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Minimum cost design of welded tubular frames for a special truck

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Abstract

An optimum design procedure is worked out in the case of a simple one-bay one story rectangular frame welded from rectangular or square hollow sections (RHS or SHS). Optimum dimensions of profiles for the columns and beam are calculated to minimize a cost function and to fulfill design constraints. The cost function includes the costs of material, welding and painting. Design constraints on static stress, flexural and local buckling as well as fatigue stress range are taken into account. The optimization is performed using British (UK) and South African (SA) cost data and profile series. Four structural versions are optimized and compared to each other as follows: for columns and beam (a) the same rectangular hollow section (RHS) profile, (b) two different RHS profiles, (c) the same square hollow section (SHS) profile, (d) two different SHS profiles.

Key words: optimum design, welded structures, trucks, hollow sections, minimum cost design, frames

Introduction

The investigated truck is a special one, the purpose of which is the transport of a tank filled with fluid. The tank is elevated and put on the truck platform by a crane trolley, which is running on a rolled I-beam. This beam is fixed on two frames (Fig.1). Our aim is to work out the optimum design of these frames. We want to show how to select the cheapest structural version considering more combinations of profiles for the frame columns and beam. Rectangular and square hollow sections (RHS and SHS, Fig.3) are selected for this purpose.

The frame corners as well as the connections of the frames and the longitudinal I-beam are welded, thus, the fatigue strength of these joints should be taken into account. Besides this fatigue stress constraint the constraints on static stresses due to normal forces and bending moments in the columns and beam should be included considering the overall buckling factors.

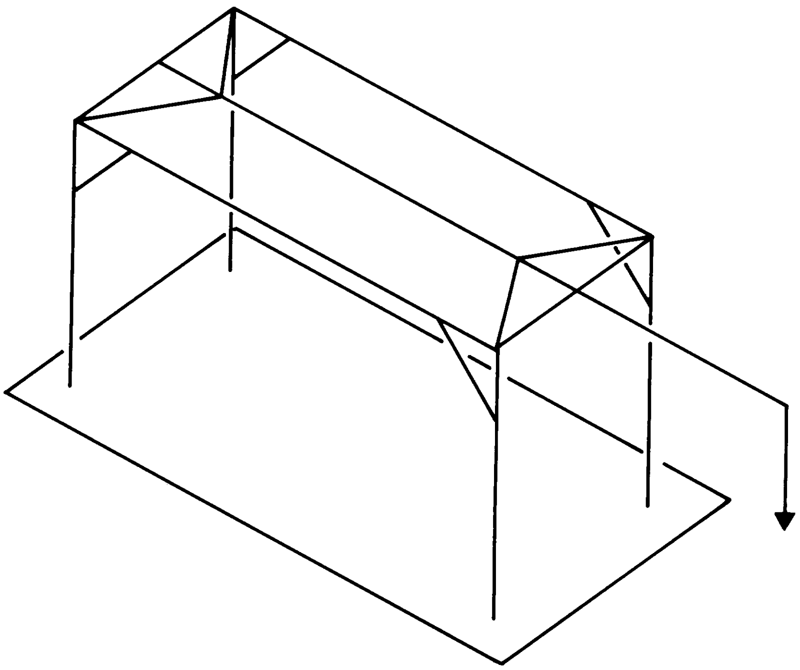

Fig.1. Scheme of the structure for the elevation of a load to a special truck

The frames are fixed on the truck platform and, to guarantee their longitudinal stability, are connected to each other by longitudinal braces (Fig.1). Thus, the frame corners can be treated as fixed in the longitudinal direction. The welded corners are constructed using intermediate splice plates. Four structural versions are considered as follows: the columns and the beam are constructed from

(a) the same RHS profile,

(b) two different RHS profiles,

(c) the same SHS profile,

(d) two different SHS profiles.

