

Parametric study of virtual stick balancing based on a delayed PD model

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Abstract: Virtual stick balancing by human subjects is analyzed with special attention to the reaction time delay and the sensory dead zones. A pendulum-cart system displayed on a computer screen is controlled via an optical computer mouse. Realization of the balancing task involves additional computational delay in the feedback loop. The measurement results are compared to numerical simulations based on a model with delayed proportional-derivative (PD) feedback. Balancing skills are measured using the critical length, i.e., the length of the shortest stick that subjects can balance. The critical length obtained by the virtual balancing tests is larger by a factor of $2 \sim 3.5$ than the critical length for mathematical model with delayed PD control. Time domain simulations for a model with dead zones both for the position and for the velocity show a limit cycle type of oscillation rather than a realistic noisy (or chaotic) motion.

Keywords: Reflex delay, computer delays, virtual stick balancing, inverted pendulum, dead zone

1. INTRODUCTION

Balancing about an unstable equilibrium is a natural everyday activity for human beings. The nature and the characteristics of the feedback process employed by the central nervous system (CNS) is still a subject of debates. Investigation of balancing tasks may help in identifying and in understanding the underlying control mechanism. Different human balancing tasks, such as simple quiet standing Maurer and Peterka (2005); Suzuki et al. (2012); Hwang et al. (2016), standing on pinned or rolling balance boards Chagdes et al. (2016); Molnar et al. (2017), stick balancing on the fingertip or on a pingpong racket Cabrera and Milton (2004); Milton et al. (2016); Yoshikawa et al. (2016) and its virtual counterpart, stick balancing on a computer screen using some interface actuator (Mehta and Schaal, 2002; Cabrera et al., 2004; Zgonnikov et al., 2014), are therefore often analyzed via simplified mechanical and mathematical models.

Human balancing can be modeled as a feedback mechanism, where the input is provided by the sensory organs, decision is made by the CNS and the actuation is performed by the musculature. Identification of the decision making mechanism, in other words, the control law employed by the CNS, is a difficult task since the mechanical model of the human body involves several uncertain elements. There are several concepts in the literature, such as the traditional proportional-derivative (PD) feedback (Mehta and Schaal, 2002; Stepan, 2009), proportional-derivative-acceleration (PDA) feedback (Insperger and Milton, 2014), predictor feedback (Milton et al., 2016), intermittent predictive controller (Gawthrop et al., 2013) or event-driven intermittent controller (Yoshikawa et al., 2016), just to mention a few.

Human balancing tasks are often analyzed experimentally using motion capture systems. These systems are expensive, typically not mobile and requires considerable time to calibrate. Performing balancing tasks in virtual environment, e.g., virtual stick balancing, reduces the costs of the experiments significantly, while it makes possible to manipulate the key parameters. For instance, the dynamics of the system (e.g., the order of the system) can easily be adjusted, or blank out tests can easily be performed (Mehta and Schaal, 2002).

In this paper, realization of a virtual stick balancing environment is investigated, where a pendulum-cart system is displayed on a computer screen and is controlled by human subjects using an optical computer mouse. Different computer-display configurations are investigated in order to reduce additional computational delays. A simple filtering concept is used to determine the acceleration of the position of the cart. Parameter identification and the concept of the critical length (the length of the shortest stick that subjects can balance) is used to characterize balancing properties.

2. DYNAMICS OF VIRTUAL BALANCING

A schematic model of the virtual balancing environment is shown in Fig. 1. The balancing task under study is a virtual stick balancing, which is modeled as a pendulum-cart model. For the sake of simplicity and limited visualization, a horizontally driven, planar inverted pendulum is analyzed as a two-degree-of-freedom mechanical system shown in Fig. 2. This is a simplified model of human stick balancing at the fingertip while it still captures the main characteristics of the task, namely, (1) an unstable position

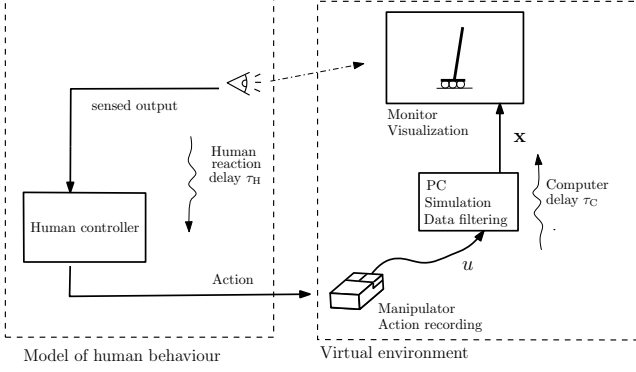


Fig. 1. Human in the loop: a schematic model of the virtual environment.

is stabilized via force feedback, and (2) the control loop involves a reaction delay.

