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Quantification of uncertainty in machining operations based on probabilistic and robust approaches

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Abstract

Reliable stability predictions of machine tool chatter have high potential in the optimization of machining processes. Industrial applications can benefit from the corresponding stability lobe diagrams, which can guarantee improvement in the performance. However, predictions and experiments often do not match during cutting tests due to uncertain dynamics and operational parameters. In order to provide safe and accurate tools for machining engineers, the importance of reliability of the stability lobe diagrams must be highlighted. There exists several solution to include uncertainty in the model used for stability predictions, such as modal parameter estimations or cutting force characteristics measurements. This paper collects the most well-known techniques that are used for the construction of robust stability lobe diagrams. In order to highlight the potential of such solutions, the results obtained by two different methods are compared to experimental cutting tests.

Keywords: Machine tool vibration; robustness; stability

1. Introduction

Productivity of manufacturing operations are characterized by the material removal rate, which depends on the spindle speed, feed rate and axial/radial immersions of the machining operations. Tuning of these parameters does not result necessarily in better cutting conditions, since unstable harmful vibrations can arise. These vibrations are referred as chatter in the literature, that spoils the surface quality, results extensive noise, possible damages in the machine components and increases toolwear. Surface regeneration effect is often considered as the main reason for this instability, which is extensively studied since the 50's, when Tobias [1] and Tlusty [2] presented the first mathematical models to describe chatter.

The model of surface regeneration effect is based on the stability analysis of delay-differential equations, where the regenerative time-delay is determined by the sequentially cutting edges of the milling cutter. Variations in the chip thickness affects the cutting force and self-excited vibrations arise. When the stationary solution loses stability, then large amplitude oscillation, i.e. chatter, arises. The stability properties are visualized on the so-called stability lobe diagrams, which plot stable (chatter-free) domains in the plane of technological parameters (usually the spindle speed and the depth of cut). These diagrams contain useful information for the machinists and give a guide to select optimal machining parameters, where unstable vibrations are completely avoided.

There exists several numerical techniques to analyze the stability of different machining operations, some of them directly utilize frequency response functions (FRFs), while others are time-domain techniques. Frequency domain based solutions, such as the single-frequency solution or zero-order approximation (ZOA), the multi-frequency solution (MFS) [3,4] or the extended multi-frequency solution (EMFS) [5] can be applied to measured FRFs directly. Time domain based techniques, such as the semi-discretization [6], the full-discretization [7], the integration method [8], the Chebyshev collocation method [9] and the spectral element technique [10] require fitted modal parameters as input.

In spite of the available efficient numerical algorithms, the stability lobe predictions are still not utilized in most industrial applications. One reason for this is the relatively high complexity of the problem and low reliability of the predictions. Since cutting parameter estimations and dynamical measurements are loaded with significant uncertainty, the model contains simplifications and unmodelled dynamics, the actual stability diagrams differ from the theoretical predictions. In this work we put the uncertainty analysis into focus.

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2. Dynamical model of milling operations

Here the dynamical model of milling operations is presented briefly. If the tool has N teeth of uniform helix angle β, then the corresponding regenerative time-delay is calculated as $\tau = 60/(N\Omega)$, where Ω (rpm) is the spindle speed. The equation of motion in the modal space can be written as

$$\ddot{\mathbf{q}}(t) + [2\zeta_k\omega_{\mathrm{n},k}]\dot{\mathbf{q}}(t) + [\omega_{\mathrm{n},k}^2]\mathbf{q}(t) = \mathbf{U}^\top\mathbf{F}(t), \tag{1}$$

where $\mathbf{q}(t) \in \mathbb{R}^n$ is the modal coordinate vector, $\mathbf{F}(t) \in \mathbb{R}^n$ is the spatial forcing vector, n is the number of degrees of freedom, $[\cdot] \in \mathbb{R}^{n\times n}$ are diagonal matrices, $\omega_{\mathrm{n},k}$ is the k^{th} natural angular frequency, ζ_k is the relative damping ratio and $\mathbf{U} \in \mathbb{R}^{m\times n}$ is the modal transformation matrix that connects the spatial coordinate vector $\mathbf{u}(t) \in \mathbb{R}^m$ and modal space as $\mathbf{u}(t) = \mathbf{U}\mathbf{q}(t)$. Inserting the regenerative forcing model into the governing equation, then assuming small perturbation $\boldsymbol{\xi}(t)$ about the periodic motion $\mathbf{q}_{\mathrm{p}}(t)$ of the stationary cutting yields the governing equation

$$\ddot{\boldsymbol{\xi}}(t) + [2\zeta_k\omega_{\mathrm{n},k}]\dot{\boldsymbol{\xi}}(t) + [\omega_{\mathrm{n},k}^2]\boldsymbol{\xi}(t) = \mathbf{U}^\top\mathbf{G}(t)\mathbf{U}\left(\boldsymbol{\xi}(t-\tau) - \boldsymbol{\xi}(t)\right), \tag{2}$$

where $\mathbf{G}(t) = \mathbf{G}(t+\tau)$ is the periodic directional matrix [6].