In the optimum design the cost function consists of material, welding and painting costs, other costs are neglected. The design constraints are formulated according to Eurocode 3 [1,2]. The computer algorithm of the Rosenbrock's hillclimb method is used for the constrained function minimization [3]. The design variables are the heights and thicknesses of the profiles. Having obtained the continuous optima, the discrete dimensions, determined by the available profiles, are calculated by a complementary search.

This research is carried out within the South African – Hungarian agreement between the Universities of Pretoria and Miskolc.

The cost function

The calculated cost includes the costs of material, welding and painting as follows:

$$K = K_m + K_w + K_p \tag{1}$$

where

$$K_m = k_m \rho V \tag{2}$$

$$V = 2HA_1 + LA_2 \tag{3}$$

$$K_w = k_w \left(\Theta_w (\kappa \rho V)^{0.5} + 1.3 \sum_i C_{wi} a_{wi}^n L_{wi} \right) \tag{4}$$

$$K_p = k_p A_s \tag{5}$$

k_m, k_w and k_p are the material, welding and painting cost factors, respectively, ρ is the material density, V is the volume of the structure, H and L are the main frame dimensions (Fig.2), A_1 and A_2 are the cross-section areas of the columns and beam, respectively, A_s is the surface of the frame to be painted. Θ_w is a difficulty factor expressing the complexity of the structure regarding the assembly and welding, κ is the number of structural parts to be assembled. The first term in Eq.4 expresses the time of preparation, assembly and tacking, the second term is the time of welding and additional works such as deslagging, changing the electrode etc. C_w is a constant, depending on the welding technology and type of welds, a_w is the weld size and L_w is the weld length.

For two RHS profiles the welding time is

$$\sum C_{wi} a_{wi}^n L_{wi} = 2C_w \left[\left(h_1 t_1^2 + h_2 t_2^2 \right) \left(\frac{2}{\cos 45^0} + 1 \right) + 3 h_1 t_1^2 \right] \tag{6}$$

for two SHS profiles it is

$$\sum C_{wi} a_{wi}^{n} L_{wi} = 2C_w \left[\left(h_1 t_1^2 + h_2 t_2^2 \right) \left(\frac{2}{\cos 45^0} + 2 \right) + 4 h_1 t_1^2 \right] \tag{7}$$

The cross-section area of a RHS profile with a height *h*, width *b* and thickness *t*, considering rounded corners of corner radius of $R = 2t$ and supposing that $b_i = h_i/2$, using the formulae given by Eurocode 3 Part 1.3 [2], can be calculated as

$$A_i = 2t_i \left(1.5h_i - 2t_i \right) \left(1 - 0.43 \frac{4t_i}{1.5h_i - 2t_i} \right) \qquad i = 1, 2 \tag{8}$$

where subscripts 1 and 2 denote values for the columns and beam, respectively.

Furthermore the surface area is

$$A_s = 3\left(2Hh_1 + Lh_2 \right) \tag{9}$$

For SHS it is

$$A_i = 4t_i \left(h_i - t_i \right) \left(1 - 0.43 \frac{2t_i}{h_i - t_i} \right) \tag{10}$$

$$A_s = 4\left(2Hh_1 + Lh_2 \right) \tag{11}$$

Design constraints

Diagrams of normal forces and bending moments due to the vertical force *F* and the horizontal force 0.1*F* can be calculated according to [4] and are given in Fig.2. It is assumed that the column bases are fixed and the beam-to-column corner connections are rigidly welded. It should be noted that the lateral-torsional buckling factor is $\chi_{LT} = 1$, since the torsional stiffness of hollow sections is large.

