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Reconstruction of motor force during stick balancing

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Abstract: Understanding the control mechanism behind human balancing is a challenging task, which becomes more and more important with the aging society. The core problem is the stabilization of a body around an unstable equilibrium in the presence of a reaction time delay and sensory uncertainties. Stick balancing on the fingertip plays an important role in human balancing research since it incorporates the above features still it has a relatively simple mechanical model, namely, the inverted pendulum subjected to delayed feedback control. The goals of stick balancing measurements are to find the control forces and to identify the nature of the human controller during. An important step of this identification is the accurate estimation of the control force. In this paper, an approach is proposed to estimate the control force based on techniques used for underactuated mechanical systems.

1. Introduction

The aim of this work is to reconstruct the control force from position measurements during stick balancing on the fingertip (see in Fig. 1). In most of the identification processes the input and the output is typically known and the model of the system should be identified [4]. In case of stick balancing, the mechanical model is known (an inverted pendulum) and the input (the control force) is unknown. A straightforward idea is to use an inverse dynamical calculation for the computation of the input. The inverse calculation requires the knowledge of the position, the velocity and the acceleration of the stick. During balancing experiments, we have used a camera based motion capturing system which gives only the positions of the measured markers in time. The numerical computation of the velocity and acceleration signal via discrete differentiation amplifies the noise of the source data [7], which strongly affects the accuracy and the reliability of the computed forces. A possible solution is the application of classical filtering techniques, such as moving average, or the Kalman filter [5]. Filtering techniques have to be accommodated to the problem in hand, which is not a straightforward tasks, since the exact signals are not available for comparison. In this paper, a different approach is proposed: a method, which is based on a control technique specially devoted to underactuated mechanical systems.

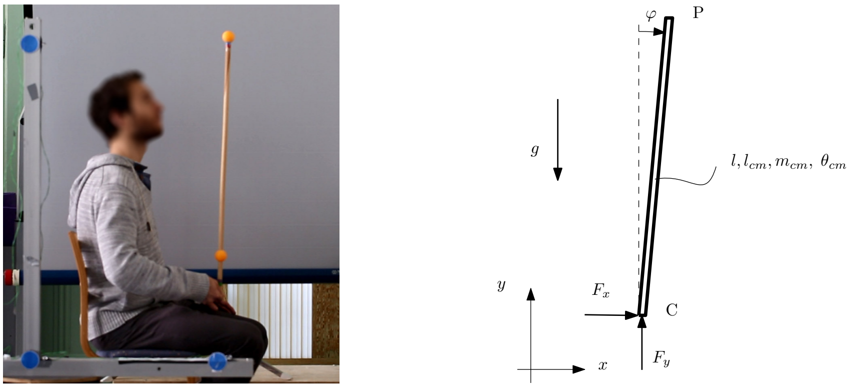

Figure 1. The measurement setup and the mechanical model.

2. Predictive force identification method

The goal of this work is to recalculate the control forces during stick balancing. This problem can be considered as a a trajectory tracking control problem. The predictive controller is applied, which was presented in [1] and [2]. The main idea of the predictive controller is based on the partition of the descriptor coordinates to actuated coordinates $\mathbf{q}_a$ that are controlled directly by the control forces and to unactuated coordinates $\mathbf{q}_u$. It is assumed that the actuated coordinates are known in time and using this assumption we approximate the unactuated coordinates with a linearized model. Then the trajectory following error $\mathbf{E}_s$ and a cost function J can be constructed, which enables us to define a minimization problem and find the optimal values for the actuated coordinates. Finally, the optimal control forces can be calculated, which can be considered as an estimation of the actual forces during balancing.

2.1. Predicting the unactuated motion of the balanced stick

We investigate stick balancing in the anterior-posterior plane only [6]. The corresponding mechanical model is shown in Fig. 1. In the estimation process first the equation of motion of the stick is generated. The vector of the generalized coordinates is $\mathbf{q}^\intercal = [x_C, y_C, \varphi]^\intercal$ (see in Fig. 1). For the sake of brevity ,the trigonometrical functions are denoted as: $c_\varphi = \cos\varphi$ and $s_\varphi = \sin\varphi$. Using Lagrange's equation the governing equation of motion can be written as:

$$\begin{bmatrix} m_{cm} & 0 & l_{cm}m_{cm}c_\varphi \\ 0 & m_{cm} & -l_{cm}m_{cm}s_\varphi \\ l_{cm}m_{cm}c_\varphi & -l_{cm}m_{cm}s_\varphi & l_{cm}^2 m_{cm} + \theta_{cm} \end{bmatrix} \begin{bmatrix} \ddot{x}_C \\ \ddot{y}_C \\ \ddot{\varphi} \end{bmatrix} + \begin{bmatrix} l_{cm}m_{cm}s_\varphi\dot{\varphi}^2 \\ m_{cm}(g - l_{cm}c_\varphi\dot{\varphi}^2) \\ -gm_{cm}l_{cm}s_\varphi \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} F_x \\ F_y \end{bmatrix}. \tag{1}$$