The system is governed by the differential equation

$$\begin{bmatrix} m_c + m_s & Hm_s \cos \varphi \\ Hm_s \cos \varphi & H^2m_s + J_C \end{bmatrix} \begin{bmatrix} \ddot{\xi} \\ \ddot{\varphi} \end{bmatrix} - \begin{bmatrix} Hm_s \dot{\varphi}^2 \sin \varphi \\ Hm_s g \sin \varphi \end{bmatrix} = \begin{bmatrix} F \\ 0 \end{bmatrix}, \quad (1)$$

where m_s and m_c are the masses of the stick and the cart, respectively, H is the distance between the pivot point O and the center C of mass of the stick, J_C is the mass moment of inertia of the stick with respect to the normal axis through point C and $g = 9.81 \text{ m/s}^2$ is the gravitational acceleration. While the manipulated variable in real stick balancing on the fingertip is the force acting at the bottom of the stick, in virtual stick balancing, the acceleration $\ddot{\xi}$ of the cart is manipulated by the subjects through some input device (a computer mouse in this case). This consideration implies that the governing equation is reduced to a one degree-of-freedom mechanical system, with the acceleration ξ of the cart as input. The corresponding equation can be written as

$$(J_C + H^2m_s)\ddot{\varphi} - Hm_s g \sin \varphi = -Hm_s \ddot{\xi} \cos \varphi. \quad (2)$$

This equation is used to simulate the motion of the virtual stick as result of the input signal $u(t) = \ddot{\xi}(t)$ produced by human subjects using a computer mouse.

There are several concepts in the control law, i.e., the connection between the input u and the state variables φ and $\dot{\varphi}$. Here, a linear PD controller is assumed, namely, the input is given as a linear combination of the pendulum angle (P) and its angular velocity (D). Two main feature

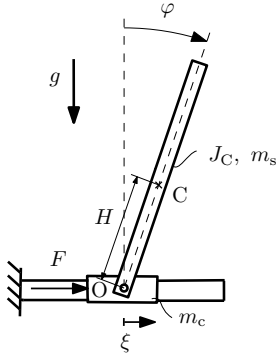


Fig. 2. The mechanical model: a horizontally driven planar inverted pendulum

of human stick balancing are the reaction time delay τ (i.e., sensory information perceived at time t induces control actions at time $t + \tau$) and the sensory dead zone (i.e., activation only takes place, when the change in the system's state becomes large enough). We assume that the angular position φ of the stick is detected only if it is larger than a position threshold φ_{dz} and the angular velocity $\dot{\varphi}$ is detected only if it is larger than a velocity threshold ω_{dz} . Then, the input can be modeled as

$$u(t) = \begin{cases} 0 & h_1 < 0, h_2 < 0 \\ -k_p \varphi(t - \tau) & h_1 < 0, h_2 > 0 \\ -k_d \dot{\varphi}(t - \tau) & h_1 > 0, h_2 < 0 \\ -k_p \varphi(t - \tau) - k_d \dot{\varphi}(t - \tau) & h_1 > 0, h_2 > 0 \end{cases} \quad (3)$$

where $h_1 = |\varphi(t - \tau)| - \varphi_{dz}$ and $h_2 = |\dot{\varphi}(t - \tau)| - \omega_{dz}$.

Linearization of equation (2) about the equilibrium $(\varphi, \dot{\varphi}) = (0, 0)$ gives

$$\ddot{\varphi} - a\varphi(t) = -bu(t), \quad (4)$$

where

$$a = \frac{Hm_s g}{J_C + H^2m_s} \quad (5)$$

is the system parameter and

$$b = \frac{Hm_s}{J_C + H^2m_s}. \quad (6)$$

For a homogeneous stick of length l and mass m_s , the system parameter is $a = \frac{3g}{2l}$.

