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Optimum design of a stiffened conical roof considering the residual welding distortions

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Abstract: It is shown how to prevent or decrease the residual welding distortions by means of structural optimization. The investigated conical roof structure is constructed from one circular and more radial stiffeners of rectangular hollow section. The deck plate elements are welded to stiffeners by two SMAW fillet welds. The cost function to be minimized contains the costs of material and fabrication. The design constraints relate to the maximum bending stresses due to snow load and to the allowable radial displacement of the roof periphery due to shrinkage of circular and radial welds. In the calculation of radial displacements the angular deformations of the inner ringbeam of box section are also considered. In the optimization the dimensions of stiffeners and the inner ring as well as the number of radial stiffeners are sought which minimize the cost function and fulfil the design constraints.

Keywords: residual welding distortions, distortion prevention, structural optimization, minimum cost design, design of welded structures, ringbeam deformations

1. Introduction

Residual welding stresses and distortions play an important role in structural integrity. Residual stresses affect the static brittle fracture, the fatigue strength and the overall and local buckling phenomena of welded structures. Residual distortions cause initial imperfections significantly affecting the safety against instability. Residual distortions affect the quality of fabrication, therefore they should be limited to guarantee the easy assembly and serviceability of structures.

The above mentioned unfavourable effects should be eliminated by research efforts to determine the main affecting parameters and the possibilities to decrease the residual stresses and distortions. The importance of this problem led to the organization of an IIW working group X/XV-RSDP (residual stress and distortion prevention). In the frame of this working group meeting in San Francisco 1997 a number of documents has been presented and discussed.

We have worked out a relatively simple method for the calculation of residual stresses and distortions due to longitudinal welds of beams [1]. This document has been recommended for publication. The aim of the present study is to apply our method to distortion calulcation of a structure composed from beam elements. For this purpose we have selected a shallow conical roof structure with one circular and more radial stiffeners of rectangular hollow section (RHS)(Fig.1). The deck plate elements are welded to the stiffeners by two SMAW (Shielded Metal Arc Welding) fillet welds. The radial stiffeners are welded to the inner ringbeam of welded box cross-section.

The circular and radial welds cause significant radial displacements of the roof periphery, which should be decreased by designing for sufficient stiffness of stiffeners and ringbeam to guarantee the easy assembly of the roof to the supporting structure. This requirement can be incorporated in the optimum design procedure, which enables designers to minimize the cost and to fulfil the design constraints.

The cost function contains the material and fabrication costs. For the calculation of fabrication cost of welded structures we have worked out a relatively simple cost function based on the welding times given by COSTCOMP software [2, 3]. This cost function is described in our book [4] and applied for several minimum cost design problems [5, 6, 7, 8].

The design constraints relate to the bending stresses due to snow load and to the limitation of radial displacement of the roof periphery due to residual welding distortions caused by circular and radial welds.

Fig.1. Conical roof structure with one circular and more radial stiffeners of RHS with an inner ring of welded box section

2. Characteristics of the roof structure

In the case of the roof structure shown in Fig.1 the shell effect is neglected, but it is assumed that the stiffeners consist of a RHS and a strip of deck plate with an effective width of $b_e = 50t_f$ (Fig.3). Stiffeners form a grid consisting of a circular and more radial beams. This grid is welded to the inner ring. This inner ring of box section has a sufficient torsional stiffness to decrease the residual distortions of the grid due to shrinkage of circular and radial welds.

2.1 The inner ringbeam

It is assumed that the dimensions of the inner ringbeam can be expressed by the only variable h i.e. by the height of webs as follows: $b = 0.7h$, $t_{f0} = b/40$, $t_{w0}/2 = h/70$. With these dimensions the cross-sectional area and the torsional moment of inertia are

$$A_0 = ht_{W0} + 2bt_{f0} = 5.357 * 10^{-2} h^2 \tag{1a}$$

$$I_{T0} = \frac{4h^2b^2}{2b/t_{f0} + 4h/t_{W0}} = \frac{h^4}{110} \tag{1b}$$

The ringbeam is loaded by bending moments due to shrinkage of circular and radial welds. The uniformly distributed bending moments of intensity m cause torsional and bending moments in the inner ring, which can be determined considering the equilibrium of a half ring (Fig.2).