As opposed to the time-domain representations, the use of FRFs and Fourier transformations gives an alternative way to analyze the stability of the system. The dynamic behavior of the system in the frequency domain is represented by $\mathbf{u}(\omega) = \mathbf{H}(\omega)\mathbf{F}(\omega)$, where $\mathbf{u}(\omega)$ and $\mathbf{F}(\omega)$ are the Fourier transforms of $\mathbf{u}(t)$ and $\mathbf{F}(t)$, while $\mathbf{H}(\omega)$ is the frequency response function matrix of the tool. According to the multi-frequency solution, the stability boundaries can be calculated from

$$\det(\mathbf{I} - \mathbf{Q}(\Omega, a_{\mathrm{p}}; \omega)) = 0, \tag{3}$$

where $\mathbf{Q}(\Omega, a_{\mathrm{p}}; \omega)$ is a truncated Hill's infinite matrix, that includes the machining parameters and FRF matrix. With the use of time-domain or frequency-domain techniques, the stability of the system can be investigated on the plane of the spindle speed Ω and depth of cut a_{p}.

Independently from the representation of the governing equation, the uncertainties in the FRF matrix $\mathbf{H}(\omega)$ or in the modal parameters $\omega_{\mathrm{n},k}$, ζ_k, $\mathbf{U}$ affect the stability boundaries. A measured FRF is presented in Fig. 1, obtained from 10 different measurements. Each measurement is indicated by solid gray lines, while the upper and lower bounds are denoted by black lines ($\pm 1\sigma$). A seven-degree-of-freedom system was fitted onto the average of the measurements. This results $7 \times 4 = 28$ modal parameters in total. As it can be seen, neither the measurement, nor the fitting process is perfect, therefore the uncertainty analysis plays an important role in stability predictions.

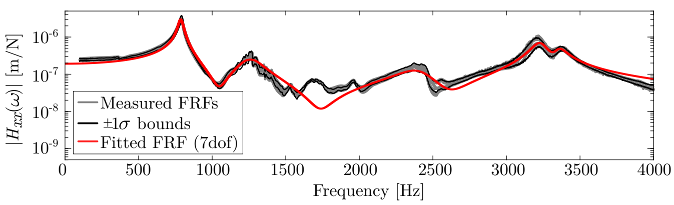

Fig. 1. Measured frequency response functions $|H_{xx}(\omega)|$ (gray), their upper and lower bounds based on 1σ standard deviation (black) and the fitted model (red).

3. Uncertainty analysis

The effect of parameter variations on the system's stability have already been analyzed by many researchers. In most of the cases the input parameters (modal parameters, cutting coefficients) are assumed to be uncertain and the stability diagram is determined with the use of statistical or robust methods. In the simplest scenario, one can determine the stability boundaries using Monte Carlo simulations [11–13] and evaluate the distribution, sensitivity and robustness afterwards. Depending on the number of parameters, this calculation process often leads to very high computational time and therefore their practical application without significant improvements is limited. In order to overcome this limitation, statical and/or robust techniques can be utilized that can speed up the calculations, typically in cost of the precision.

3.1. Statistical methods

In reliability analysis the reliability index and reliability probability are two important parameters that quantitatively reflect the effect of uncertainty on the design. The definition of reliability includes a multi-dimensional integral in the form

$$P = \int\cdots\int_{g(\mathbf{x})>0} f_{\mathbf{X}}(\mathbf{x})\mathrm{d}\mathbf{x}, \tag{4}$$

where $\mathbf{x}$ is the characteristic parameter vector, $\mathbf{X}$ is the random parameter vector corresponding to $\mathbf{x}$, $f_{\mathbf{X}}(\mathbf{x})$ is the joint probability density function (PDF) of $\mathbf{X}$, and $g(\mathbf{x})$ is the limit state function. Here, $g(\mathbf{x}) > 0$ indicates the cases, when the system is stable, then P gives the probability of stability. Since the evaluation of a multidimensional integral is highly complicated numerically, several solutions exist to overcome this limitation.