Stabilizability properties are often described in terms of a critical feedback delay or in terms of a critical length. In case of zero thresholds ($\varphi_{dz} = 0, \omega_{dz} = 0$), the system can only be stabilized by PD feedback for a fixed stick length l if $\tau < \tau_{crit}$, where

$$\tau_{crit} = \sqrt{\frac{4l}{3g}} \quad (7)$$

is the critical delay (Stapan, 2009). Alternatively, for a fixed feedback delay τ , the stick can only be stabilized by PD feedback if the length of the stick is $l > l_{crit}$, where

$$l_{crit} = \frac{3g\tau^2}{4} \quad (8)$$

is the critical length.

3. THE VIRTUAL ENVIRONMENT

The software for the virtual stick balancing task was developed in JAVA environment. A fourth-order Runge-Kutta method was used with an adaptive time step such that the simulation was running in real time. The vertical dimension was scaled such that the whole stick was displayed on the computer screen. Sticks of different lengths all appeared to be 11 cm long. The displacements in the horizontal direction were not scaled, so the horizontal deviation between the top and the bottom of the stick on the screen was the real deviation of the underlying dynamical model.

Since the digital effects and the additional computational delays might change the qualitative behavior of the balancing task, we tried to minimize the computer delay τ_C (i.e., the time between an input produced by a computer mouse and its appearance on the screen). The computer delay for different computer-screen configurations was measured

Table 1. Computer delays for different configurations

Computer-display	Measured delay [ms]
Lenovo X260 (built-in display)	54
Dell Vostro (built-in display)	83
Lenovo X260 – TV screen	127

using a light sensor system, which detects black and white transition time of the screen synchronized to the mouse input. Based on the results shown in Table 1, the Lenovo Thinkpad X260 was selected to be the computing unit. The response time between a mouse input and its full representation on the screen was measured to be $\tau_{C,\text{response}} = 54$ ms. The input device is a conventional optical computer mouse.

During the visualization of the pendulum-cart system, the screen refresh rate was 60 FPS, therefore the frequency of the simulation was also set to 60 Hz in order to be synchronized to the screen refresh rate. The sampling frequency of a mouse is typically in the range of 1000 Hz, which, in our analysis is sampled at 60 Hz.

The computation of the acceleration of the cart (mouse) and its effect on the stick was performed within a single sampling period of length $\Delta t = 16.7$ ms. This sampling effect introduces an extra delay varying linearly between 0 and Δt with average of $\tau_{C,\text{sampling}} = \Delta t/2 = 8.3$ ms (Insperger et al., 2015).

The governing equation (2) contains the horizontal acceleration of the cart $\ddot{\xi}$, which is equivalent to the acceleration of the mouse moved by the subjects. In order to visualize the instantaneous position ξ of the cart on the screen, the exact position of the mouse is also required. The displacement of the mouse can be determined based on number of pixels the mouse passes over on the screen. For this, the cursor nonlinearity has to be turned off in the pointer precision settings of the operating system. The acceleration can be determined via numerical derivation of the pixel-based position. Due to the finite number of pixels, this results in a noisy acceleration signal. Therefore, the pixel-based acceleration signal has to be filtered before feeding back in the simulation. For this purpose, we used a simple resampling filter, which takes multiple time steps into account. The acceleration signal was computed as

$$u_i = K \frac{x_i - 2x_{i-k} + x_{i-2k}}{\Delta t^2}, \quad (9)$$

where x_i is the position of the mouse measured in pixels at the i^{th} time instant, $k \in \mathbb{N}$, $k \geq 1$ is the filtering parameter and K [kgm²/pixel] is a gain parameter. This filter can easily be implemented but, on the other hand, it produces a large artificial delay $\tau_{\text{filter}} = k\Delta t$ as we increase the filtering parameter k . During the tests the optimal value for the filtering parameter was found to be $k = 3$, which gives the additional delay $\tau_{\text{filter}} \cong 50$ ms. Thus, the overall computer delay was estimated to be

$$\tau_C = \tau_{C,\text{response}} + \tau_{C,\text{sampling}} + \tau_{\text{filter}} \approx 112\text{ms} \quad (10)$$

during the regular virtual stick balancing tests.