(a) The stress constraint for the beam (point E, Fig.2) according to Eurocode 3 Part 1.1 [1] is

$$\frac{H_A + H_{D1}}{\chi_{2.\min} A_2 f_{y1}} + \frac{k_{M2} M_E}{W_{x2} f_{y1}} \le 1; \qquad f_{y1} = \frac{f_y}{\gamma_{M1}} \tag{12}$$

where f_y is the yield stress, $\gamma_{M1} = 1.1$ is a safety factor,

$$H_A = \frac{3M_A}{H}; M_A = \frac{M_B}{2}; M_B = \frac{FL}{4(k+2)}; k = \frac{I_{x2} H}{I_{x1} L} \tag{13}$$

$$H_{D1} = \frac{0.1F(k+1)}{2(k+2)}; M_E = \frac{FL}{4} - M_B \tag{14}$$

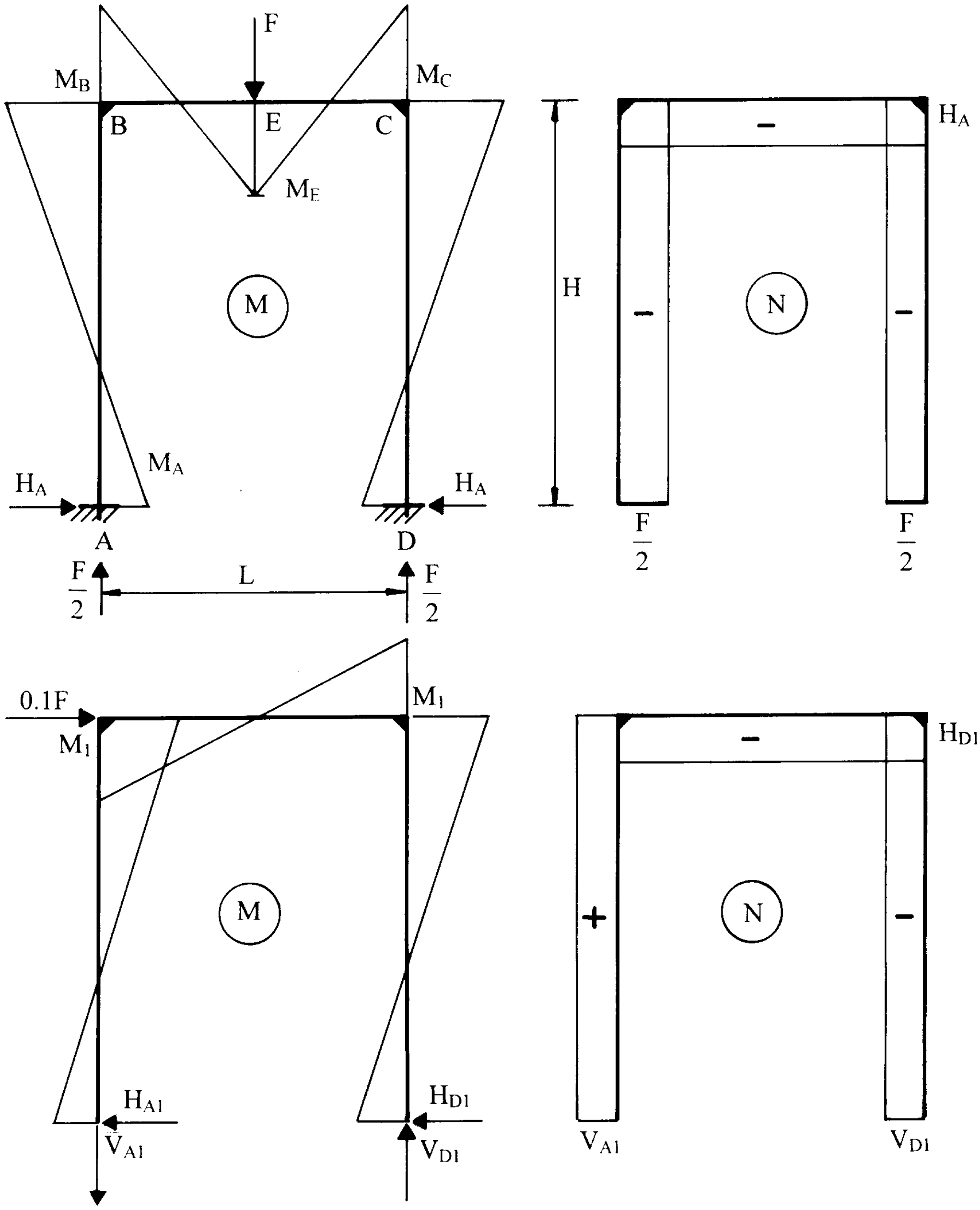

Fig.2. Diagrams for the bending moments and normal forces of a simple frame

For RHS the second moments of area are as follows (Fig.3)