Probabilistic methods are based on the estimation of the PDF of the absolute value of the largest characteristic exponent or the spectral radius of the monodromy operator. When uncertainties are assumed to be 'small' (or their effect is 'small'), and the PDFs of the input random variables are known, then the local sensitivity techniques, such as the local partial derivatives [12,14] or first-order second moment methods and their improved versions [15–17] can give an accurate result for the probability of stability. For milling operations, where the spectral radius of the approximated monodromy operator determines the stability properties, local sensitivity methods provide less accurate solutions due to the more complex topology of stability diagrams. [18] used neural networks to speed up the calculation process, while [19] presented an approximating numerical method, which provides confidence levels of stability boundaries for high number of uncertain parameters. [20] introduced the RCPM method, that is based on discretization of the PDF and evaluation of stability on a discrete grid in the space of uncertain system parameters. Although this is one of the most accurate solution, it is limited to small number of parameters only. A different solution is used by [21] and [22] by introducing Fuzzy arithmetics to characterize the effect of uncertainty, but the calculation procedure is still limited to few parameters due to the high numerical complexity.

A more sophisticated method is proposed by [23] using the dimension reduction method and saddlepoint approximation (DRM-SPA), which is effective in calculation time and precision, however, gives less accurate solution where Hopf and flip stability boundaries meet. This is a technique of function approximation for the moment estimation of probability density functions. Therefore the complete evaluation of (4) simplifies to one-dimensional integrations, which can be done with quadrature rules (such as Gauss-Hermite). Once the first four moments are approximately known, the saddlepoint approximation can be used to generate the probability density function and cumulative distribution function (CDF). In this work, we utilize this solution and compare the results to experiments in Sec. 4

3.2. Robust methods

While statistical methods are useful when information about the probability distribution is required, robust techniques are found to be very effective if the distributions are omitted. In case of robust stability, the stability of the system is guaranteed for bounded perturbations in the prescribed parameters. There exists several very different ways to obtain these boundaries, some of them are already applied for machining operations. The edge theorem combined with the zero exclusion method is presented in [24,25] and compared to the results obtained by linear matrix inequalities in [26]. [27] used the stability radius method to give the exact robust stability boundaries of a single-degree-of-freedom system and provides and approximation of robust stability for multiple-degrees-of-freedom systems with uncertain modal parameters.

Up to this point, each technique was based on fitted modal parameters and cutting coefficients. In order to avoid modal parameter estimation and significantly reduce the number of uncertainties, [27] presented a frequency-domain approximation by means of envelope fitting around the measured FRFs. In [28], this concept was extended further using structured singular values and the extended multi-frequency solution (MFS-SSV). The advantage of the technique is the small computational time and direct applicability of measured FRFs, however, it does not take into account the variations in the cutting coefficients. The solution obtained by the structured singular value analysis is also presented in Sec. 4 through a case study.

4. Experiment

An experiment was carried out on a CNC machine. The FRF matrix of the tool-tip was measured in XY directions (10 hit each), the result corresponding to the main direction x can be seen in Fig. 1. Then a seven-degree-of-freedom system was fitted onto the average of the measured functions. The same fitting process was repeated for several combination of the measured FRFs in order to characterize the uncertainty in the fitting and measuring procedure. Due to the high number of parameters, these modal data are not listed here.

Cutting tests were performed to identify the empirical specific force characteristics in radial (r) and tangential directions

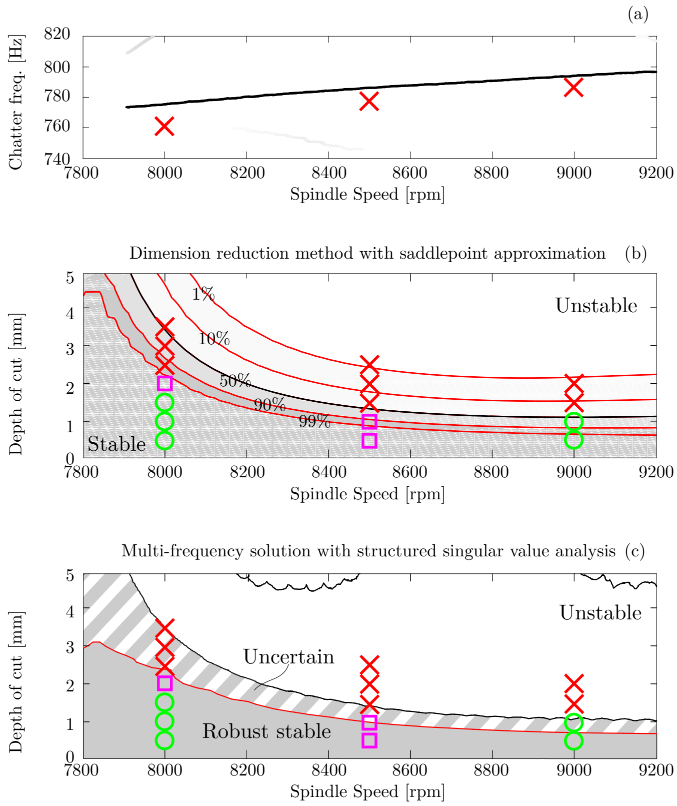

Fig. 2. Theoretical stability chart with measurements: (a) Chatter frequency; (b) DRM-SPA method [23]; (c) MFS-SSV method [28]. Notation: ○-stable, □-marginal, ×-unstable.