As a first step, the coordinates are partitioned in to two groups. The vector of actuated coordinates is $\mathbf{q}_a = [x_C \; y_C]^\mathsf{T}$ (x_C and y_C are controlled directly by F_x and F_y) and the coordinate which is not actuated directly is $\mathbf{q}_u = [\varphi]$. Thus, the equation of motion (1) can be written in the following compact form:

$$\begin{bmatrix} \mathbf{M}_{aa} & \mathbf{M}_{au} \\ \mathbf{M}_{ua} & \mathbf{M}_{uu} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}}_a \\ \ddot{\mathbf{q}}_u \end{bmatrix} + \begin{bmatrix} \mathbf{c}_a \\ \mathbf{c}_u \end{bmatrix} = \begin{bmatrix} \mathbf{H}_a \\ \mathbf{0} \end{bmatrix} \boldsymbol{\tau}. \tag{2}$$

The relation between the unactuated accelerations $\ddot{\mathbf{q}}_u$ and the actuated accelerations $\ddot{\mathbf{q}}_a$ can be expressed from the second row of the above equation. This relation can be linearized about a given configuration $\mathbf{q}_0$, $\dot{\mathbf{q}}_0$, $\ddot{\mathbf{q}}_0$ as:

$$\boldsymbol{\alpha}_u \ddot{\mathbf{q}}_u + \boldsymbol{\beta}_u \dot{\mathbf{q}}_u + \boldsymbol{\gamma}_u \mathbf{q}_u = \boldsymbol{\alpha}_a \ddot{\mathbf{q}}_a + \boldsymbol{\beta}_a \dot{\mathbf{q}}_a + \boldsymbol{\gamma}_a \mathbf{q}_a + \boldsymbol{\delta}. \tag{3}$$

Equation (3) is a system of linear differential equations, which has the same number of equations as the number of the unactuated coordinates, therefore, it is possible to determine the values of the unactuated coordinates. For the solution it is assumed that the actuated coordinates are known in the following form:

$$\mathbf{q}_a = \mathbf{P}_a \boldsymbol{\varphi}_n = \left(\sum_{i=1}^{N_a} p_{a,i} \mathbf{P}_{a,i} \right) \boldsymbol{\varphi}_n, \tag{4}$$

where the matrix $\mathbf{P}_a$ contains the yet unknown $p_{a,i}$ coefficients. In the sum $N_a = m\,(n-1)$, where m denotes the number of the actuated coordinates and n is equal to the order of the polynomial base $\boldsymbol{\varphi}_n$. The basic goal of the whole optimization process is to determine these coefficients.

The unactuated coordinates $\mathbf{q}_u$ can be determined by solving Eq. (3). First the homogeneous part of the equation should be solved. For this, we rewrite the homogeneous part of Eq. (3) in first order form:

$$\frac{d}{dt} \underbrace{\begin{bmatrix} \mathbf{q}_{u,h} \\ \dot{\mathbf{q}}_{u,h} \end{bmatrix}}_{\mathbf{z}_h} = \underbrace{\begin{bmatrix} \dot{\mathbf{q}}_{u,h} \\ \ddot{\mathbf{q}}_{u,h} \end{bmatrix}}_{\dot{\mathbf{z}}_h} = \underbrace{\begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\boldsymbol{\alpha}_u^{-1} \boldsymbol{\gamma}_u & -\boldsymbol{\alpha}_u^{-1} \boldsymbol{\beta}_u \end{bmatrix}}_{\mathbf{A}_h} \begin{bmatrix} \mathbf{q}_{u,h} \\ \dot{\mathbf{q}}_{u,h} \end{bmatrix}. \tag{5}$$

The solution can be given in matrix exponential form [3] as:

$$\mathbf{z}_h = e^{\mathbf{A}_h t} \mathbf{C}. \tag{6}$$

Here $\mathbf{C}$ is an unknown constant coefficient, which depends on the initial values. The vector $\mathbf{q}_{u,h}$ can be expressed using a constant selector matrix, i.e, $\mathbf{q}_{u,h} = \mathbf{K}_u \mathbf{z}_h$.

