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Optimum design of welded stiffened plates loaded by hydrostatic normal pressure*

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Abstract
The optimal positions of horizontal stiffeners are computed considering the condition that the base plate parts, having equal thicknesses and loaded by bending moments, should be stressed to yield strength. The trapezoidal stiffeners are designed for bending using the stress and local buckling constraints. The optimal number of stiffeners is determined on the basis of material and fabrication cost calculations. It is shown by a numerical example that the non-equidistant stiffener arrangement gives 26% weight and 19-21% cost savings.

Keywords
Stiffened plates, minimum cost design, welded structures, hydrostatic pressure

1 Introduction

Stiffened plates can be loaded by hydrostatic pressure in several structures, for instance in bunkers and gates (Fig.1). In the book of Wickert and Schmausser (1971) a solution is given to obtain the optimal distances of horizontal stiffeners using the condition that all the stiffeners should be equally loaded. This solution disregards the dimensioning of the base plate, thus, it does not give a realistic optimum. McGrattan (1985) has worked out a weight and cost optimization for equally spaced longitudinal and orthogonal stiffeners. This study does not consider the non-equidistant arrangement of stiffeners. Kravanja et al. (1998) have used a mixed-integer nonlinear programming method for cost optimization of hydraulic steel gate structures. In their article the problem is described generally for non-equidistant horizontal stiffeners and the numerical example of the orthogonally stiffened Intake Gate (Aswan, Egypt) structure is solved showing 28% cost savings compared with the actual solution.

In the present study the stiffened plate consists of a square base plate of constant thickness and horizontal stiffeners. Vertical stiffeners are also treated for cost comparison. This arrangement is selected for its relative simplicity regarding the fabrication. When the horizontal stiffeners are equally spaced only the lowest part of the base plate can be stressed to yield strength, thus, these solutions are not optimal.

The optimum position of stiffeners can be calculated on the basis of the condition that all parts of the base plate, having the same thickness, should be stressed to yield strength.

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For the calculation of the maximum bending moments in simply supported base plate parts due to hydrostatic pressure the values given in tabulated form in (Vaynberg 1970) are approximated by nonlinear functions of the ratio of side dimensions *b/a* (Fig.2). The optimum stiffener positions characterized by x_j (Fig.3) are calculated from a set of nonlinear equations using the MathCAD software. Trapezoidal stiffeners are considered (Fig.4) and designed for bending considering also the local buckling constraint.

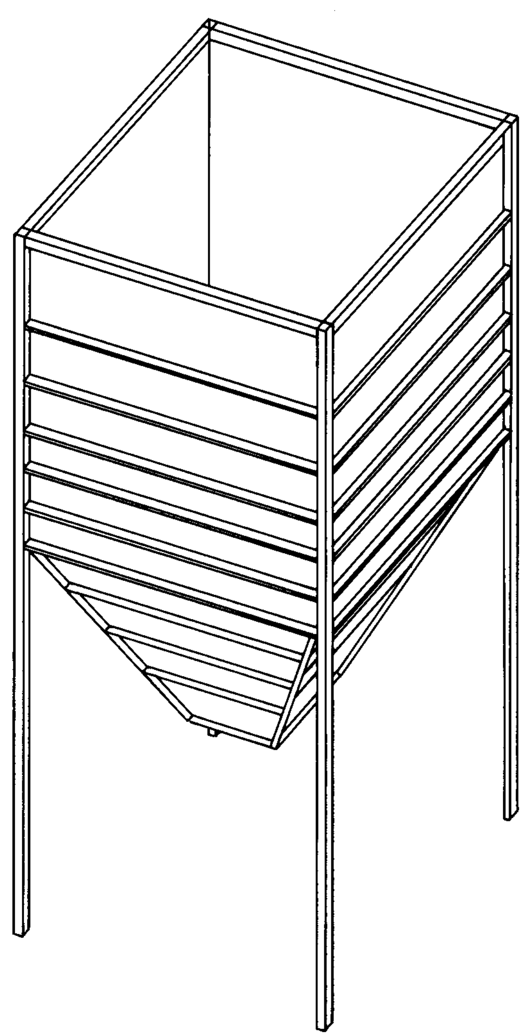

Figure 1. A bunker constructed from stiffened plates

The cost function consists of material as well as fabrication costs and is calculated in the function of number of stiffeners for equidistant and non-equidistant (optimal) stiffener positions. It is shown that, using optimally spaced stiffeners, significant mass and cost savings can be achieved.