The described virtual environment was suitable for performing virtual stick balancing tests by human subjects. The positions of the stick and the cart can be stored for further analysis, e.g., for comparisons with different

models. As an initial test series, four subjects have been completed the virtual stick balancing tasks.

4. VIRTUAL BALANCING TESTS

Two types of balancing tests were performed, normal stick balancing tests and blank out tests.

4.1 Normal stick balancing

The virtual stick balancing environment is able to test the balancing skills of human subjects. The dynamical behavior of the pendulum-cart model is computed and visualized by the computer. The test subjects are able to interact with the system using a conventional optical computer mouse. At first, the human subjects were asked to practice some time in order to get comfortable with the virtual environment. After they managed to balance a stick of length $l = 3$ m for 60 seconds, they started the balancing trials. The first length was $l = 3$ m. Stick balancing for a given length was declared to be successful if the subject was able to balance it for 60 seconds at least once out of 40 trials. If the subject successfully balanced a stick of a given length, then the length of the stick was decreased by $\Delta l = 0.2$ m and he/she started new series of 40 trials. The critical length l_{crit} for a subject was the one, for which the balancing task was successful, but for the stick of length $l_{\text{crit}} - \Delta l$, it was not successful. The parameters of the simulation were fixed, except the length of the pendulum. The movement of the pendulum and the reaction of the test subjects via the computer mouse was stored for all the trials.

After reaching the critical length, the subjects were asked again to balance a stick of length $l = 3$ m, and their reaction time was measured by blank out tests.

4.2 Blank out tests

In order to determine the reaction times of the test subjects, we implemented the so-called blank out tests (Mehta and Schaal, 2002; Milton et al., 2016). This method applies a visual disturbance during the balancing. The task is the same for the subjects, namely, balancing a stick of length $l = 3$ m, but at an unknown time instant the visualization on the screen was turned off for 500 ms. During the blank out period, the pendulum was not shown on the screen, but the subjects were instructed to keep on balancing. When the visual feedback returns at the end of the blank out, the subjects try to compensate the effects of the signal loss, which results in a intensive mouse motion. This way the overall delay can be estimated by measuring the difference between the end of the blank out and the initiation of the corrective motion. During the blank out tests, the filter delay does not appear, because the movement of the mouse is directly stored without any filtering. Thus, the computer delay in case of the blank out tests was

$$\tau_{C,\text{blankout}} = \tau_{C,\text{response}} + \tau_{C,\text{sampling}} \approx 62\text{ms}. \quad (11)$$

5. PARAMETER IDENTIFICATION

Evaluation of human balancing skills is not a trivial task. Many different parameters can be defined which are

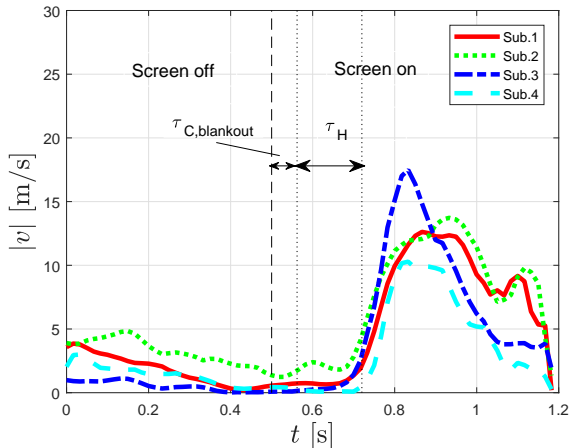


Fig. 3. Results of the blank-out tests. The averaged absolute velocity of the hand movement is calculated from 10 blank out trials for each subject. The additional computer delay $\tau_{C,\text{blankout}}$ and the human sensory delay τ_H can be measured from the figure. The dashed line indicates the time when the visual feedback is returned.

Table 2. Results of the blank out tests

Subject	Human reaction delay τ_H [ms]
1.	154
2.	137
3.	121
4.	171
Average	146

related to the balancing skill of the subject. Here, we use the standard deviation of the time history of the angular position of the stick in terms of the stick length and the subject’s reaction delay for a PD feedback model.

5.1 Human reaction delays

Each subject has accomplished 10 successful blank-out tests after the normal stick balancing trials. The start of the blank out was set to a random time instant in order to eliminate the effects of the human learning. The length of the visual signal loss was set to 500 ms for each test. The reaction of the test subjects was analyzed using the absolute values of their hand’s velocity recorded by the mouse position. The time series of the 10 blank-out tests were averaged for the individual subjects. The results of the measurements for all test subjects are shown in Fig. 3.