Fig.2. Equilibrium of the half inner ringbeam

From the equation

$$2X = \int_0^{\pi} mR_0 \sin\varphi d\varphi = 2mR_0$$

the bending moment is $X = mR_0$ (2)

and from $$M_A = 2R_0 Y = \int mR_0^2 \sin^2 \varphi d\varphi + \int mR_0^2 (1+\cos\varphi)\cos\varphi d\varphi = mR_0^2 \pi$$

the torsional moment is $Y = mR_0 \pi / 2$ (3)

The angular deformation of the ringbeam due to bending moments m is

$$\phi_0 = \frac{mR_0\pi}{2}\frac{2R_0\pi}{GI_{T0}} = \frac{mR_0^2\pi^2}{GI_{T0}} \tag{4}$$

When the bending moment from a radial stiffener is M and the number of radial stiffeners is n, then

$$m = \frac{nM}{2R_0\pi} \quad \text{and} \quad \phi_0 = \frac{nMR_0\pi}{2GI_{T0}} \tag{5}$$

where G is the shear modulus $G = E/2.6$.

2.2 Formulae for stiffeners

RHS are used according to prEN 10219-2 (1992) [9]. We select RHS with $b_{R,C} = h_{R,C}/2$ only. Subscripts R and C denote radial and circular, respectively. The corner radius is taken as $2t_{R,C}$. For the calculation of cross-sectional area and moment of inertia the approximate formulae proposed by DASt Richtlinie 016 (1986) [10] are used as follows:

Fig.3. Cross-section of stiffeners with the effective width of deck plate

$$A_{RHS} = 2t(1.5h_{R,C} - 2t)\left(1 - 0.43\frac{4t}{1.5h_{R,C} - 2t}\right) \tag{6}$$

$$I_{RHS} = \left[\frac{\left(h_{R,C} - t\right)^3 t}{6} + \frac{t}{2}\left(\frac{h_{R,C}}{2} - t\right)\left(h_{R,C} - t\right)^2\right]\left(1 - 0.86\frac{4t}{1.5h_{R,C} - 2t}\right) \tag{7}$$

The characteristics of the whole stiffener cross-section are as follows (Fig.3):

$$A_{R,C} = A_{RHS} + b_e t_f \qquad b_e = 50t_f \tag{8}$$

$$y_G = \frac{A_{RHS} h_{R,C} / 2}{A_{RHS} + b_e t_f} \tag{9}$$

$$I_{R,C} = I_{RHS} + A_{RHS}\left(\frac{h_{R,C}}{2} - y_G\right)^2 + b_e t_f y_G^2 \tag{10}$$

and the section modulus is given by

$$W_{R,C} = \frac{I_{R,C}}{h_{R,C} / 2 + y_G} \tag{11}$$

2.3 Welding parameters

For one stiffener two fillet welds of dimension $a_W = 0.7t_f$ are used for which the heat input is

$$Q_T = 2 * 78.8 a_W^2 \quad \text{(J/mm)} \tag{12}$$

The specific shrinkage in the gravity centre for steels is [1]

$$\varepsilon_{GR,C} = 0.844 * 10^{-3} \frac{Q_T}{A_{R,C}} \tag{13}$$

and the curvature is given by

$$C_{R,C} = 0.844 * 10^{-3} \frac{Q_T y_T}{I_{R,C}} \qquad y_T = y_G - t_f/2 \tag{14}$$

2.4 The radial displacement of the roof periphery ΔR_B due to shrinkage of welds

The radial displacement ΔR_{BR} due to shrinkage of radial welds consists of the following parts (minus sign denotes direction to the roof centre):

effect of ε_{GR}:

$$\Delta R_B(\varepsilon_{GR}) = -\left(R_B - R_0\right)\varepsilon_{GR} \tag{15}$$

since a specific shrinkage of a circular weld of radius R causes a decrease of the radius $\Delta R = R\varepsilon$,

effect of C_R:

$$\Delta R_B\left(C_R\right) = C_R h_B (s_1 + s_2) / 2 \tag{16}$$

effect of angular deformation of inner ringbeam ϕ_0 due to C_R using (5) for $M = C_R E I_R$

$$\Delta R_B(\phi_0) = \phi_0 h_B = \frac{n R_0 \pi C_R E I_R h_B}{2 G I_{T0}} \tag{17}$$

A radial force F_R acts on the circular stiffener, which can be determined from a displacement equation (Fig.4) expressing that the radial displacements of the radial and circular stiffeners at point A are the same

$$\Delta R_{AR}(C_R) - \Delta R_{AR}(\varepsilon_{GR}) + \Delta R_{AR}(\phi_0) - \Delta R_{AR}(F_R) = \Delta R_{AC}(F_R) \tag{18}$$

or more detailed using the bending moment diagrams shown in Fig.4

$$\frac{C_R h_A s_1}{2} - \left(R_A - R_0\right)\varepsilon_{GR} + \frac{nR_0 \pi C_R EI_R h_A}{2GI_{T0}} - \frac{F_R h_A^2 s_1}{3EI_R} - \frac{nR_0 \pi F_R h_A^2}{2GI_{T0}} = \frac{nF_A R_A}{2A_C E} \tag{19}$$