2 Optimum position of horizontal stiffeners

The thickness of each base plate part can be calculated from the stress constraint

$$\frac{6M}{t_F^2} \le f_y \qquad (1)$$

where M is the factored bending moment, t_F is the base plate thickness, f_y is the yield stress.

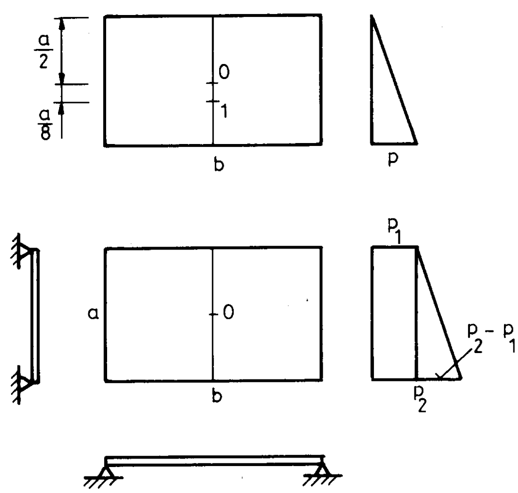

Figure 2. Points where the maximum bending moment arises

The maximum bending moment in a simply supported rectangular plate due to hydrostatic pressure can be obtained using tables given in the book of Vaynberg (1970) in the form of

$$M_{\max} = k_1 p a^2 \qquad (2)$$

where k_1 is given by tabulated values depending on the ratio b/a.

In a plate stiffened by horizontal stiffeners the upper plate part is loaded by a distributed load of triangular shape, in which the maximum bending moment arises at point I in distance of $a/8$ from the center point O (Fig.2). This bending moment is given by k_1. Tabulated values of k_1 can be approximated in function of $x_0 = b/a$ in the following form

$$10^2 k_1 = \frac{c_1}{1 + d_1 \exp(-f_1 x_0)} \; ; \quad c_1 = 6.2877, \quad d_1 = 12.2974, \quad f_1 = 2.1664 \qquad (3)$$

The maximum bending moment in the plate parts loaded by a distributed load of trapezoidal shape, given by p_1 and p_2, can be calculated adding two moments as follows: one moment is due to uniform loading p_1 and the second is due to triangular loading p_2 - p_1. Calculation shows that the resulting moment is larger for point O than that for point I, thus we use tabulated values for point O.

$$M = k_2 p_1 a^2 + k_3 (p_2 - p_1) a^2 \tag{4}$$

where k_2 and k_3 can be approximated by functions as follows

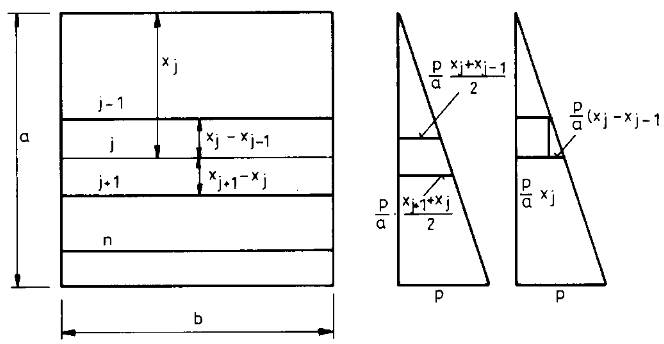

Figure 3. The distances of stiffeners and the pressures for the calculation of bending moments acting on stiffeners

$$10^2 k_2 = \frac{c_2}{1 + d_2 \exp(-f_2 x_0)} \; ; \quad c_2 = 12.3044, \quad d_2 = 13.7725, \quad f_2 = 2.1695 \tag{5}$$

$$10^2 k_3 = \frac{c_3}{1 + d_3 \exp(-f_3 x_0)} \; ; \quad c_3 = 6.1496, \quad d_3 = 13.8096, \quad f_3 = 2.1709 \tag{6}$$

