

CONSTITUTIVE EQUATIONS FOR ROCKS UNDER THREE-POINT BENDING LOAD

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Forty samples were tested for three-point bending load. A pre-crack was notched in the middle of all samples on the lower side in order to determine the starting point of the crack. This way the crack mouth opening displacement (dilatation) can be measured, beside the applied force and the deflection. The aim of this paper is to investigate the influence of the confining pressure on the tensile strength and deformations and to determine the constitutive relations of the three-point bending test with pre-crack.

1. Introduction

In the engineering practice the knowledge of the behaviour of rocks or rocklike materials under tension is very important for designing of underground constructions, solving geotechnical and rock engineering problems, etc. Usually the Brazilian test is used to determine the tensile strength but sometimes we need more information about development of failure.

The influence of the hydrostatic pressure for three-point bending test with pre-crack was investigated by Schmidt and Huddle (1977) and Abou-Sayed (1977). The aim of their research was to determine the influence of the confining pressure on fracture toughness in opening mode. They indicated that the apparent fracture toughness of Indiana limestone increases linearly with increasing confining pressure. The anisotropic rocks were examined by Afassi (1991) and Vásárhelyi (1997). They calculated the fracture toughness for different applied hydrostatic pressure and determined that there is a linear relation between them.

The aim of present paper is to investigate the fracture development of samples under three-point bending load. The results of the experiments indicate that tensile stress and crack mouth opening displacement are both functions of the confining pressure and deflection. Based on the interpretation of the experimental results (Vásárhelyi, 1995), a constitutive law for three-point bending

tests was developed. The constitutive equations allow consideration of the nonlinearity of the material behaviour. In our calculations we used the theory of Lechnitz (1981, 1985) who defined constitutive relations for shearing tests.

2. Specimen preparation, test equipment and experimental results

The material used in this research is an anisotropic gneiss. This metamorphic rock consists of small grains; during its formation the minerals deform in one direction as can be seen with the unaided eye. *Table 1* shows the physical properties of the rock relative to the axes shown in *Figure 1*.

Table 1. Mechanical properties of the measured gneiss

Physico-mechanical properties	Index	Unit	Result
Modulus of elasticity	E_1	GPa	41.69
	$E_2 = E_3$	GPa	70.05
Poisson's ratio	$\nu_{12} = \nu_{13}$	—	0.166
	$\nu_{21} = \nu_{31}$	—	0.262
	$\nu_{23} = \nu_{32}$	—	0.247
Uniaxial compressive strength	σ_1	MPa	238
	$\sigma_2 = \sigma_3$	MPa	267