$$I_{xi} = \left[\frac{(h_i - t_i)^3 t_i}{6} + \frac{t_i}{2}\left(\frac{h_i}{2} - t_i\right)(h_i - t_i)^2\right]\left(1 - 0.86\frac{4t_i}{1.5h_i - 2t_i}\right) \tag{15}$$

$$I_{yi} = \left[\frac{(0.5h_i - t_i)^3 t_i}{6} + \frac{t_i}{2}\left(\frac{h_i}{2} - t_i\right)^2 (h_i - t_i) \right]\left(1 - 0.86\frac{4t_i}{1.5h_i - 2t_i}\right) \tag{16}$$

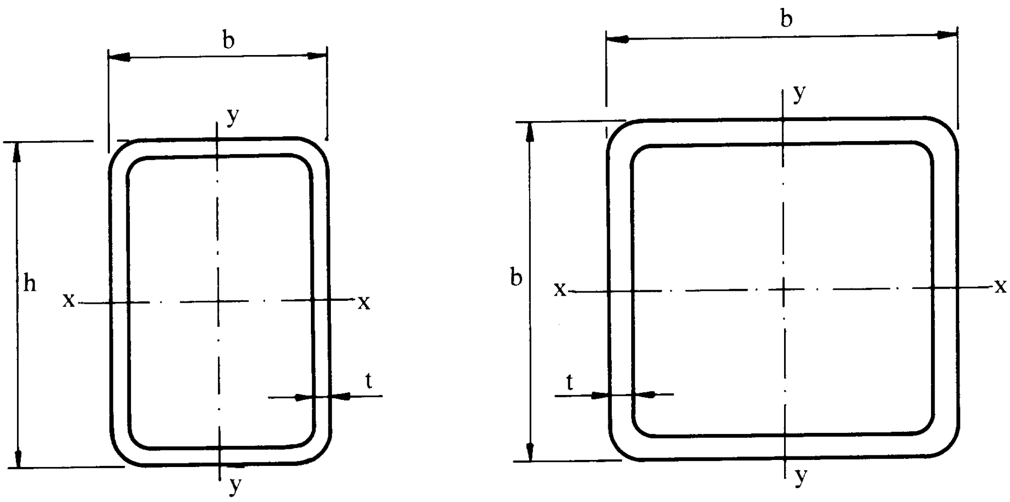

Fig.3 Dimensions of RHS and SHS profiles

For the beam i = 2 and for the columns i = 1.

For SHS

$$I_{xi} = I_{yi} = \left[\frac{(h_i - t_i)^3 t_i}{6} + \frac{t_i}{2}(h_i - t_i)^3 \right]\left(1 - 0.86\frac{2t_i}{h_i - t_i}\right) \tag{17}$$

The elastic section modulus is

$$W_{xi} = \frac{2I_{xi}}{h_i} \tag{18}$$

The flexural buckling factor is

$$\chi_i = \frac{1}{\phi_i + \left(\phi_i^2 - \bar{\lambda}_i^2\right)^{0.5}}; \qquad \phi_i = 0.5\left[1 + 0.34\left(\bar{\lambda}_i - 0.2\right) + \bar{\lambda}_i^2\right] \tag{19}$$

$$\bar{\lambda}_{x2} = \frac{K_{x2}L}{r_{x2}\lambda_E}; \qquad \text{the effective length factor is} \quad K_{x2} = 0.5 \tag{20}$$

$$r_{x2} = \left(\frac{I_{x2}}{A_2}\right)^{0.5}; \lambda_E = \pi\left(\frac{E}{f_y}\right)^{0.5}; \qquad E \text{ is the elastic modulus} \tag{21}$$

$$\bar{\lambda}_{y2} = \frac{K_{y2}L}{r_{y2}\lambda_E}; K_{y2} = 0.5; r_{y2} = \left(\frac{I_{y2}}{A_2}\right)^{0.5} \tag{22}$$