The next step is to determine the particular solution of the inhomogenous equation Eq. (3). For this we search the unactuated coordinates in the following form:

$$\mathbf{q}_{u,ih} = \mathbf{P}_u \boldsymbol{\varphi}_n. \tag{7}$$

Here the notations are similar to the ones used in Eq. (4).

Substituting the coordinates and their derivatives defined in Eq. (4) and in Eq. (7) into Eq. (3) and considering that the elements of the functions $\boldsymbol{\varphi}_n$ are linearly independent we get the following expression:

$$\boldsymbol{\alpha}_u \mathbf{P}_u \mathbf{D}^2 + \boldsymbol{\beta}_u \mathbf{P}_u \mathbf{D} + \boldsymbol{\gamma}_u \mathbf{P}_u = \boldsymbol{\alpha}_a \mathbf{P}_a \mathbf{D}^2 + \boldsymbol{\beta}_a \mathbf{P}_a \mathbf{D} + \boldsymbol{\gamma}_a \mathbf{P}_a + \boldsymbol{\delta} \mathbf{K}_{1,n}, \tag{8}$$

where $\mathbf{D}$ is the matrix of the differential operator which satisfies the relation $\dot{\boldsymbol{\varphi}} = \mathbf{D}\boldsymbol{\varphi}$. Here we introduce a row vector $\mathbf{K}_{1,n}$ which gives $\mathbf{K}_{1,n}\boldsymbol{\varphi}_n = 1$. If the the number of unactuated coordinates is one then the coefficients of $\mathbf{P}_u$ are scalars, which can be collected directly. Thus $\mathbf{P}_u$ can be determined as:

$$\mathbf{P}_u = (\alpha_u \mathbf{D}^2 + \beta_u \mathbf{D} + \gamma_u \mathbf{I})^{-1} (\boldsymbol{\alpha}_a \mathbf{P}_a \mathbf{D}^2 + \boldsymbol{\beta}_a \mathbf{P}_a \mathbf{D} + \boldsymbol{\gamma}_a \mathbf{P}_a + \boldsymbol{\delta} \mathbf{K}_{1,n}). \tag{9}$$

It should be noted that in case of higher number of unactuated coordinates $\mathbf{P}_u$ can also be given using additional transformations.

Now the solution of Eq. (3) can be formalized. The unknown coefficient $\mathbf{C}$ can be determined from the initial values $\mathbf{q}_u(0)$ and $\dot{\mathbf{q}}_u(0)$ and the complete solution can be written in the form:

$$\mathbf{q}_u = \mathbf{K}_u e^{\mathbf{A}t} \underbrace{\left(\mathbf{z}|_{t=0} - \begin{bmatrix} \mathbf{P}_u \\ \mathbf{P}_u \mathbf{D} \end{bmatrix} \boldsymbol{\varphi}|_{t=0} \right)}_{\mathbf{C}} + \mathbf{P}_u \boldsymbol{\varphi}. \tag{10}$$

After this the general coordinates $\mathbf{q}$ can be computed.

2.2. Constraint with the measurement results

During the measurement the lower and the upper endpoints of the stick can be measured. The motion of the stick can be characterized with the motion of the lower endpoint (point C) and with the inclination of the stick. The corresponding generalized coordinates are:

$$\mathbf{r} = \begin{bmatrix} x_C & y_C & x_P - x_C \end{bmatrix}^\mathsf{T} = \begin{bmatrix} x_C & y_C & l \sin\varphi \end{bmatrix}^\mathsf{T}. \tag{11}$$

These coordinates can be also computed from the measured values as:

$$\mathbf{r}_m = \begin{bmatrix} x_{C,m} & y_{C,m} & x_{P,m} - x_{C,m} \end{bmatrix}^\mathsf{T}. \tag{12}$$

The idea behind the identification process is that we are looking for an input, which induces a motion, which is the closest to the measured motion. Therefore, we define an error vector as:

$$\mathbf{E}_s = \begin{bmatrix} s_0\mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & s_1\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & s_2\mathbf{I} \end{bmatrix} \left(\begin{bmatrix} \mathbf{r} \\ \dot{\mathbf{r}} \\ \ddot{\mathbf{r}} \end{bmatrix} - \begin{bmatrix} \mathbf{r}_m \\ \dot{\mathbf{r}}_m \\ \ddot{\mathbf{r}}_m \end{bmatrix} \right). \tag{13}$$

