

Identification of the model of stick balancing using the cepstral analysis

Dalma J. Nagy, László Bencsik, Tamás Insperger

Abstract: Stick balancing on the fingertip is one of the simplest human balancing tasks, still it represents the key features of more complex balancing tasks, namely, an unstable equilibrium should be stabilized in the presence of a reaction time delay. In order to understand the mechanism of human balancing, first we have to identify the control concept employed by the human brain during stick balancing. There are several possible concepts in the literature to model this neural control mechanism. Here, we assume a delayed proportional-derivative-acceleration (PDA) feedback. This concept assumes that, besides the inclination and the angular velocity, humans are able to estimate the angular acceleration of the stick from the pressure distribution perceived by mechanoreceptors at the fingertip. Because of the acceleration feedback, the mathematical model is a neutral delayed differential equation (NDDE). For systems governed by NDDEs, cepstral analysis can be used to identify the time delay, and to gain information about the neutral behaviour of the model. In a proposed experimental study, sticks with different weights have to be balanced. In case of sticks with larger masses the pressure at the fingertip during balancing is larger and it is supposed that the acceleration gains are also of higher value. In this work we verify this phenomenon using cepstral analysis of signals obtained by time-domain simulations.

1. Introduction

How humans balance themselves physically can be viewed as a complex feedback control system. The neurons transmit the information to and from the brain that controls muscle groups in a synchronized process that maintains balance and prevents falling. Although the exact mechanism of this complex control system is not known, there are some elements, which can easily be identified. One of the most important elements of the control mechanism is the reaction delay, which is the result of the finite-time information transmission and processing of the neurons. The simplest model for human balance control is the proportional-derivative (PD) controller [6, 9, 16], which accounts for the perception of the position and the velocity of the controlled object. Since mechanoreceptors are able to measure forces and pressure distributions, it is a straightforward idea that this information is used by the nervous system for motor control mechanisms. Since force is related to acceleration, control models which involves the feedback of the acceleration have been analyzed extensively in the

literature [7, 15]. In this study, we assume a proportional-derivative-acceleration feedback (PDA) controller to model stick balancing on the fingertip. Due to the reaction delay and the acceleration feedback, the mathematical model is a neutral delayed differential equation (NDDE). In case of a signal generated by a NDDE, the time delay can be estimated using cepstral analysis [4, 8]. The goal of this study is to show how the control gain for the acceleration feedback is related to the result of the cepstral analysis. Several nonlinear numerical simulations are presented with different acceleration gains and the corresponding cepstrum is analyzed with respect to the feedback delay.

The outline of this paper is the following. In Section 2, the dynamic model of stick balancing together with the assumed control concept and the corresponding equation of motion is introduced. In Section 3, the cepstral analysis is described briefly with some examples and a possible application to the stick balancing model identification process is demonstrated. Section 4 presents the detailed numerical study.

2. Dynamic model of stick balancing

The principles of general human balancing tasks is aimed to understand through stick balancing on the fingertip shown in Fig. 1. In this study, stick balancing in the anterior-posterior plane [10] is investigated. In order to simplify the problem, the vertical motion is neglected. Thus we have a two-degree-of-freedom mechanical model, which is illustrated on the right hand side of Fig. 1.

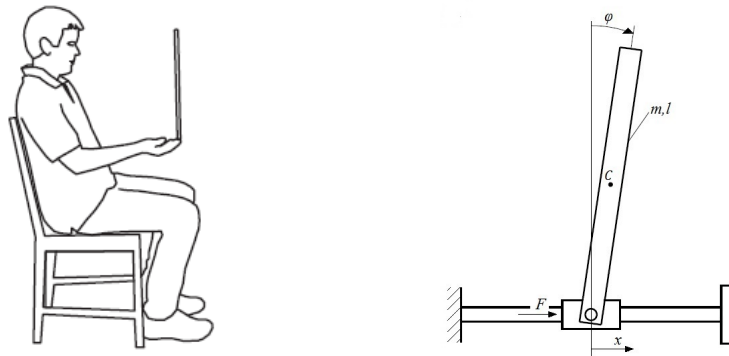


Figure 1. Stick balancing and the inverted pendulum as mechanical model.

In the mechanical model, the tilt angle of the stick is denoted with φ , and x represents the horizontal displacement of the massless cart, which represents the motion of the palm.