$\chi_{2.\min}$ is calculated for $\bar{\lambda}_{2.\max} = \max\left(\bar{\lambda}_{x2}, \bar{\lambda}_{y2}\right)$.

$$k_{M2} = 1 + \frac{1.2\bar{\lambda}_{x2}\left(H_A + H_{D1}\right)}{\chi_{x2} A_2 f_y} \tag{23}$$

(b) Constraints on local buckling for the beam

For the compression flange of a RHS:

$$\frac{0.5h_2 - 3t_2}{t_2} \le 42\varepsilon_2 ; \varepsilon_2 = \left(\frac{235}{\sigma_{\max.2}}\right)^{0.5} ; \sigma_{\max.2} = \frac{H_A + H_{D1}}{A_2} + \frac{M_E}{W_{x2}} \tag{24}$$

for the SHS flange

$$\frac{h_2 - 3t_2}{t_2} \le 42\varepsilon_2 \tag{25}$$

for RHS and SHS webs

$$\psi_2 = -\frac{\dfrac{M_E}{W_{x2}} - \dfrac{N_2}{A_2}}{\dfrac{M_E}{W_{x2}} + \dfrac{N_2}{A_2}} ; N_2 = H_A + H_{D1} \tag{26}$$

when $\psi_2 > -1; \dfrac{h_2 - 3t_2}{t_2} \le \dfrac{42\varepsilon_2}{0.67 + 0.33\psi_2}$ (27a)

when $\psi_2 \le -1; \dfrac{h_2 - 3t_2}{t_2} \le 62\varepsilon_2 \left(1 - \psi_2\right)\left(-\psi_2\right)^{0.5}$ (27b)

(c) Stress constraint for columns (point C, Fig.2)

$$\frac{N_1}{\chi_{1.\min} A_1 f_{y1}} + \frac{k_{M1}\left(M_1 + M_C\right)}{W_{x1} f_{y1}} \le 1 \tag{28}$$

where

$$N_1 = \frac{F}{2} + V_{D1} ; V_{D1} = \frac{2M_1}{L} ; \; M_1 = 0.1FH \frac{3k}{2(6k+1)} \tag{29}$$

I_{x1}, I_{y1}, W_{x1} are calculated with Eqs 15, 16, 17, 18 using i = 1.

$$r_{x1} = \left(\frac{I_{x1}}{A_1}\right)^{0.5} ; r_{y1} = \left(\frac{I_{y1}}{A_1}\right)^{0.5} ; \bar{\lambda}_{x1} = \frac{K_{x1} H}{r_{x1}\lambda_E} ; K_{x1} = 2.19 ; \bar{\lambda}_{y1} = \frac{K_{y1} H}{r_{y1}\lambda_E} ; K_{y1} = 0.5 \tag{30}$$

$$\bar{\lambda}_{1.\max} = \max\left(\bar{\lambda}_{x1}, \bar{\lambda}_{y1}\right) \tag{31}$$

$$\chi_{1.\min} = \frac{1}{\phi_1 + \left(\phi_1^2 - \bar{\lambda}_{1.\max}^2\right)^{0.5}} ; \phi_1 = 0.5\left[1 + 0.34\left(\bar{\lambda}_{1.\max} - 0.2\right) + \bar{\lambda}_{1.\max}^2\right] \tag{32}$$

$$k_{M1} = 1 - \frac{0.3\bar{\lambda}_{x1} N_1}{\chi_{x1} A_1 f_y}; \quad \chi_{x1} \text{ is calculated with } \bar{\lambda}_{x1}. \tag{33}$$