This vector contains the errors at position, at velocity and at acceleration level. These components are weighted by the constants $s_0,\ s_1,\ s_2$. The quadratic error for the whole motion can be defined as:

$$J = \int_{t_1}^{t_2} \mathbf{E}_s^{\intercal}\mathbf{E}_s\ dt = \langle \mathbf{E}_s,\ \mathbf{E}_s \rangle = \|\mathbf{E}_s\|^2. \tag{14}$$

This integral have to be minimized and the motion of the system which is closest to the measured one can be selected. For this we substitute the solution of the unactuated coordinates $\mathbf{q}_u$ (see Eq. (10)) and the supposed form of the actuated coordinates $\mathbf{q}_a$ (see Eq. (4)) into Eq. (14). Therefore we get:

$$J = \left\| \boldsymbol{\Psi} - \sum_{i=1}^{N_a} p_{a,\,i} \boldsymbol{\psi}_i \right\|^2 \tag{15}$$

where $\boldsymbol{\psi}_i$ contains the coefficients of $p_{a,\,i}$, which are the coefficients in the assumed solution form Eq.(4) of the actuated coordinates and $\boldsymbol{\Psi}$ contains the remaining terms. In the optimization the following initial conditions have to be satisfied:

$$\begin{aligned} -\mathbf{q}_a|_{t=0} + \underbrace{\sum_{i=1}^{N_a} p_{a,i}\mathbf{P}_{a,i}\boldsymbol{\varphi}|_{t=0}}_{\mathbf{q}_a(0)} = \mathbf{0},\ -\dot{\mathbf{q}}_a|_{t=0} + \underbrace{\sum_{i=1}^{N_a} p_{a,i}\mathbf{P}_{a,i}\mathbf{D}\boldsymbol{\varphi}|_{t=0}}_{\dot{\mathbf{q}}_a(0)} = \mathbf{0}, \\ -\ddot{\mathbf{q}}_a|_{t=0} + \underbrace{\sum_{i=1}^{N_a} p_{a,i}\mathbf{P}_{a,i}\mathbf{D}^2\boldsymbol{\varphi}|_{t=0}}_{\ddot{\mathbf{q}}_a(0)} = \mathbf{0}. \end{aligned} \tag{16}$$

In order to determine the unknown constants $p_{a,\,i}$, we use the method of Lagrange multipliers, which results in the modified cost function as:

$$J_L = \left\| \boldsymbol{\Psi} - \sum_{i=1}^{N_a} p_{a,\,i} \boldsymbol{\psi}_i \right\|^2 + (\mathbf{a}_0 + \sum_{i=1}^{N_a} c_i\mathbf{a}_i)^{\intercal}\boldsymbol{\lambda}. \tag{17}$$

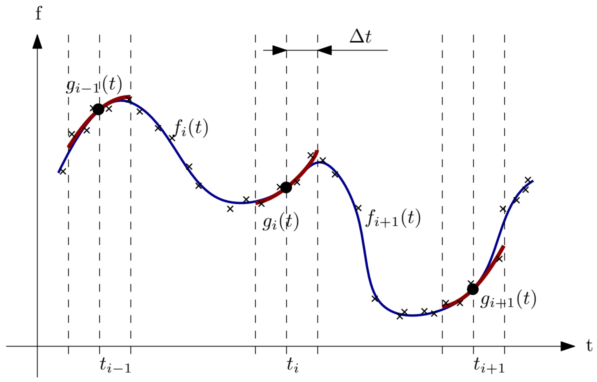

Figure 2. The idea behind the fitting of the polynomials.

Here $\mathbf{a}_0$ and $\mathbf{a}_i$ can be calculated from the initial values shown in Eq. (16). Then the coefficients $p_{a,i}$ and the optimal actuated coordinates $\mathbf{q}_a$ can be calculated, which are needed to determine the applied balancing forces from Eq. (2) as:

$$\boldsymbol{\tau} = \mathbf{H}_a^{-1}(\mathbf{M}_{aa}\ddot{\mathbf{q}}_a - \mathbf{M}_{au}\mathbf{M}_{uu}^{-1}(\mathbf{M}_{ua}\ddot{\mathbf{q}}_a + \mathbf{c}_u) + \mathbf{c}_a). \tag{18}$$

With this computed torque the measured motion can be re-simulated based on the equation of motion (1).