In the case of n stiffeners the position of the jth stiffener is characterized by x_j (j = 1,...,n) (Fig.3) and the maximum bending moments in the base plate parts can be given as follows:

$$M_1 = k_1 p x_1^3 / a; \quad \text{in } k_1 \quad x_0 = b/x_1 \tag{7}$$

$$M_j = k_2 p x_{j-1} (x_j - x_{j-1})^2 / a + k_3 p (x_j - x_{j-1})^3 / a; \; \text{in } k_2 \text{ and } k_3 \quad x_0 = b/(x_j - x_{j-1}) \tag{8}$$

$$M_{n+1} = k_2 p x_n (a - x_n)^2 / a + k_3 p (a - x_n)^3 / a; \quad \text{in } k_2 \text{ and } k_3 \quad x_0 = b/(a - x_n) \tag{9}$$

Using these formulae n equations can be written expressing that these bending moments should be equal to each other, since the thickness of the base plate parts is the same.

The set of equations for unknowns x_j is the following:

$$k_1x_1^3 = k_2x_1(x_2 - x_1)^2 + k_3(x_2 - x_1)^3 \qquad (10)$$

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$k_1x_1^3 = k_2x_{j-1}(x_j - x_{j-1})^2 + k_3(x_j - x_{j-1)})^3$; in k_2 and k_3 $\quad x_0 = b/(x_j - x_{j-1})$ (11)

.

$$k_1x_1^3 = k_2x_n(a - x_n)^2 + k_3(a - x_n)^3 \qquad (12)$$

This set of equations is solved by MathCAD software.

3 Design of stiffeners

Having obtained the optimal stiffener positions, the required dimensions of trapezoidal stiffeners can be calculated. The maximum bending moment in the *j*th stiffener is given by (Fig.3)

$$M_{Sj} = \frac{pb^2}{64a}\left(x_{j+1} - x_{j-1}\right)\left(x_{j+1} + 2x_j + x_{j-1}\right) \qquad (13)$$

Considering trapezoidal stiffeners according to Stahlbau Handbuch (1985) with given dimensions a_1 = 90, a_3 = 300 mm, applying the local buckling constraint for the inclined webs according to Eurocode 3

$$a_2 \le 38t_S\varepsilon; \varepsilon = \sqrt{235/f_y} \qquad (14)$$

Taking equation (14) as equality, the section characteristics of a stiffener can be expressed by the unknown thickness t_S. The cross-sectional area is (Fig.4)

$$A = (90 + 76t_S\varepsilon)t_S \qquad (15)$$

the height of the stiffener and the angle of the web is

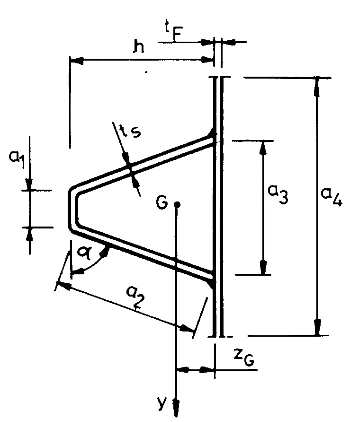

Figure 4. Dimensions of a trapezoidal stiffener

$$h = \left[(38t_S\varepsilon)^2 - 105^2\right]^{1/2}; \qquad \sin^2\alpha = 1 - \left(\frac{105}{38t_S\varepsilon}\right)^2 \qquad (16)$$

The distance of the gravity center G is

$$z_G = ht_S \frac{a_1 + a_2}{a_4 t_F + (a_1 + 2a_2)t_S} \tag{17}$$

and the moment of inertia is

$$I_y = a_4 t_F z_G^2 + a_1 t_S (h - z_G)^2 + \frac{a_2^3 t_S \sin^2 \alpha}{6} + 2a_2 t_S \left(\frac{h}{2} - z_G\right)^2 \tag{18}$$

The stress constraint for a stiffener is expressed by

$$M(h - z_G)/I_y \le f_y / \gamma_{M1}; \gamma_{M1} = 1.1 \tag{19}$$

from which the required stiffener thickness t_S can be calculated.

