



## **THE FEASIBILITY OF MODELLING ROCKS IN ENGINEERING APPLICATIONS WITH THE USE OF DISCRETE ELEMENT METHOD**

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**Abstract**

This paper represents the motivation behind modelling rocks and the technique. Several possible agricultural engineering and other applications are introduced. The need for tracing each grain makes the discrete element method (DEM) ideal for the task. There are several types of rock aggregates, which need different models and parameter sets. Grains were represented with clumps and crushed rocks were approximated with convex polyhedral particles. The behaviours of the models were tested in simulations.

**Keywords**

discrete element modelling, gravel, crushed rock, clump, polyhedron

**1. Introduction**

Numerical computational simulations provide an excellent way to examine the behaviour of complex processes. A proper model can provide data, which are hardly accessible with measurements or would have high cost requirement. The simulation also gives a way to examine the effect of process parameters on the behaviour of the material independently. There are many cases where engineering structures interact with rock aggregates. The modelling of these processes creates the opportunity to test the machines virtually, creating the possibility to improve the design, decrease the necessary manufactured prototype variants, and thus reduce development cost and time.

Several examples can be found for stone-tool interaction in the field of agricultural engineering. The modelling possibilities of soil-tillage interaction

was already examined [1, 2], which can be extended to study the effect of rocky soil on the cultivation tools. Rock crushers are often used to reclaim fields and also rock pickers for collecting stones. Internal parts of forage and combine harvesters can be damaged by rocks getting into the machine. In combine harvesters, there is a stone trap to avoid rock reaching sensitive parts, which indicates that this issue cannot be neglected.

A proper rock model also can be used in other fields, for example railway ballast conditioning, mining and roadwork.

The first phase of creating a rock-tool interaction model is defining a proper rock model. The finite element method (FEM) describes bodies by dividing them into elements and creating the so-called finite element mesh. These elements have prescribed number of common nodes, with defined number of degrees of freedom. In case of contacting bodies, the proper definition of every contact surface is needed. This makes very difficult and time-consuming to model aggregates with considerable amount of grains, therefore the discrete element method (DEM) was applied instead, as it was developed to model granular media.

DEM involves particles (elements) with independent motional and rotational degrees of freedom and arising forces between them [3, 4], so it creates the possibility to model and trace the grains independently. The displacements of the particles are computed via numerical integration, relying on the principle laws of dynamics. The time between two successive iteration steps is called the timestep.

The definition of the technique makes it ideal for modelling the mechanical behaviour of granular media e.g. soil [5], rock aggregates, fractured solid

stones [6] different seeds [7, 8], crops [9] and even buildings made of blocks [10]. The main challenge in the creation of a DEM model is that it needs calibration [11] for each material model.

The oldest and simplest element shape in DEM is the sphere, however more complex particles are used in our research, which can simulate the sliding of grains and interlocking effect between rocks more realistically.

## 2. Properties of rock aggregates

Different types of rock aggregates are distinguished based on the material, size and shape of the grains. Widely used construction materials are e.g. andesite, limestone, dolomite, quartzite and basalt. The desired gradation of a yielded aggregate is obtained by sorting it with different sizes of sieves. The properties of the aggregate are highly influenced by its origin. The naturally formed (river) gravels (Figure 1. a) have smooth surface with round edges and corners unlike manufactured crushed rocks (Figure 1.b) which have rough surface and sharp edges and corners. The grain shape theoretically can be classified as equant, flat, elongated or flat-elongated relying on the ratio of the length, width and height dimensions. However, only equant and flat classes are used in practice due to economic reasons and the features of separation processes. These dimensions are defined by the size of the imaginary bounding cuboid around the grain. This introduction to the different aggregates and grains shows that there is a wide variety of properties, that must be taken into account during the creation process of their numerical model, especially in defining the particle shape.