The equation of motion was derived using Lagrange's equation of the second kind as:

$$\begin{bmatrix} m & \frac{1}{2}ml \cos \varphi \\ \frac{1}{2}ml \cos \varphi & \frac{1}{3}ml^2 \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{\varphi} \end{bmatrix} + \begin{bmatrix} -\frac{1}{2}ml\dot{\varphi}^2 \sin \varphi \\ -\frac{1}{2}mgl \sin \varphi \end{bmatrix} = \begin{bmatrix} F \\ 0 \end{bmatrix}, \quad (1)$$

where m is the mass of the stick, l is the length of the stick, C refers to the center of gravity and the stick is assumed to be a homogeneous body. On the right hand side of the equation F represents the control force.

Since x is a cyclic coordinate [3], it can be eliminated from the equation. Thus the equation of the essential motion transforms to:

$$\ddot{\varphi} = -\frac{Fl \cos \varphi}{2(\frac{1}{3}ml^2 - \frac{1}{4}ml^2 \cos^2 \varphi)} - \frac{(\frac{1}{4}ml^2 \dot{\varphi}^2 \cos \varphi - \frac{1}{2}mgl) \sin \varphi}{(\frac{1}{3}ml^2 - \frac{1}{4}ml^2 \cos^2 \varphi)}. \quad (2)$$

2.1. Modelling the control mechanism

As it was mentioned in the Introduction, in this study a PDA type controller is assumed to model the control mechanism of the brain during stick balancing. Thus, the control force F has the following form:

$$\begin{aligned} F_{PDA}(\varphi, \dot{\varphi}, \ddot{\varphi}) &= K_p \varphi(t - \tau) + K_d \dot{\varphi}(t - \tau) + K_a \ddot{\varphi}(t - \tau) = \\ &= K_p \varphi_D + K_d \dot{\varphi}_D + K_a \ddot{\varphi}_D, \end{aligned} \quad (3)$$

where K_p , K_d and K_a stand for the proportional, derivative and acceleration gains, respectively. Substituting F_{PDA} into Eq. (2) results a NDDE [5].

A delay differential equation (DDE) is a differential equation, in which the rate of change of state depends on present and past states of the system, too. If the DDE involves a time-delay in the derivative term of the highest order, then it is a neutral delay differential equation (NDDE). There exists several techniques to solve and to analyze DDEs [1], [2], [14]. An important property of NDDEs is the accumulation of the discontinuities which are introduced into the solution through the initial data. This property is utilized in the model identification process. The initial data in case of an NDDE is called the history function for the interval $t < 0$ [11].

2.2. Determination of the stability charts

For the further analysis of the described system stability analysis should be carried out in order to find the parameters of the stable operation. Stability charts are diagrams constructed in the plane of some system parameters, which present the stable and unstable parameter regions. Here, the stability charts are constructed using the D-subdivision method [7, 13]. In case of NDDEs, the so called strong stability is a necessary condition for stability.

The physical parameters in this study were set as follows. The length of the stick was $l = 1$ [m], the delay parameter was $\tau = 0.2$ [s], and the gravitational acceleration was $g = 9.81$ [m/s²]. For the stability investigation the equation of the essential motion (2) was linearized and the dimensionless time was introduced as $t^* = t/\tau$. Thus, the dimensionless control gains are:

$$k_p = \frac{6\tau^2 K_p}{ml}, k_d = \frac{6\tau K_d}{ml}, k_a = \frac{6K_a}{ml}. \quad (4)$$

Fig. 2 shows the stability charts and the stable domains (shaded areas) with varying values of the dimensionless acceleration gain k_a . The strong stability condition is that $|k_a| < 1$. If $|k_a| > 1$, then the NDDE (2) has infinitely many characteristic roots with positive real parts, therefore the solution is unstable (see [13]). For the forthcoming numerical study the control parameters are selected as follows: the dimensionless proportional gain is $k_p = 2.4$ and the dimensionless derivative gain is $k_d = 2.5$.