4 The cost function

We have developed on the basis of COSTCOMP (1990) software (Bodt 1990) a cost function containing the material and fabrication cost (Farkas-Jármai 1997, Jármai-Farkas 1999)

$$\frac{K}{k_m} = \rho V + \frac{k_f}{k_m}\left[C_1 \Theta_d (\kappa \rho V)^{0.5} + 1.3T_2\right] \tag{20}$$

where ρ is the material density, k_f and k_m are the fabrication and material cost factors, respectively, κ is the number of structural elements to be assembled, V is the volume of the structure, Θ_d is the difficulty factor expressing the complexity of a structure (planar or spatial, using simple plates or profiles), the coefficient for the preparation time is C_1 = 1.0 min/kg$^{0.5}$. To give internationally usable results, the ratio of k_f/k_m is varied in a wide range. For steel k_m = 0.5-1.2 kg, for fabrication including overheads k_f= 0-60\$/manhour = 0-1\$/min, thus, the ratio may vary in the range of 0 - 2 kg/min, the value of 0 corresponds to the minimum weight design. The welding time is given by

$$T_2 = \sum_i C_{2i} a_{Wi}^n L_{Wi} \tag{21}$$

the factor of 1.3 expresses that the additional time for chipping, deslagging and changing the electrode is approximated by $T_3 = 0.3T_2$. The formulae for $C_2 a_W^n$ are given for various welding technologies and weld types. a_W is the weld size, L_W is the weld length.

5 Numerical example

To illustrate the optimization procedure the following data are taken: $a = b = 6$ m, the intensity of the factored hydrostatic load is $p = \gamma p_0 = 1.5x0.036 = 0.054$ N/mm^2, f_y = 235 MPa. It is assumed that the base plate is butt-welded from plate strips of width

1500 mm. A stiffener is welded to the base plate with 2 fillet welds of size $a_W = 0.7t_S$. The welding times are calculated with the following data: use GMAW-M welding technology (Gas Metal Arc Welding with mixgas). For (21) the following formulae are used:

a_W = 10-40 mm X butt welds $C_2 a_W^n = 0.1433 a_W^{1.9035}$
a_W= 4-15 mm V butt welds $C_2 a_W^n = 0.1861 a_W^2$
a_W= 0-15 mm fillet welds $C_2 a_W^n = 0.3258 a_W^2$
and L_W is calculated in mm. The difficulty factor is chosen for $n = 0$ (base plate without stiffeners) $\Theta_d = 2$, and for $n>0$ $\Theta_d = 3$.

The results of computations are summarized in Tables 1, 2 and 3 and Fig. 5.

Table 1. Results of the numerical example. Optimum stiffener positions, thicknesses and costs for different numbers of stiffeners

n	x_j (m)	t_S (mm)	t_F (mm)	K/k_m(kg) for k_f/k_m=0	K/k_m(kg) for k_f/k_m=1	K/k_m(kg) for k_f/k_m=2
0	0	0	36	10174	12943	15712
1	3.6754	12	23	7066	9299	11532
2	2.8747	10	18	5967	7907	9847
	4.5758	11				
	2.3261	9				
3	3.7494	10	14	5085	7204	9323
	4.9397	10				
	1.9868	8				
4	3.2173	9	11	4428	6332	8236
	4.2447	9				
	5.1599	10				

Table 2. Results of the numerical example (continuation of Table 1.)

n	x_j (m)	t_S (mm)	t_F (mm)	K/k_m(kg) for k_f/k_m=0	K/k_m(kg) for k_f/k_m=1	K/k_m(kg) for k_f/k_m=2
	1.7519	7				
	2.8418	8				
5	3.7514	9	9	3996	5804	7612
	4.5616	9				
	5.3053	9				
	1.5773	7				
	2.5603	8				
6	3.3806	8	8	3911	5786	7661
	4.1111	8				
	4.7816	9				
	5.4080	9				