As mentioned in the previous section, the virtual environment introduces an additional artificial delay $\tau_{C,\text{blankout}}$. The blank-out test is able to measure the overall reaction delay. Thus, the human reaction delay can be computed as the difference of the overall and computer delay ($\tau_H = \tau - \tau_{C,\text{blankout}}$). The measured human delays for the test subjects are listed in Table 2.

In the literature, the reaction delay during real stick balancing tasks is estimated to be around 170 – 230 ms (Mehta and Schaal, 2002; Milton et al., 2016). Our result are slightly smaller than these ones.

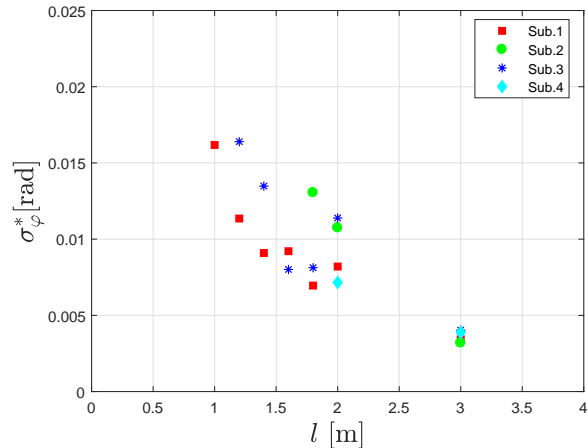


Fig. 4. The standard deviation of the vertical angle for different stick lengths. The tendency shows, that the more difficult the balancing task is the larger the deviations are.

5.2 Balancing skill

Many stabilometry parameters can be defined to describe and compare balancing skills, such as largest amplitude during balancing, standard deviation, length of center of mass trajectory, mean power frequency (Nagymáté and Kiss, 2016; Nagymáté et al., 2018). Here, we used the standard deviation σ_φ^* of the angle φ for the normal balancing trials of duration 60 s with different stick lengths. The results are shown in Fig. 4. As can be seen, the standard deviation clearly increases for shorter sticks, which corresponds to the trend that it is more difficult to balance short sticks than long ones. This shows that the length of the pendulum has a major effect on the robustness of the human controller.

Figure 4 also shows the critical length for the subjects, i.e., the length of the shortest sticks that they were able to balance for 60 s at least once out of 40 trials. These values can be compared to the theoretical values associated with a PD controller given by equation (8) using the corresponding overall reaction time $\tau = \tau_H + \tau_C$ for each subject. The theoretical values $l_{\text{crit,PD}}$ and the experimentally obtained ones $l_{\text{crit,tests}}$ are given in Table 3. It can be seen that there is a significant difference between the theoretical values and the experimentally observed ones. Namely, the experimentally observed l_{crit} is larger by a factor of 2 ~ 3.5. The explanation for this difference can be the uncertainties in the sensory information, which were not considered in equation (8). It is also possible that the human CNS employs more sophisticated control concepts than delayed PD feedback. For instance, delayed PDA feedback (Insperger and Milton, 2014), predictor feedback (Milton et al., 2016), event-driven intermittent controller (Yoshikawa et al., 2016), act-and-wait (Insperger, 2006) or drift-and-act controllers (Milton et al., 2009) can be possible candidates to the control mechanism.

5.3 Sensory dead zone

A method for predicting the dead-zone parameters of the controller is used following Milton et al. (2016). A

Table 3. Theoretical critical stick length $l_{\text{crit,PD}}$ and the experimentally determined critical length $l_{\text{crit,tests}}$.