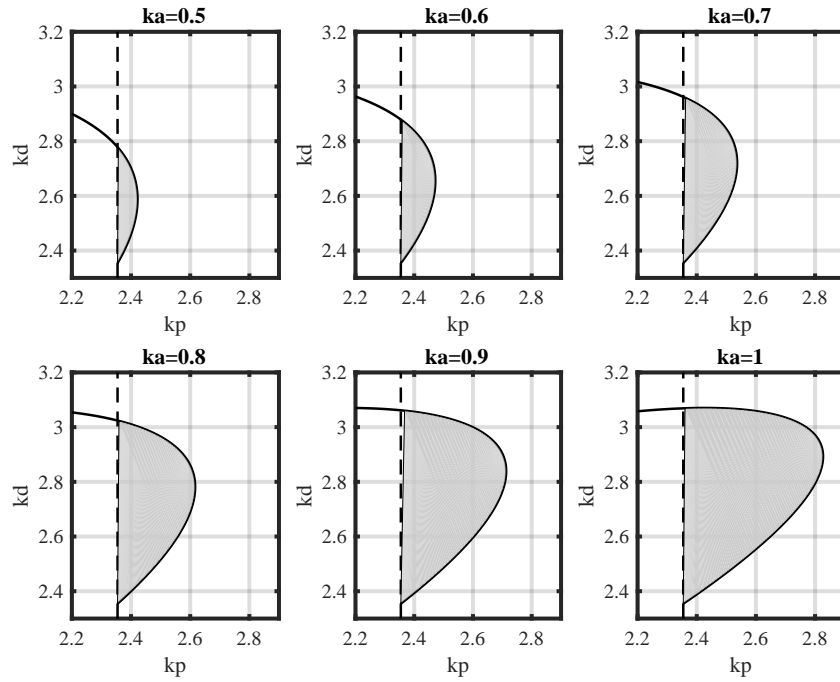


Figure 2. Stability charts for delayed PDA control. Stable regions are indicated by gray shading.

3. The cepstrum and its application in signal processing

The cepstrum was first defined as the power spectrum of the logarithm of the power spectrum [12]. Later, a new type of cepstrum was defined as the inverse Fourier Transform of the complex logarithm of the complex spectrum. Thus the formula for computing the cepstrum is:

$$\hat{f} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \log[\phi(\omega)] e^{i\omega n} d\omega. \quad (5)$$

where $\phi(\omega)$ is the Fourier Transform of the function f , and n is the independent variable of f . Therefore, the cepstrum is the spectrum of the spectrum, and transforms the spectrum back in the time domain. The independent variable of the cepstrum is quefrency, which is of time unit [s].

The original application of the cepstrum was the detection of echoes in seismic signals, and one of the other earliest applications was speech analysis. Cepstral analysis is also used in machinery fault identification: to detect gear eccentricity, general wear of gears and identifying local faults. [4]

If there is time-delay in a system described by a NDDE, then the cepstrum of the solution changes because of the accumulation of discontinuities. As a consequence of the time-delay a peak appears in every equal interval of the domain of the cepstrum. This interval is equal to the delay of the system [8]. This property of the cepstrum is used in this study in the case of stick balancing.

4. Numerical study

Instead of an built-in solver routine like `ddeNsd` in MatLab, for the nonlinear numerical simulations a self-developed solver was used. In order to consider the time delay the routine is based on the concept of semi-discretization, which was introduced for time-delay systems in [14]. However, instead of the exponential mapping for the integration a 4th order Runge-Kutta method was applied. The values of the proportional and derivative control gains were selected as it was discussed in Section 2.2, and the acceleration gain k_a was varied between 0.5 and 0.99. First, the NDDE was solved, and we checked, whether with the chosen control gains the solution is indeed stable. During the nonlinear simulations, the initial data were those specified in Section 2.2 (length, delay parameter, gravitational acceleration).

Since the mathematical model of the analysed problem (2) is an NDDE, we expect peaks to appear in the cepstrum in every 0.2 [s] of the quefrency domain. The proportional and derivative gains were hold constant, and the change in the cepstrum was examined if the acceleration gain was changed. Naturally, the position of the peaks should not alter, because the delay parameter τ is not changed.