(t) as a function of chip thickness h: $f_x(h) = K_{c,x}h + K_{e,x}(1 - \mathrm{e}^{-E_x h})$, $x = t, r$, where $K_{c,t} = 1049$ MPa, $K_{e,t} = 13.3$ N/mm, $E_t = 668\,\mathrm{mm}^{-1}$, $K_{c,r} = 532$ MPa, $K_{e,r} = 15$ N/mm, $E_r = 453\,\mathrm{mm}^{-1}$. Other cutting parameters are: $N = 2$, $\beta = 30°$, $f_Z = 0.05$ mm and 50% down-milling operation.

In order to determine the probability of stability, the DRM-SPA method of [23] was utilized. The total number of stability chart calculations required $28 \times 4 + 1 = 113$ repeated function evaluations at each point on the stability diagram. The results are indicated in Fig. 2(a-b),where panel (a) shows the theoretical dominant chatter frequencies and (b) indicates the probability of stability at levels 1-10-50-90-99%, respectively.

The robust boundaries were determined by the MFS-SSV method [28] also, where the envelope of uncertainty is described by the 1σ standard deviation of the measurements. This calculation requires slightly more computational effort than a single stability diagram (here it was approx. two-times longer). The corresponding solution is presented in Fig. 2(c), where black curves indicate the nominal boundary, red curve indicates the robust stability boundary.

The experiment was carried out at different spindle speeds, green circles indicate stable operation, magenta squares denote marginal points (undecided) and red crosses stand for unstable operation. The experimental setup is presented in Fig. 3. The stability properties were determined based on the surface quality, signals of the accelerometers and spectrum of the industrial microphones. Two sample spectra of the microphone signals are presented in Fig. 4(a-b). In panel (b) the dominant chatter frequency is marked with red cross.

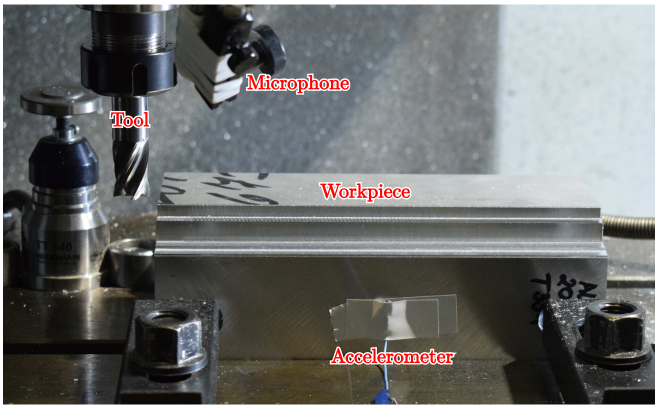

Fig. 3. Experimental setup.

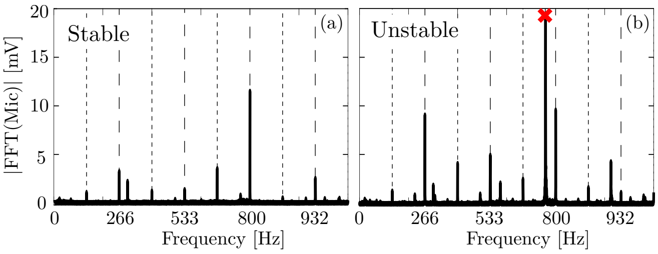

Fig. 4. Spectrum of the measured signal of the microphone: (a) Stable (Ω = 8000 rpm, a_p = 1.5 mm); (b) Unstable (Ω = 8000 rpm, a_p = 3 mm).

5. Conclusions

While uncertainties in conventional stability calculations often lead to unreliable predictions, statistical and robust methods can overcome these limitations. This paper collects the most well-know time and frequency-domain techniques listed by the literature. Two different algorithms are highlighted, namely the dimension reduction method with saddlepoint approximation and the multi-frequency solution with structured singular value analysis. The methods are tested in experiments that validated the necessity of uncertainty analysis in stability lobe diagram calculations.

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