Fig.4. Moment diagrams for the calculation of F_R

From (19) one obtains

$$F_R = \frac{1}{D}\left[\frac{C_R h_A s_1}{2} - \left(R_A - R_0\right)\varepsilon_{GR} + \frac{nR_0 \pi C_R EI_R h_A}{2GI_{T0}}\right] \tag{20}$$

where

$$D = \frac{h_A^2 s_1}{3EI_R} + \frac{nR_0 \pi h_A^2}{2GI_{T0}} + \frac{nR_A}{2A_C E} \tag{21}$$

effect of F_R: $\Delta R_B(F_R) = -\frac{F_R h_A s_1 (3h_B - h_A)}{6EI_R} - \frac{nR_0 \pi F_R h_A h_B}{2GI_{T0}}$ (22)

The radial displacement ΔR_{BC} due to shrinkage of circular welds is calculated as follows. The force F_C (Fig.5) can be calculated on the basis of a displacement equation

$$\Delta R_{AC}(F_C) - \Delta R_{AC}(\varepsilon_{GC}) = -\Delta R_{AR}(F_C) - \Delta R_{AR}(\phi_0) \tag{23}$$

or more detailed

$$\frac{nF_C R_A}{2A_C E} - R_A \varepsilon_{GC} = -\frac{F_C h_A^2 s_1}{3EI_R} - \frac{nR_0 \pi F_C h_A^2}{2GI_{T0}} \tag{24}$$

Fig.5. Moment diagrams for the calculation of F_C

From (24) we get

$$F_C = \frac{R_A \varepsilon_{GC}}{D} \tag{25}$$

With F_C the value of ΔR_{BC} can be calculated

$$\Delta R_{BC} = -\frac{F_C h_A s_1 (3h_B - h_A)}{6EI_R} - \frac{nR_0 \pi F_C h_A h_B}{2GI_{T0}} \tag{26}$$

3. The cost function

As mentioned in the introduction we have developed on the basis of COSTCOMP software a cost function containing the material and fabrication cost

$$\frac{K}{k_m} = \rho V + \frac{k_f}{k_m}\left[C_1 \Theta_d (\kappa \rho V)^{0.5} + 1.3 T_2\right] \tag{27}$$

where ρ is the material density, k_f and k_m are the fabrication and material cost factors, respectively, $\kappa = 4n + 4$ is the number of structural elements to be assembled, V is the volume of the structure, Θ_d is the difficulty factor, C_1 = 1.0 min/kg$^{0.5}$. The welding time is given by

$$T_2 = \sum_i C_{2i} a_{Wi}^n L_{Wi} \tag{28}$$

For SMAW fillet welds of dimension a_W = 2 - 5 mm

$$T_2 / L_W = 4a_W * 10^{-3} \quad \text{(min/mm)} \tag{29a}$$

and the weld length is, neglecting the welds of the inner ringbeam,

$$L_W = 2\left[2R_A \pi + n(s_1 + s_2)\right] \tag{29b}$$

The volume of roof structure is

$$V = \pi (R_B + R_0)(s_1 + s_2) t_f + nA_{RHS,R}(s_1 + s_2) + 2R_A \pi A_{RHS,C} + 2R_0 \pi A_0 \tag{30}$$

where $s_1 = \dfrac{R_A - R_0}{\cos\psi}$ $\qquad s_2 = \dfrac{R_B - R_A}{\cos\psi}$

4. The design constraints

The stiffeners are subject to bending due to snow load p_S (Fig.6).

The approximate stress constraint for radial stiffeners can be formulated as

$$\frac{\gamma M_R}{W_R} = \frac{\gamma p_S (R_B - R_0)^2}{8W_R} \frac{2R_A \pi}{n} \le \frac{f_y}{\gamma_{M1}} \tag{31}$$

where the partial safety factors are $\gamma = 1.5$ and $\gamma_{M1} = 1.1$. f_y is the yield stress.

Fig.6. Forces and bending moments in a radial stiffener due to snow load

Stress constraint for a circular stiffener of span length $2R_A\pi / n$ is given by

$$\frac{\gamma M_C}{W_C} = \frac{\gamma p_S}{8}\left(\frac{2R_A\pi}{n}\right)^2 \frac{R_B - R_0}{2} \le \frac{f_y}{\gamma_{M1}} \tag{32}$$

The constraint on radial displacement of the roof periphery due to weld shrinkage can be expressed as $\left|\Delta R_{BR} + \Delta R_{BC}\right| \le \Delta_{allow}$ (33)

where Δ_{allow} is the allowable displacement.