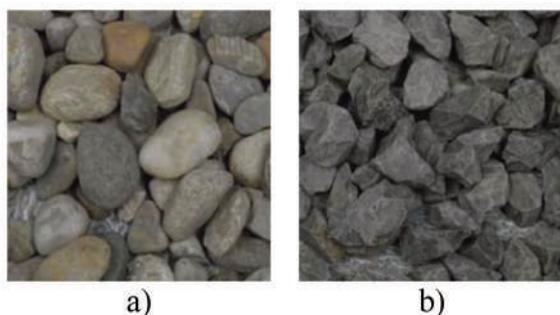


Figure 1. Aggregates made of a) gravels and b) crushed rocks

## 3. Definition of particle shapes

The gravel and crushed stone aggregates were modelled with different approaches in our research. The smooth gravels are approximated with clumps, which are particles made up by spheres with rigid connection between them. The elements that

represent equant (Figure 2. a) and flat (Figure 2. b) gravels, have spheres with different size ratio and constellation.

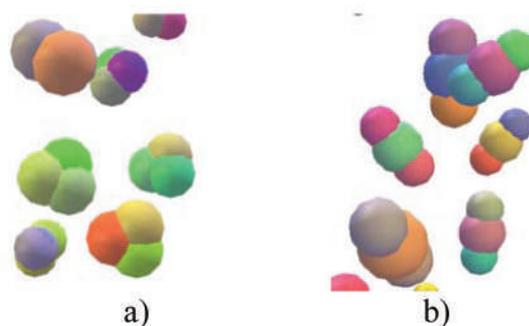


Figure 2. a) Equant and b) flat clump particles

Crushed rocks were approximated by randomly generated [12] convex polyhedra, with predefined size and shape index, which allows the creation of equant (Figure 3. a) and flat (Figure 3. b) particles. Polyhedra can simulate the significant interlocking between grains effectively, and effect of rough surface can be modelled with a properly set coefficient of friction.

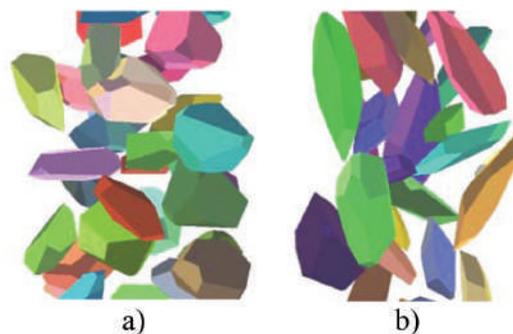


Figure 3. a) Equant and b) flat polyhedral particles

## 4. Interaction laws of arising forces

The interaction (constitutional) law computes the arising forces between the particles. Two of them was chosen from the many existing laws, one for the clump elements and one for polyhedra. The definitions of laws belonging to the corresponding particle types necessarily differ, but the aim was to find such ones that have the same base in some point.

Following from the behaviour of the rock aggregates, forces only need to be risen, when the particles come into contact, so the models have to be cohesionless. For clumps, the model of Cundall and Strack [13] was chosen and the law of J. Eliáš [14] was used for polyhedral particles. Both are implemented in Yade DEM software [15]. The particles are ideally rigid in both models, and their stiffness is represented by definition of the interaction law.

## Clumps

The model of Cundall and Strack [13] is originally applied to spheres, but it can be also adapted to clumps. In the clump model, two kinds of forces arise between the particles during contact: normal (repulsive) and shear forces. The magnitude of the normal force ( $F_n$ ) is linearly proportional (Equation 1) to the compression ( $u_n$ ) of the imaginary linear spring between particles with normal stiffness  $k_n$  (Figure 4.).

$$F_n = k_n u_n \quad (1)$$

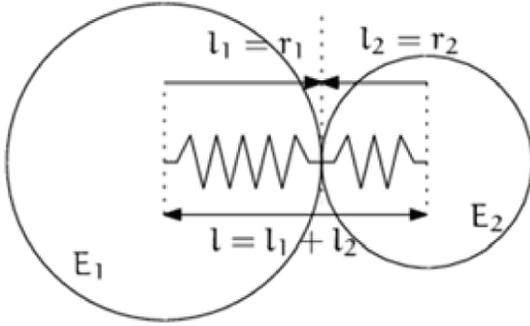


Figure 4. Visualization of normal force definition [15]

The shear force ( $F_s$ ) is linearly proportional to the displacement ( $u_s$ ) from the relative rotation and translation of the elements (Equation 2), and its maximum value ( $F_s^{\text{max}}$ ) is regulated by the Coulomb friction law (Equation 3).