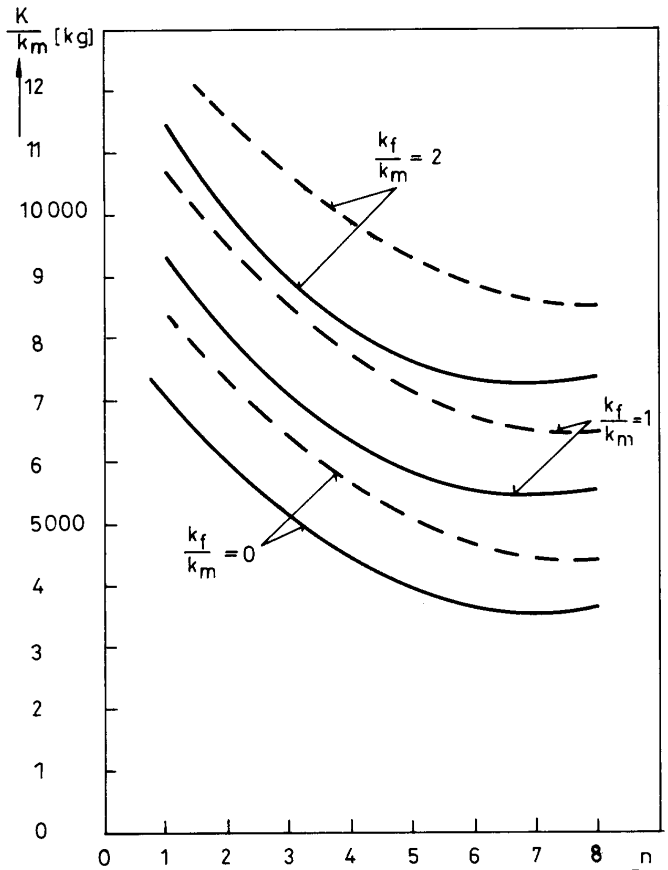

Figure 5. Costs in the function of number of stiffeners in the case of k_f/k_m=0,1 and 2

Table 3. Results of the numerical example(continuation of Table 2.)

n	x_j (m)	t_S (mm)	t_F (mm)	K/k_m(kg) for k_f/k_m=0	K/k_m(kg) for k_f/k_m=1	K/k_m(kg) for k_f/k_m=2
	1.4410	7				
	2.3398	7				
	3.0898	8				
7	3.7575	8	7	3705	5566	7427
	4.3705	8				
	4.9431	8				
	5.4843	8				
	1.3301	6				
	2.1615	7				
	2.8544	7				
8	3.4714	8	6	3446	5344	7242
	4.0377	8				
	4.5668	8				
	5.0668	8				

5.5432	8

It should be noted that, in the case of 8 stiffeners equidistantly arranged, the K/k_m -values for k_f/k_m = 0, 1 and 2 are as follows: 4349, 6472 and 8595, respectively. It means that the non-equidistant arrangement results in savings of 26, 19 and 21%, respectively.

6 Comparison with vertical stiffeners

For another comparison, the vertical arrangement of stiffeners is designed as well (Fig.6). According to Vaynberg (1970), the maximum bending moment arises in the point P. When the number of stiffeners is greater than 5, the base plate thickness can be calculated as a simply supported strip. Thus, from the stress constraint