5. Optimum design procedure for a numerical example

Data: R_B = 5000, R_A = 2750, R_0 = 500, t_f = 4, a_W = 3 mm, $\psi = 30^0$, $\rho = 7.85*10^{-6}$ kg/mm^3, p_S = 1 kN/m^2 = 10^{-3} N/mm^2, f_y = 235 MPa, E = 2.1*10^5 MPa, Θ_d = 3, Δ_{allow} = 30 mm; $h_A = \left(R_A - R_0\right)\tan\psi = 1299$, $h_B = \left(R_B - R_0\right)\tan\psi = 2598$, $s_1 = s_2$ = 2598 mm.

Variables: h_R, t_R, h_C, t_C, h, n. Ranges of their values: h_R, h_C, h = 40...400, t_R, t_C = 2...12 mm and n = 6, 8, 10.

Design of deck plate segments for bending due to snow load: assume that trapezoidal plate segments can be approximately calculated as simply supported rectangular ones having side lengths of s_2 = 2598 mm and $\pi(R_B + R_A)/n$. The bending moments are given by Timoshenko and Woinowsky-Krieger [11]. Intensity of the uniform normal snow load is p_S =

1 kN/m^2 = 10^{-3} N/mm^2 with a partial safety factor of 1.5. For a yield stress f_y = 235 MPa and bending moment $M = 1.5*10^3 a^2\left(M/qa^2\right)^{0.5}$ (a is the smaller side length in m) the required deck plate thickness is $t_f = 6.1885a\left(M/qa^2\right)^{0.5}$. The thicknesses for various numbers of radial stiffeners are given in Table 1.

Table 1. Deck plate thicknesses for various numbers of radial stiffeners

n	24347/n(mm)	a(mm)	b/a	M/qa^2	t_f(mm)	rounded t_f
6	4058	2598	1.56	0.0842	4.66	5.0
8	3043	2598	1.17	0.0605	3.95	4.0
10	2435	2435	1.07	0.0531	3.47	3.5
12	2029	2029	1.28	0.0681	3.28	3.5
14	1739	1739	1.49	0.0806	3.06	3.5

In the minimum cost design the optimum values of variables are sought, which minimize the cost function (27) and fulfil the design constraints (31), (32) and (33). The computations are performed using the Rosenbrock's Hillclimb mathematical programming method.

It should be mentioned that in computation the size limitation of $h \geq h_R$ is considered due to fabrication requirements. Furthermore, since the size limitations of $t_{R,C} = h_{R,C}/2$ are also considered in formulae, the selection of RHS are restrained to such profiles only.

The computational results are summarized in Tables 2 and 3.

Table 2. Optimization results for Δ_{allow} = 30 mm: optimum dimensions in mm and values of K/k_m in kg for cost

n	t_f	h_R	t_R	h_C	t_C	h	k_f/k_m	K/k_m

6	5	160	4	80	2	160	1	4067
8	4	120	4	80	2	140	0	1415
8	4	120	4	80	2	140	1	3806
8	4	120	4	80	2	140	2	6197
10	3.5	120	3	80	2	130	1	3758
12	3.5	120	2.5	80	2	120	1	4087
14	3.5	120	2.5	40	2	120	1	4458

Table 3. Optimization results for Δ_{allow} =20 mm: optimum dimensions in mm and K/k_m values in kg for cost in the case of $k_f/k_m = 1$

n	t_f	h_R	t_R	h_C	t_C	h	K/k_m
6	5	160	4	80	2	160	4067
8	4	160	4	80	2	160	4009
10	3.5	160	4	80	2	160	4178
12	3.5	160	4	40	2.5	160	4665
14	3.5	160	4	40	2	160	5176

6. Conclusions

The results show that the optimum number of radial ribs for $k_f/k_m = 1$ is $n_{opt} = 8$ for both values of Δ_{allow}. For this optimum number, in Table 2, values for $k_f/k_m = 0$ and 2 are also given. It can also be seen that the costs of the structure in the case of smaller allowable displacement are larger, since this smaller displacement should be achieved using larger stiffeners. The cost difference between the best and worst solution, in the case of $\Delta_{allow} = 30$ mm and $k_f/k_m = 1$, is 100(4458-3806)/3806 = 17%, which shows that significant cost savings can be achieved by optimum design.

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