Subject	$l_{\text{crit,PD}}$ [m]	$l_{\text{crit,tests}}$ [m]
1.	0.521	1.0
2.	0.456	1.8
3.	0.399	1.2
4.	0.589	2
Average	0.491	1.5

window of length t_w is swept over the history of φ and $\omega = \dot{\varphi}$ with time step $\Delta t = 16.7$ ms. This way, the signals were investigated step-by-step over the intervals $[(j-1)\Delta t, (j-1)\Delta t + t_w]$ with $j = 1, 2, \dots$, and the maximum values $\varphi_{\text{max},j}$ and $\omega_{\text{max},j}$ were stored for each step j . Then the minimum of the maximum values were considered as an upper estimate for the dead zone Π_φ for the angular position and for the dead zone Π_ω for the angular velocity. The variation of Π_φ and Π_ω with increasing window length t_w is shown in Fig. 5. As can be seen, there is a plateau both for Π_φ and Π_ω when $t_w > 6 \sim 8$ s. This means that there are periods of length $6 \sim 8$ s where $|\varphi| < \Pi_\varphi = 0.00301$ rad and where $|\dot{\varphi}| < \Pi_\omega = 0.0259$ rad/s. These suggest that the thresholds for φ and $\dot{\varphi}$ are less than these values, since it is unlikely that the stick does not fall without control for $6 \sim 8$ s. A better estimate of φ_{dz} and ω_{dz} can be given by considering the reaction delay.

In order to account with the reaction delay, it was assumed that the stick is falling freely for a delay period after leaving the dead zone with initial condition $(\varphi(t_{\text{dz}}), \dot{\varphi}(t_{\text{dz}})) = (\varphi_{\text{dz}}, 0)$ and $(\varphi(t_{\text{dz}}), \dot{\varphi}(t_{\text{dz}})) = (0, \omega_{\text{dz}})$. Using the dynamics of a freely falling pendulum, one get

$$\varphi(t_{\text{dz}} + \tau) = \frac{1}{2} \left(e^{-\sqrt{a}\tau} + e^{\sqrt{a}\tau} \right) \varphi(t_{\text{dz}}), \quad (12)$$

$$\dot{\varphi}(t_{\text{dz}} + \tau) = \frac{1}{2} \left(e^{-\sqrt{a}\tau} - e^{\sqrt{a}\tau} \right) \dot{\varphi}(t_{\text{dz}}). \quad (13)$$

Using the concept that $\varphi(t_{\text{dz}} + \tau) \cong \Pi_\varphi$ and $\dot{\varphi}(t_{\text{dz}} + \tau) \cong \Pi_\omega$, the dead zones are estimated as

$$\varphi_{\text{dz}} = \varphi(t_{\text{dz}}) = 2 \left(e^{-\sqrt{a}\tau} + e^{\sqrt{a}\tau} \right)^{-1} \Pi_\varphi, \quad (14)$$

$$\omega_{\text{dz}} = \dot{\varphi}(t_{\text{dz}}) = 2 \left(e^{-\sqrt{a}\tau} - e^{\sqrt{a}\tau} \right)^{-1} \Pi_\omega. \quad (15)$$

Considering a measurement of Subject 1 with stick length 3 m, the dead zones were estimated to $\varphi_{\text{dz}} = 0.0026$ rad and $\omega_{\text{dz}} = 0.022$ rad/s.

5.4 Control gain identification

The control gains were identified by comparing the standard deviation of the oscillations of the stick during virtual stick balancing and obtained by the simulations for a series of control parameters pairs (k_p, k_d) . Comparison was performed by means of the objective function

$$J = \left(\frac{\Delta\text{std}(\varphi)}{\varphi_{\text{dz}}} \right)^2 + \left(\frac{\Delta\text{std}(\dot{\varphi})}{\omega_{\text{dz}}} \right)^2, \quad (16)$$

where Δstd is the difference between the standard deviation of the measured time series of the angle φ and that of the simulated data. The selected control gain pair was the one, for which the value of J was the smallest. For $\tau = 266$ ms, the gains were found to be $k_p = 8.8421$

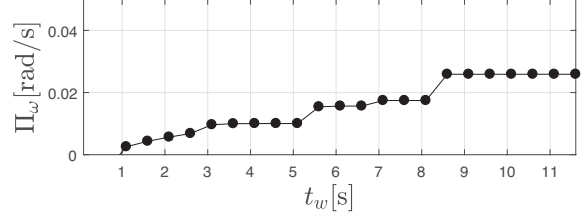
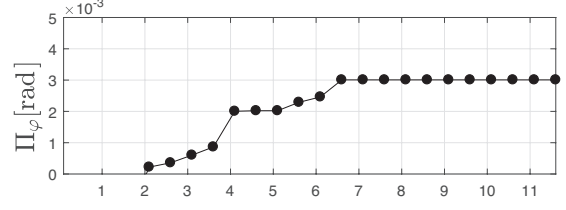


Fig. 5. The dead-zone prediction for Subject 1 with stick length of 3 m using the sweeping window method.