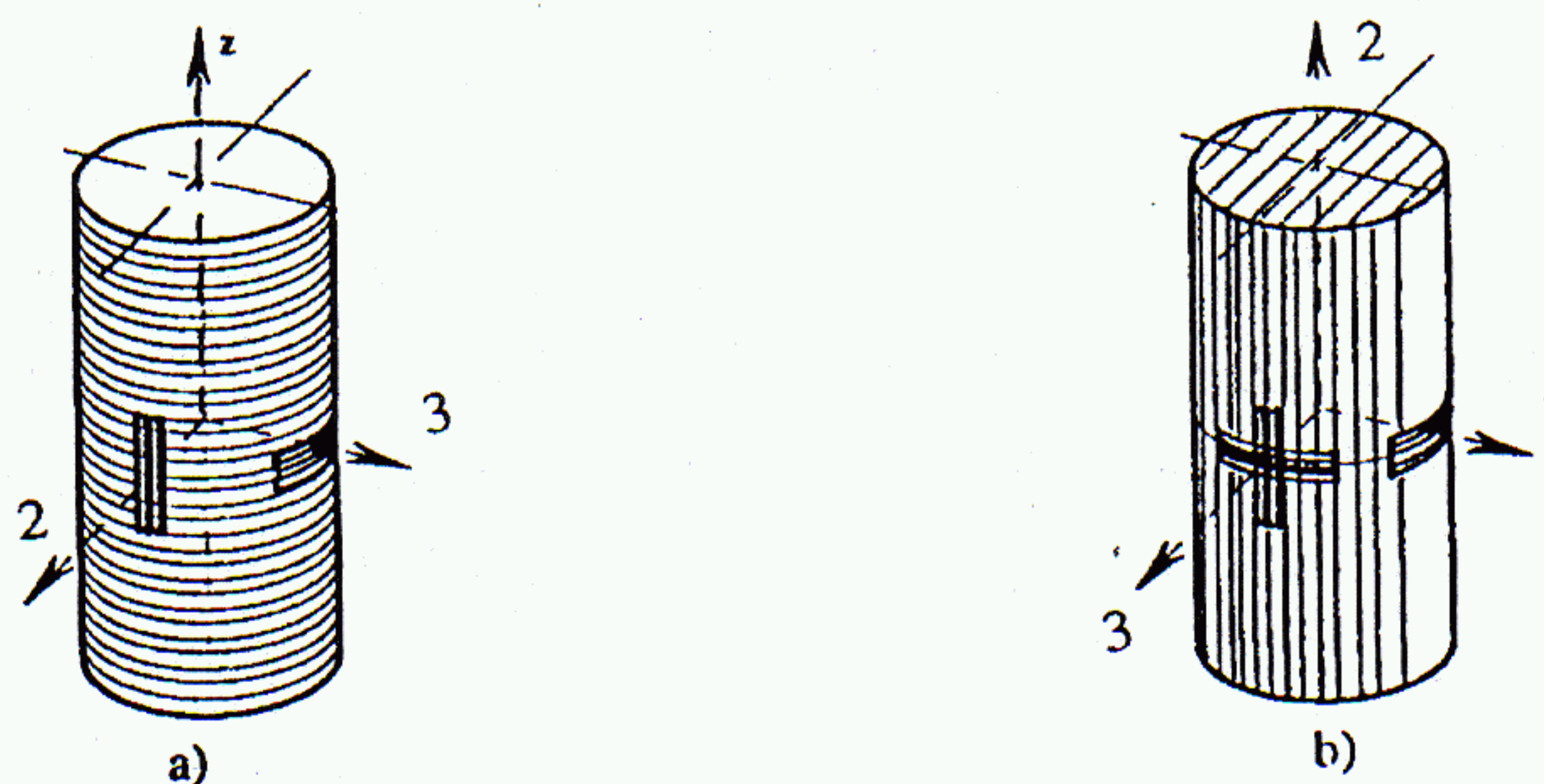


Figure 1. Orthotropic samples for uniaxial compression test with the axes

Extensive specimen preparation is necessary for performing confined three-point bend tensile tests (ASTM, 1972). This includes initial specimen machining, pre-cracking, final machining, instrumenting and jacketing. For the best result the specimens were cut from one block. The experimental measurements were performed at the rock mechanics laboratories of the Technical University of Lorraine in Nancy, France.

Forty notched three-point bending specimens of gneiss were dry-machined. All experiments were performed on 2 specimens. Each specimen was 25 mm (± 0.1) high (h) and 15 mm (± 0.1) wide (w) and the distance between the two fixed points (L) was always 100 mm. All tests were performed on specimens of equal size with nearly equal crack length a (5 mm long) in the middle of the bottom of the specimen. *Figure 2* shows the dimensions of the samples. The pre-crack was cut by electro-controlled cutting machine.

