-1} 2^{j(2-p)}
\end{aligned}$$

Since $\displaystyle\sum_{j=0}^{n-m-1} 2^{j(2-p)} \leq \frac{2^p}{2^p - 4}$ we have

$$\|T_n(|x|) - T_m(|x|)\| \leq 2^{m(2-p)} \cdot \frac{2}{2^p - 4}\theta\|x\|^p$$

The assumption $p > 2$ implies that $\{T_n(|x|)\}$ is a Cauchy sequence. Since Y is complete we can define $T : X \to Y$,

$$T(|x|) = \lim_{n\to\infty} T_n(|x|) \tag{2.9}$$

for any $x \in X$. Clearly, T is a cone-related operator. Now, assume $p > 4$, we must show T is maximum preserving. Let $\tau = \eta = 2^{-n}$ in (2.6):

$$\|F(2^{-n}(|x| \vee |y|)) - 2^{-2n}(F(|x|) \vee F(|y|))\| \leq \theta(\|x\|^p + \|y\|^p)$$

Substituting x with $2^{-n}x$ and y with $2^{-n}y$ in the last inequality:

$$\|F(4^{-n}(|x| \vee |y|)) - 4^{-n}(F(2^{-n}|x|) \vee F(2^{-n}|y|))\| \leq \theta 2^{-np}(\|x\|^p + \|y\|^p)$$

Thus

$$\|4^{2n}F(4^{-n}|x|\vee|y|)-4^n(F(2^{-n}|x|)\vee F(2^{-n}|y|))\| \le 2^{n(4-p)}\theta(\|x\|^p+\|y\|^p)$$

By letting $n\to\infty$ we get for all $x,y\in X$ the equality

$$\|T(|x|\vee|y|)-T(|x|)\vee T(|y|)\|=0$$

or equivalently

$$T(|x|\vee|y|)=T(|x|)\vee T(|y|)$$

because,

$$\lim_{n\to\infty}4^{2n}F(2^{-2n}|z|)=\lim_{m\to\infty}4^mF(2^{-m}|z|),\quad z\in X$$

For the proof of $T(r|x|)=r^2T(|x|)$ for all $x\in X$ and $r\in[0,\infty)$, use equation (2.6) with $\eta=\tau$, $y=0$ and substituting τ with $2^{-n}\tau$:

$$\|F(2^{-n}\tau|x|)-(2^{-n}\tau)^2F(|x|)\|\le\theta\|x\|^p$$

If we replace x with $2^{-n}x$ in this inequality, then

$$\|F(4^{-n}\tau|x|)-4^{-n}\tau^2F(2^{-n}|x|)\|\le\theta\|x\|^p2^{-np}$$

which implies:

$$\|4^{2n}F(4^{-n}\tau|x|)-4^n\tau^2F(2^{-n}|x|)\|\le\theta\|x\|^p2^{n(4-p)}$$

Since $p>4$:

$$\lim_{n\to\infty}4^{2n}F(4^{-n}\tau|x|)=\tau^2\lim_{n\to\infty}4^nF(2^{-n}|x|)=\tau^2T(|x|).$$

Taking $z=\tau|x|$, we have

$$r^2T(|x|)=\lim_{n\to\infty}4^{2n}F(4^{-n}\tau|x|)=\lim_{n\to\infty}4^{2n}F(4^{-n}|z|)=T(|z|)=T(\tau|x|).$$

Thus $T(\tau|x|)=\tau^2T(|x|)$.

Letting $n\to\infty$ in (2.8) proves the validity of inequality (2.7). To show the uniqueness of T assume there exist cone-related maps S and T from X to Y such that

$$\|T(|x|)-F(|x|)\|\le a_1\|x\|^p,$$
$$\|S(|x|)-F(|x|)\|\le a_2\|x\|^p,$$

for all $x\in X\backslash\{0\}$; Since $T(2^{-n}|x|)=4^{-n}T(|x|)$ and $S(2^{-n}|x|)=4^{-n}S(|x|)$ we have

$$\begin{aligned}\|T(|x|)-S(|x|)\|&=4^n\|T(2^{-n}|x|)-S(2^{-n}|x|)\|\\&\le4^n(\|T(2^{-n}|x|)-F(2^{-n}|x|)\|+\|S(2^{-n}|x|)-F(2^{-n}|x|)\|)\\&=2^{n(2-p)}(a_1+a_2)\|x\|^p\end{aligned}$$

since $p>4$,when $n\to\infty$ we have

$$T(|x|)=S(|x|).$$

To end the proof, we simply mention that the moreover part can be carried out exactly the same way its counterpart in Theorem 1 was proved in [3]. □

Remark 1. The condition $p > 4$ is used for the technique of the proof and we think it is necessary. However we could not find any good example (see [9]).

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