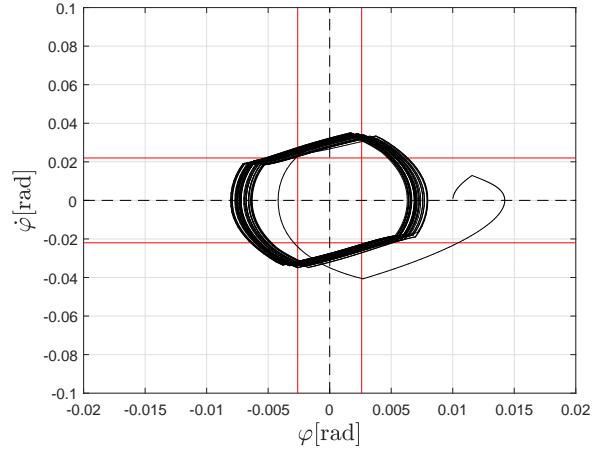
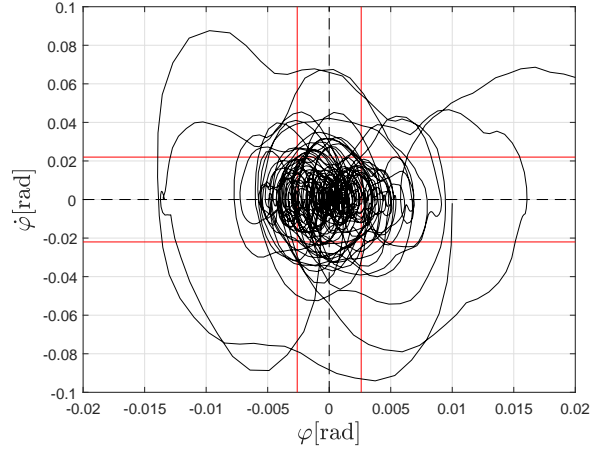


Fig. 6. Experimentally obtained phase portrait (top) and the result of the simulations after parameter identification (bottom) for the parameters of Subject 1 balancing a virtual stick of length 3m. Red lines indicates the boundaries of the dead zones.

and $k_d = 6.3158$. Simulation for these parameters and the phase portrait obtained for the measurement are shown in Fig. 6.

6. CONCLUSION

The virtual environment was established for testing the balancing skills of human subjects. An inverted pendulum displayed on a computer screen was balanced online using a conventional optical computer mouse as input. A main difficulty during the design of the tests was the additional time delay by the computer processing and visualization process. In order to get an accurate acceleration signal for the hand motion, the position signal had to be filtered, which resulted in an additional delay in the feedback loop. Balancing trials performed by human subjects for stick of different lengths showed that virtual stick balancing is different from real stick balancing on the fingertip. Due to the increased feedback delay, the shortest stick that a subject was able to balance was 1 m, while in real stick balancing, skilled subjects are able to balance sticks of length 30 ~ 40 cm (Milton et al., 2016).

Parameter identification was performed based on a PD control model. Reaction times were determined using blank out tests. The measured reaction time were slightly smaller than those ones in the literature (Mehta and Schaal, 2002; Milton et al., 2016). The dead zones for the angular position and for the angular velocity of the stick was determined using a sweeping window technique. Control gain parameters were estimated such that the amplitude of the oscillations are the closest to those obtained during the virtual balancing tests. While the measured signal shows clearly a complex (maybe chaotic) nature, the ones obtained by the simulations is rather a limit cycle oscillation generated by the dead zones. This means that the model should be modified. For instance, time periodic nature of the control mechanism can be modeled similarly to the sampling effect of digitally controlled machines. Another possibility is to involve noisy terms into the model.

The model under analysis was subjected to a delayed PD feedback. An extension of the analysis in the future can be the application of other types of control mechanism, such as delayed PDA feedback (Insperger and Milton, 2014), predictor feedback (Milton et al., 2016), event-driven intermittent controller (Yoshikawa et al., 2016), act-and-wait (Insperger, 2006) or drift-and-act controllers (Milton et al., 2009).

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