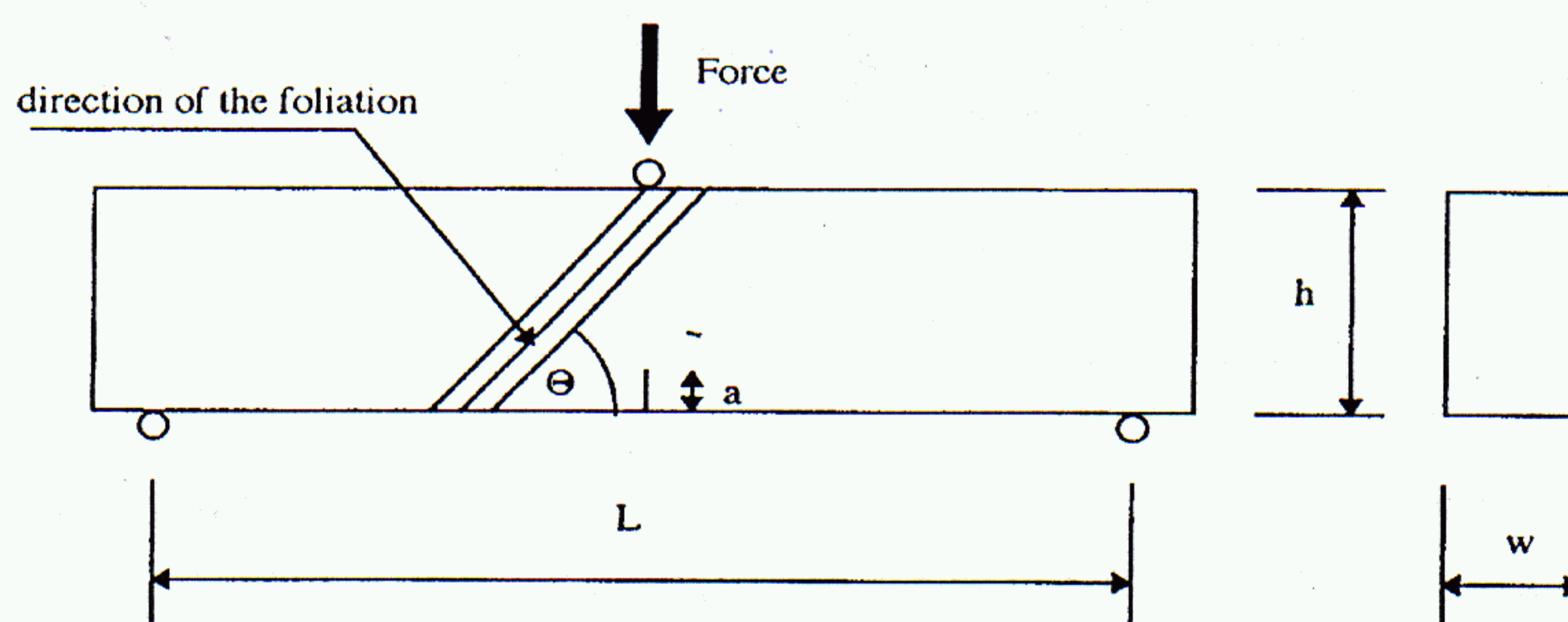


Figure 2. Dimensions of the samples

To perform tests under confining pressure, the specimens had to be jacketed with flexible material that would cover all machined surfaces including the crack, but not enter the crack. All specimens were air-dried and tested at room temperature.

A pressure vessel was specifically designed for confining pressure tests (*Figure 3*). The test chamber is 15 cm in diameter and 25 cm high with a working pressure of 100 MPa. The confining pressures of 0, 10, 30 and 60 MPa were supplied and controlled by a servo-controlled intensifier. Light hydraulic oil was used as the pressure medium. The load was measured with a proving ring equipped with an extensometer. This instrument was located between the loading piston and the specimen. The load was applied by increasing the displacement at 0.3 $\mu\text{m/s}$ vertical deformation rate. Vertical displacement and the crack mouth opening displacement was measured with linear-variable differential transformers (LVDT). The sensitivity of the crack mouth opening displacement LVDT was 0.001 mm.

The measurements were elaborated in cases of anisotropy in the directions 0° , 30° , 45° , 60° , and 90° to the horizontal line. To perform tests under confining pressure, the specimens had to be jacketed with flexible material that would cover all machined surfaces including the notch, but not enter the crack.