(d) Constraints on local buckling for columns

Formulae are the same as in (b), but

$$\sigma_{\max.1} = \frac{N_1}{A_1} + \frac{M_1 + M_C}{W_{x1}} \tag{34}$$

and

$$\psi_1 = -\frac{\dfrac{M_1 + M_C}{W_{x1}} - \dfrac{N_1}{A_1}}{\dfrac{M_1 + M_C}{W_{x1}} + \dfrac{N_1}{A_1}} \tag{35}$$

(e) Fatigue stress constraint for the beam (point E, Fig.2)

$$\frac{H_A + H_{D1}}{A_2} + \frac{M_E}{W_{x2}} \le \frac{\Delta\sigma_{N2}}{\gamma_{Mf}} \tag{36}$$

According to the IIW Recommendations [5], the fatigue stress range for the number of cycles N = 2x10^6 in the case of a transverse attachment thicker than the main plate, is $\Delta\sigma_C = 71$ MPa. Using the formula of

$$\log \Delta\sigma_N = \frac{1}{3} \log \frac{2x10^6}{N} + \log \Delta\sigma_C \tag{37}$$

one obtains for a smaller number of cycles of N = 10^5 $\Delta\sigma_N = 1927$ MPa, which is more realistic in the case of the investigated truck structure. Since a static safety factor of 1.5 is not used in the calculation of force F for the fatigue constraint, this fatigue stress range value should be multiplied by 1.5 and divided by the fatigue safety factor of 1.25. Thus, we obtain

$$\frac{\Delta\sigma_{N2}}{\gamma_{Mf}} = 231 \text{ MPa.}$$

(f) Fatigue stress constraint for the columns (point C, Fig.2)

$$\frac{N_1}{A_1} + \frac{M_1 + M_C}{W_{x1}} \le \frac{\Delta\sigma_{N1}}{\gamma_{Mf}} \tag{38}$$

According to [5], the fatigue stress range for a splice of RHS or SHS with single-sided fillet welds, toe crack and wall thickness smaller than 8 mm, is $\Delta\sigma_C = 45$ MPa. Using Eq.37, one obtains for N = 10^5 $\Delta\sigma_N = 122$ MPa. Multiplying by 1.5 and dividing by 1.25 we obtain

$$\frac{\Delta\sigma_{N1}}{\gamma_{Mf}} = 146 \text{ MPa.}$$

Numerical data

The load at the end of the longitudinal rolled I-beam (Fig.1) is 420 kg. The reaction force acting on the frame is 420x3/2 = 630 kg = 6.3 kN. For the static stress constraints this force should be multiplied, according to [1], by a static safety factor of 1.5. Besides this safety factor we multiply by a dynamic factor of 1.2. Thus according to the static constraints F = 6.3x1.5x1.2 = 11.34 kN.
The yield stress of steel is taken as f_y = 235 MPa. The material density is $\rho = 7850$ kg/m^{3,} the elastic modulus is $E = 2.1\text{x}10^5$ MPa.

British (UK) cost data and profile series:
The material cost factor for hot-formed RHS and SHS, according to the Price List of the British Steel Tubes and Pipes [6] is k_m = 1.0 \$/kg.
The welding cost factor is taken as k_w = 1.0 \$/min; $\Theta_w = 3; \kappa = 7$ since there are 3 bars, 2 splice plates and 2 base plates to be assembled. For fillet welds made by hand welding C_w = $0.7889\text{x}10^{-3}$ (SMAW = shielded metal arc welding), according to [7].
A painting cost factor of 14.4 \$/m^2 is given by Tizani [8].
Rounded optimal dimensions of RHS and SHS are selected according to available profiles given by The Steel Construction Institute [9] for hot finished hollow sections.

South African (SA) cost data and profile series:
for steel f_y =300 MPa (this is the *normal* steel in SA) the material cost is k_m = 1.35 \$/kg (11 Rand/kg); for steel f_y =235 MPa k_m = 1.28 \$/kg (normal f_y =300 MPa steel price minus 5%); k_w = 0.24 \$/min (1.95 Rand/min); k_p = 2.53 \$/m^2, RHS and SHS profiles are used according to [10]. Other data are the same as for UK.