$$F_s = k_s u_s \quad (2)$$

$$F_s^{\text{max}} = F_n \tan(\varphi) \quad (3)$$

where:

|                    |                               |       |
|--------------------|-------------------------------|-------|
| $F_s$              | shear force                   | [N]   |
| $F_s^{\text{max}}$ | maximum shear force           | [N]   |
| $F_n$              | normal force                  | [N]   |
| $k_s$              | normal stiffness              | [N/m] |
| $u_s$              | relative displacement         | [m]   |
| $\varphi$          | inter-particle friction angle | [rad] |

## Polyhedra

The polyhedral interaction law has several similarities to the clump law, as it only differs in the definition of normal force. The magnitude of normal force ( $F_{nv}$ ) is linearly proportional to the size of the  $V_c$  intersecting volume (Equation 4). The same law is visualized by Figure 5. in profile. Because of the volumetric behaviour of the definition, the so-called volumetric normal stiffness ( $k_{nv}$ ) is used as the proportional factor between the normal force and mutual volume, with the unit  $N/m^3$ .

$$F_{nv} = k_{nv} V_c \quad (4)$$

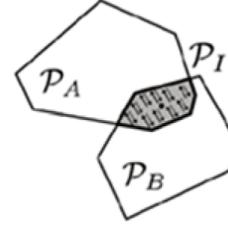


Figure 5. Visualization of normal force definition [14]

The computational method of the shear force is the same as seen in case of the clump constitutional law (Equation 2 and 3).

## Damping

The applied interaction laws do not involve any kind of damping, which is essential to dissipate kinetic energies and to reach equilibrium state. Considering that, a so-called global damping [16] was applied in both models. Equation 5 refers to each particle separately.  $F_{dw}$  is a numerical, artificial, non-viscous damping, which decreases the forces that cause velocity increases and vice versa, with a damping coefficient factor which is between 0 and 1. Global damping has a restriction that it can only provide reliable results in quasi-static simulations.

$$\frac{(\Delta F)_{dw}}{F_w} = -\lambda \cdot \text{sgn} F_w \left( \dot{u}_w^{i-1} + \frac{\ddot{u}_w^i \Delta t}{2} \right) \quad (5)$$

where:

|              |  |                     |
|--------------|--|---------------------|
| $F$          | force  | [N]                 |
| $u^{i-1}$    | velocity of particle in the previous timestep    | [m/s]               |
| $\ddot{u}^i$ | acceleration of particle in the current timestep | [m/s <sup>2</sup> ] |
| $w$          | index of dimension (x, y, z)                     |                     |
| $\Delta F_d$ | damping force                                    | [N]                 |
| $\Delta t$   | duration of a timestep                           | [s]                 |
| $\lambda$    | damping coefficient (0-1)                        | [-]                 |

## 5. Simulation test setup

The behaviour of the clump and polyhedral models were tested in a uniaxial compression simulation. The aggregate is placed in a cylinder (Figure 6. and 7.) and a top plate applied a normal force to the aggregate up to 600 kN with constant velocity, then unloading began. The normal force on the plate and its displacement was registered.

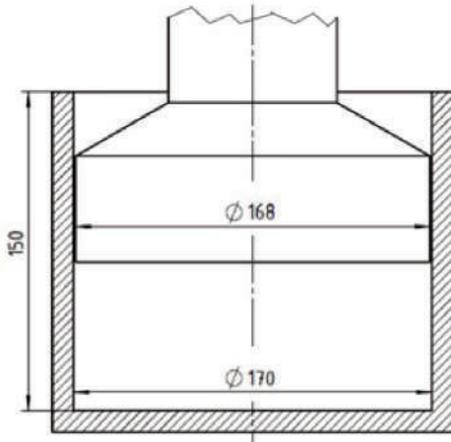


Figure 6. Sketch and dimensions of the test setup



Figure 7. Initial state of the polyhedral pack before the pressing process

The applied micromechanical characteristics of the polyhedral model were:

|           | stones | plate + cylinder |                      |
|-----------|--------|------------------|----------------------|
| $\rho$    | 2600   | 7800             | [kg/m <sup>3</sup> ] |
| $k_{nv}$  | 2·1013 | 2·1014           | [N/m <sup>3</sup> ]  |
| $k_s$     | 2·108  | 2·109            | [N/m]                |
| $\varphi$ | 0,6    | 0,4              | [rad]                |