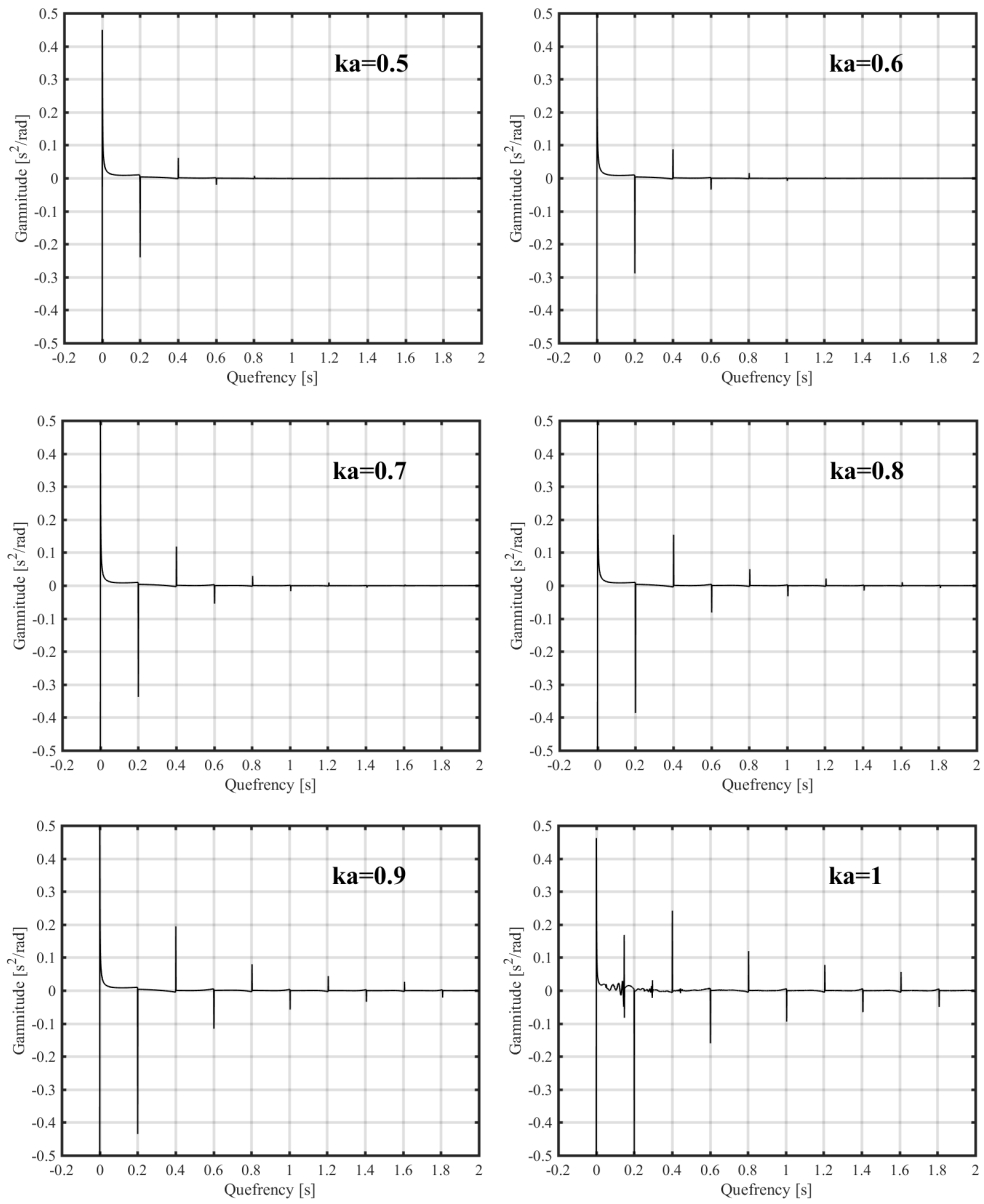


Figure 3. Results of the cepstral analysis.

The results of the cepstral analysis are shown in Fig. 3. The value of the dimensionless ka acceleration gain is displayed in each diagram. It can be seen, that the peaks of the cepstrum

appear at every 0.2 [s] interval of the quefrequency domain, as was expected. Furthermore, the peaks are getting larger (in absolute value) when the value of the acceleration gain is increased. In the last diagram, when $k_a = 0.99$, the solution contains several harmonics because it is a solution close to the stability boundary. This is why the cepstrum has peaks other than the peaks indicating the defined delay of the system.