Optimization and results

In the optimization procedure the total cost is selected as the objective function to be minimized. Design constraints on static stress, local buckling and fatigue stress range are taken into account. Unknown variables are the profile dimensions as follows: in the case of two RHS or SHS profiles the number of variables is n = 4, in other cases n = 2. As an effective mathematical programming method the Rosenbrock's hillclimb direct search algorithm is used. Having obtained the unrounded optima, an additional discretization is performed to determine the rounded values according to the mentioned standard.

The optimal unrounded (continuous) and rounded (discrete) dimensions and the related minimal costs in the case of UK and SA cost data are summarized in Table 1-4, respectively. The optima are marked by bold letters.

Table 1. Optimal dimensions in mm using UK cost data (f_y=235 MPa)

profiles	continuous solution		discrete solution	
	dimensions	cost in $	dimensions	cost in $
one RHS	88.2x44.1x2	73.9	80x40x3	90.3
two RHS	80.9x40.45x2 94.3x47.15x2	**71.6**	80x40x3, 80x40x3	90.3
one SHS	69.7x69.7x2	76.8	70x70x3	102.1
two SHS	59.4x59.4x2 80.1x80.1x2	73.2	50x50x2.5, 80x80x3	**82.0**

Table 2. Optimal dimensions in mm using SA cost data (f_y=235 MPa)

profiles	continuous solution		discrete solution	
	dimensions	cost in $	dimensions	cost in $
one RHS	88.2x44.1x2	34.5	101.6x50.8x2	39.4
two RHS	75.8x37.9x2 101.7x50.85x2	**32.7**	76.2x38.1x2 101.6x50.8x2	**32.8**
one SHS	69.7x69.7x2	36.2	76.2x76.2x2	39.3
two SHS	50.5x50.5x2 96.3x96.3x2	33.1	50x50x2 101.6x101.6x2	33.6

Table 3. Optimal dimensions in mm using SA cost data (f_y=235 MPa, cold formed)

profiles	continuous solution		discrete solution	
	dimensions	cost in $	dimensions	cost in $
one RHS	88.2x44.1x2	34.5	100x50x2	38.8
two RHS	71.8x35.9x1.6 122.8x61.4x2	**30.7**	76.2x38.1x1.6 120x60x2	**31.3**

one SHS	69.7x69.7x2	36.2	76.2x76.2x2	39.3
two SHS	56.5x56.5x2 97.1x97.1x2	32.6	57.2x57.2x1.6 100x100x2	32.6

Table 4. Optimal dimensions in mm using SA cost data (f_y=300 MPa)

profiles	continuous solution		discrete solution	
	dimensions	cost in $	dimensions	cost in $
one RHS	86x43x2	34.9	80x40x2.5	39.1
two RHS	71.8x35.9x1.6 122.8x61.4x2	**31.6**	76.2x38.1x1.6 120x60x2	**32.3**
one SHS	69.7x69.7x2	36.2	76.2x76.2x2	39.3
two SHS	57.2x57.2x1.6 96.1x96.1x2	33.1	57.2x57.2x1.6 100x100x2	33.7

Fig. 4. The truck equipped with the frame

Figure 4 shows a practical example of the truck at the University of Pretoria. It is a removable version, with bolted connections to the truck. The frame structure is not exactly the same as we calculated, due to the original frame in front of the truck.

Conclusions

(a) The difference between the unrounded and rounded solutions is caused by the fact that the series of available profiles, mainly for RHS profiles with $b = h/2$, is rough.
(b) The continuous values give in both cases minimum cost for two different RHS profiles.
(c) The discrete values give different optimum solutions: with UK cost data for two different SHS, with SA cost data for two different RHS profiles.
(d) The UK cost data are much higher than the SA ones mainly for fabrication (welding) and painting cost.
(e) The cost difference between the best and worst solutions indicated in tables is 24.5% for UK and 25.5% for SA data, thus, it is worth to use an optimum design process.
(f) The example shows, that this supporting frame can be applied in serial production, in which the cost savings are significant.

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