3. Numerical study

The measured data usually contains high frequency noisy vibrations. If we fit a polynomial, which approximates the signal with the required precision, then we have to use a polynomial of high order, which is numerically expensive and due to the higher order terms could lead to numerical instability. Therefore, we do the fitting with lower-order polynomials on shorter intervals. In this approach the measured signal is divided into $N_{int} = 35$ segments of length of 1 s, namely, over the intervals $[t_i, t_i + 1]$, $i = 0, 1, \cdots, N_{int} - 1$, and the polynomials are fitted over these intervals. This enables us to use fairly low degree polynomials, where the consecutive polynomials have to be C^2 continuous in the connection points. For the sake of better numerical results, we fit the second order polynomials $g_i(t)$ around every division points t_i in the intervals $[t_i - \Delta t, t_i + \Delta t]$, $i = 0, 1, \cdots, N_{int}]$ (see the red curves in Fig. 2). In Fig. 2, the black crosses denote the measured data. These polynomials will determine the conditions to obtain the above mentioned continuity. Then the polynomials $f_i(t)$ have to be fitted (see the blue curves in Fig. 2).

Using the proposed technique the balancing force is recalculated during stick balancing experiments. Here the results of one particular measurement is presented and the control force in the anterior-posterior plane is plotted in Fig. 3. Using this force, the equation of motion is integrated with the same initial conditions. The results of the numerical simulations are shown in Fig. 4, where the results are compared with the measured results. Based on Fig. 4, it can be concluded that the agreement between the measured and the recalculated signal is good. The quadratic error of the estimation process was $RMS = 0.0056[m]$.

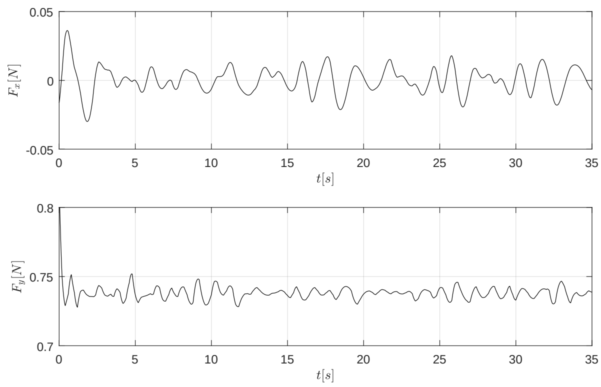

Figure 3. The reconstructed contact forces.

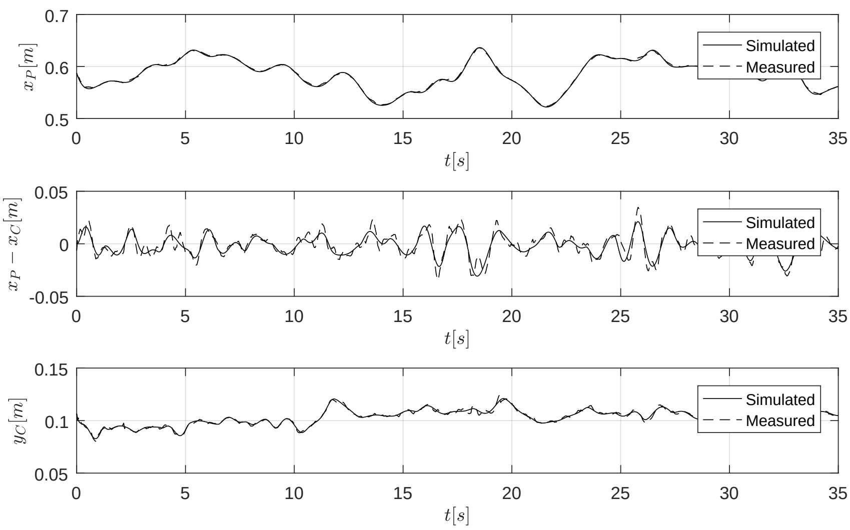

Figure 4. The measured and the simulated coordinates.

4. Conclusions

In this study for the investigation of a special human balancing task, namely, stick balancing on the fingertip was analyzed. In order to understand the mechanism of balancing, first, the control forces have to be estimated. In this paper, a novel approach was presented. We have extended a predictive control technique where the goal was to find a control force which results a motion which is appropriately close the captured motion. We applied the technique to the balancing a stick on the fingertip in the anterior-posterior plane. Since the proposed technique is general in a future work we apply it in the investigation of spatial balancing problems. This research has been supported by the ÚNKP-2017 New National Excellence Program of the Ministry of Human Capacities.

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