5. Conclusions

In this study the problem of stick balancing was modelled as a planar cart-pole system. It is likely, that humans can measure forces and thus acceleration during stick balancing. Therefore, it is supposed that during the human stick balancing in addition to the proportional and derivative feedback the acceleration feedback is also used. This type of controller is known as a PDA type controller and the resulting mathematical model is a neutral delay differential equation.

In the numerical study cepstral analysis was applied to examine the effect of the changing of the acceleration gain. It was demonstrated, that from the cepstrum the delay parameter τ can be detected. Furthermore, as the value of the acceleration gain was increased the peaks of the cepstrum were larger in absolute value. This indicates that by increasing the mass of the stick, the role of the acceleration gain also increases in the neutral feedback.

In the future we intend to perform measurements with subjects balancing sticks of different masses in order to verify the results of the current study. In order to trigger the examined person to apply different acceleration gains the tests will be carried out with sticks with different masses.

References

- [1] A. BALACHANDRAN, T. KALMÁR-NAGY, D. E. G. *Delay Differential Equations, Recent Advances and New Directions*. Springer-Verlag New York, 2009.
- [2] ERNEUX, T. *Applied Delay Differential Equations*. Springer-Verlag New York, 2009.
- [3] GOLDSTEIN, H. *Classical Mechanics*. 2nd ed. Addison-Wesley, 1980.
- [4] H. KONSTANTIN-HANSEN, H. H. Envelope and cepstrum analyses for machinery fault identification. *Sound and Vibration*, May (2010), 3pp.
- [5] HAIRER, E., AND WANNER, G. *Solving Ordinary Differential Equations II: Stiff and Differential-Algebraic Problems*. Springer-Verlag, Berlin (2nd ed.), 1996.
- [6] INSPERGER, T., AND MILTON, J. Sensory uncertainty and stick balancing at the fingertip. *Biological Cybernetics* 108, 1 (2014), 85–101.

- [7] INSPERGER, T., MILTON, J., AND STEPAN, G. Acceleration feedback improves balancing against reflex delay. *Journal of the Royal Society Interface* 10, 79 (2013), Article No. 20120763.
- [8] J. XU, Y. S. Parameter identification for linear controlled with multi time-delay. In *The 4th International Conference on Dynamics, Vibration and Control* (2014).
- [9] MEHTA, B., AND SCHAAL, S. Forward models in visuomotor control. *Journal of Neurophysiology* 88 (2002), 942953.
- [10] NORDIN, M., AND FRANKEL, V. H. *Basic biomechanics of the musculoskeletal system*. Lea & Febiger, 1989.
- [11] PAUL, C. A. H. Treatment of derivative discontinuities in differential equations. In *Journal* (Year).
- [12] RANDALL, R. Cepstrum analysis and gearbox fault diagnosis. Tech. rep., Bruel & Kjaer, Application notes, 1975.
- [13] STÉPÁN, G. *Retarded Dynamical Systems*. Longman, 1989.
- [14] T. INSPERGER, G. S. *Semi-Discretization for Time-Delay Systems, Stability and Engineering Applications*. Springer-Verlag New York, 2011.
- [15] WELCH, T., AND TING, L. A feedback model reproduces muscle activity during human postural responses to support-surface translations. *Journal of Neurophysiology*, 99 (2008), 10321038.
- [16] YOSHIKAWA, N., SUZUKI, Y., KIYONO, K., AND NOMURA, T. Sensory uncertainty and stick balancing at the fingertip. *Frontiers in Computational Neuroscience* 10 (2016), 34.

Dalma J. Nagy, BSc (MSc student): Department of Applied Mechanics, Budapest University of Technology and Economics and MTA-BME Lendület Human Balancing Research Group, H-1111, Budapest Muegyetem rkp 5., Hungary (nagy.dalma22@gmail.com).

László Bencsik, PhD: MTA-BME Lendület Human Balancing Research Group, H-1111, Budapest Muegyetem rkp 5., Hungary (bencsik@mm.bme.hu). The author gave a presentation of this paper during one of the conference sessions.

Tamás Insperger, PhD, DSc: Department of Applied Mechanics, Budapest University of Technology and Economics and MTA-BME Lendület Human Balancing Research Group, H-1111, Budapest Muegyetem rkp 5., Hungary (insperger@mm.bme.hu).