The parameters of the clump model were:

|           | gravels | plate + cylinder |                      |
|-----------|---------|------------------|----------------------|
| $\rho$    | 2600    | 7800             | [kg/m <sup>3</sup> ] |
| $k_n$     | 3·1010  | 2,1·1011         | [N/m]                |
| $k_s$     | 3·106   | 2,1·107          | [N/m]                |
| $\varphi$ | 0,6     | 0,4              | [rad]                |

Where  $\rho$  is the volumetric mass density. The value of the damping coefficient ( $\lambda$ ) was 0,3 in every case.

## 6. Results

Figure 8. and 9. shows the results of compression simulations. Normal force, which was registered on the load plate, is shown in respect of the negative

displacement of the plate. Equant and flat aggregates were tested both in clump and polyhedral cases. The results in the case of clumps (Figure 8.) show little difference between the two particle shapes, although the flat grains can be compressed slightly better. However, there is a significant difference in case of polyhedra (Figure 9.). Studying the load curves of the polyhedral particles, peak forces can be observed. This behaviour is presumably due to the interlocking and stick slip of certain elements and it is assumed that it can be neglected.

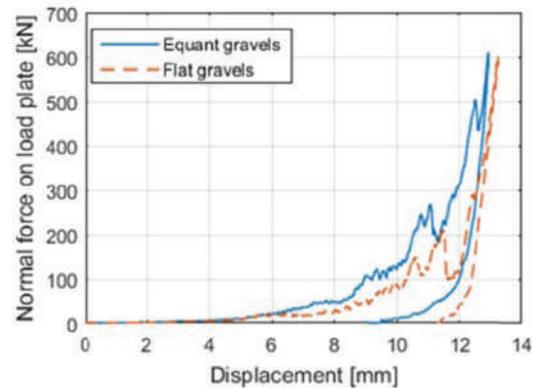


Figure 8. Load curve of the clump aggregates

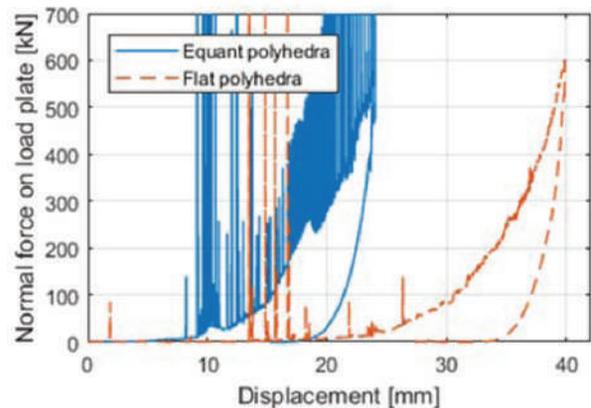


Figure 9. Load curve of the polyhedral aggregates

## 7. Conclusion

The paper discussed the motivation of the research on creating the numerical model of rocks and its the usability in the field of agricultural machinery engineering. The chosen tool was the discrete element method, which is suitable for modelling the distinct rock grains. Models for simulating gravels and crushed rocks with equal and flat shapes were introduced and studied.

In DEM models, the definition of particles and the law of interactions have to be defined. Our focus was on creating a model with a complex element shape, which is, in contrast with the simple sphere, able to model the interlocking and sliding between particles

accurately. The behaviour of the models, especially the effect of grain shape, was tested in the simulations of a uniaxial compressing test setup.

The evolution of the normal force acting on the top plate, in respect of the displacement of this plate, showed same characteristics in each test and fulfils the theoretical expectations. The load curves show the typical, highly nonlinear behaviour of rock aggregates. In the early phase of the process, the force raises slowly, as the result of the relatively high initial porosity of the aggregate. As interlocking and consolidation occurs, the aggregate becomes stiffer.

The particle shape, besides the application of the same constitutional law and parameters, have a noticeable effect on the compressibility of clump aggregates, and shows a significant influence in case of polyhedral bulk materials. This nature confirms the benefit of using complex element shapes in situations, where the influence of the grain shape on the behaviour on the whole aggregate cannot be neglected.

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