$$\frac{0.75p[b/(n+1)]^2}{8t_F^2/6} \le \frac{f_y}{\gamma_{M1}} \tag{22}$$

the required base plate thickness is

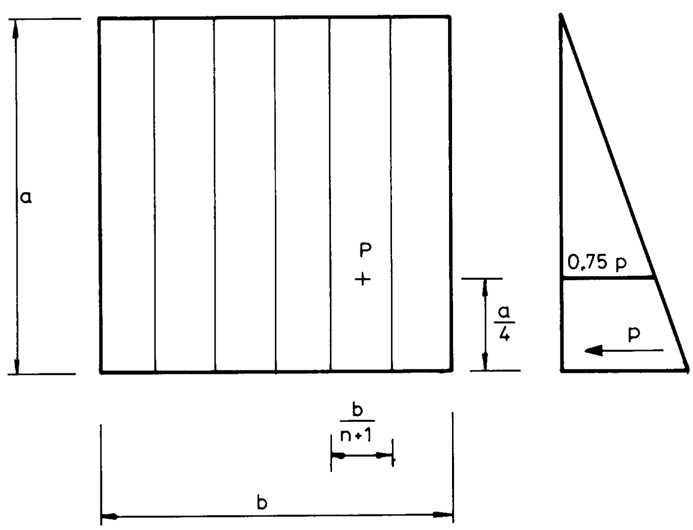

Figure 6. Solution with vertical stiffeners

$$t_F \ge \left(\frac{p}{f_y/\gamma_{M1}}\right)^{1/2} \frac{0.75b}{n+1} \tag{23}$$

In the case of our numerical example, for n = 5 and 8 t_F = 12 and 8 mm, respectively.

The maximum bending moment in a simply supported vertical stiffener is

$$M_{\max} = 0.06415pa^2 \frac{b}{n+1} \tag{24}$$

For this moment the required stiffener thickness can be calculated in the same manner as in the case of horizontal stiffeners. For n=5 and 8 t_S=8 and 7 mm, respectively. The corresponding cost values, in the case of k_f/k_m=0, 1 and 2 kg/min, are as follows: for

n=5 4710, 6752 and 8794 kg, for n=8 3901, 5805 and 7709 kg. This means that, for optimal arranged horizontal stiffeners these values are 4-18% smaller.

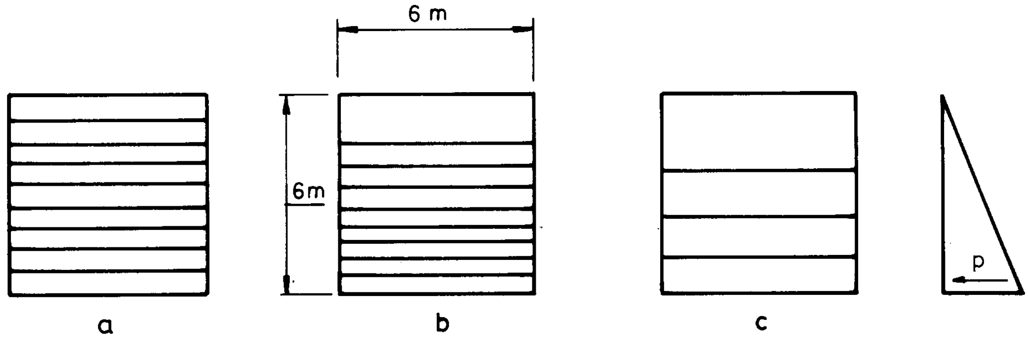

Figure 7. Values of K/k_m (kg) for a stiffened steel plate loaded by hydrostatic pressure for k_f/k_m = 1: (a) 6472 for 8 stiffeners in equidistant position, (b) 5344 for 8 stiffeners in optimized position, (c) 7204 for 3 stiffeners in optimized position

7 Conclusions

The optimum horizontal stiffener positions can be calculated on the basis of the condition that each base plate part, having the same thickness and loaded by bending, should be stressed to yield strength. This non-equidistant stiffener distribution is more economic than the equidistant or vertically arranged ones. This economy can be verified by cost calculations (Fig.7).

It is shown by a illustrative numerical example that the optimum number of stiffeners is 7 or 8 and the non-equidistant stiffener arrangement is 26% lighter and 19-21% cheaper than the equidistant one.

The difference between the costs of non-equidistant solutions with 3 or 8 stiffeners for k_f/k_m = 1 is (7204-5344)/5344x100 = 35%, thus, the search for the optimum number of stiffeners results in significant cost savings.

Acknowledgement

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