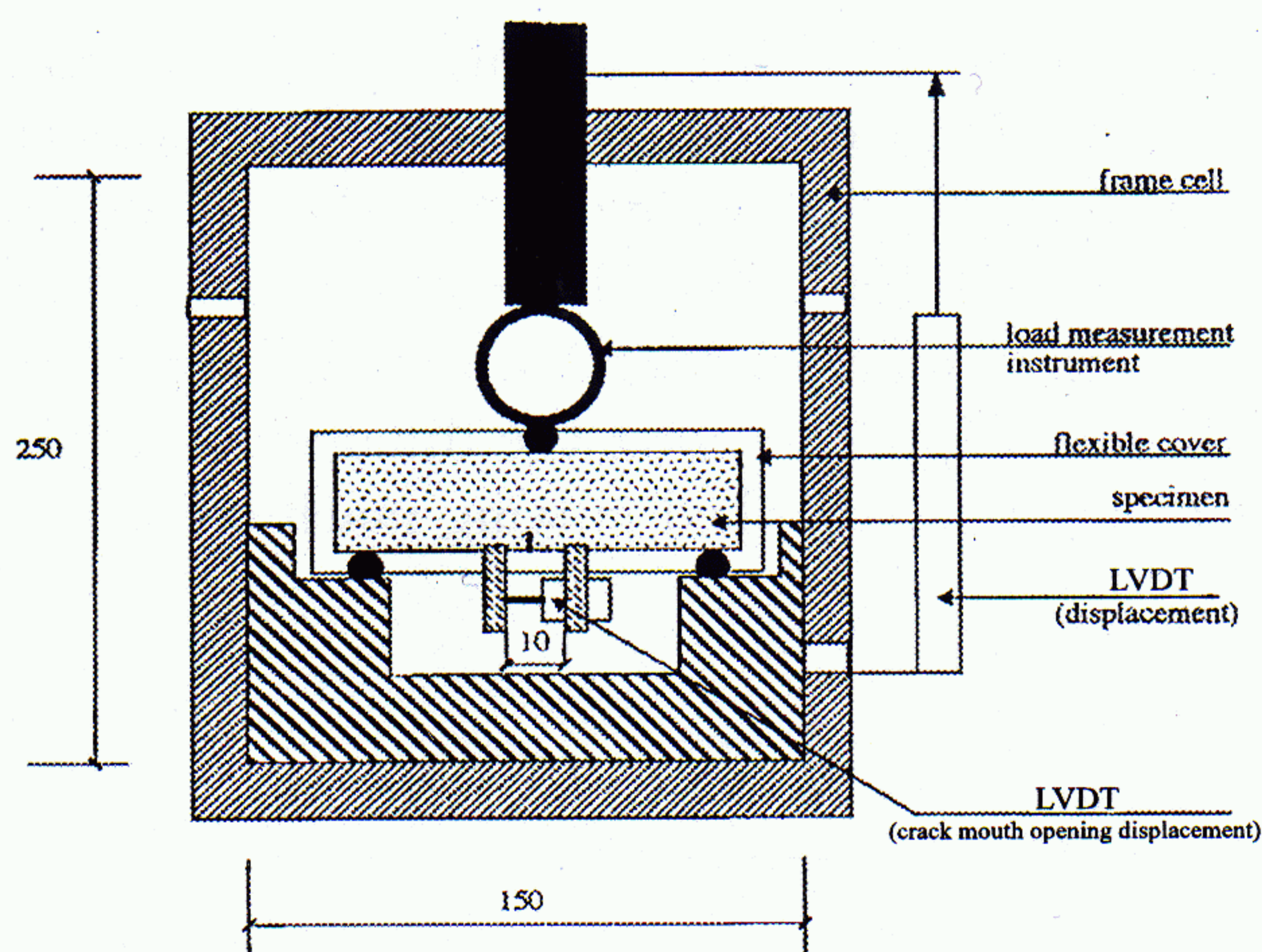


Figure 3. Schematic test configuration

The force-deflection and crack mouth opening displacement-deflection diagrams for tests under different confining pressure are shown in *Figures 4–7*. The test results (*Figure 8*) show that there is a linear relationship between the tensile strength and the confining pressure, and anisotropy has no considerable influence on it.

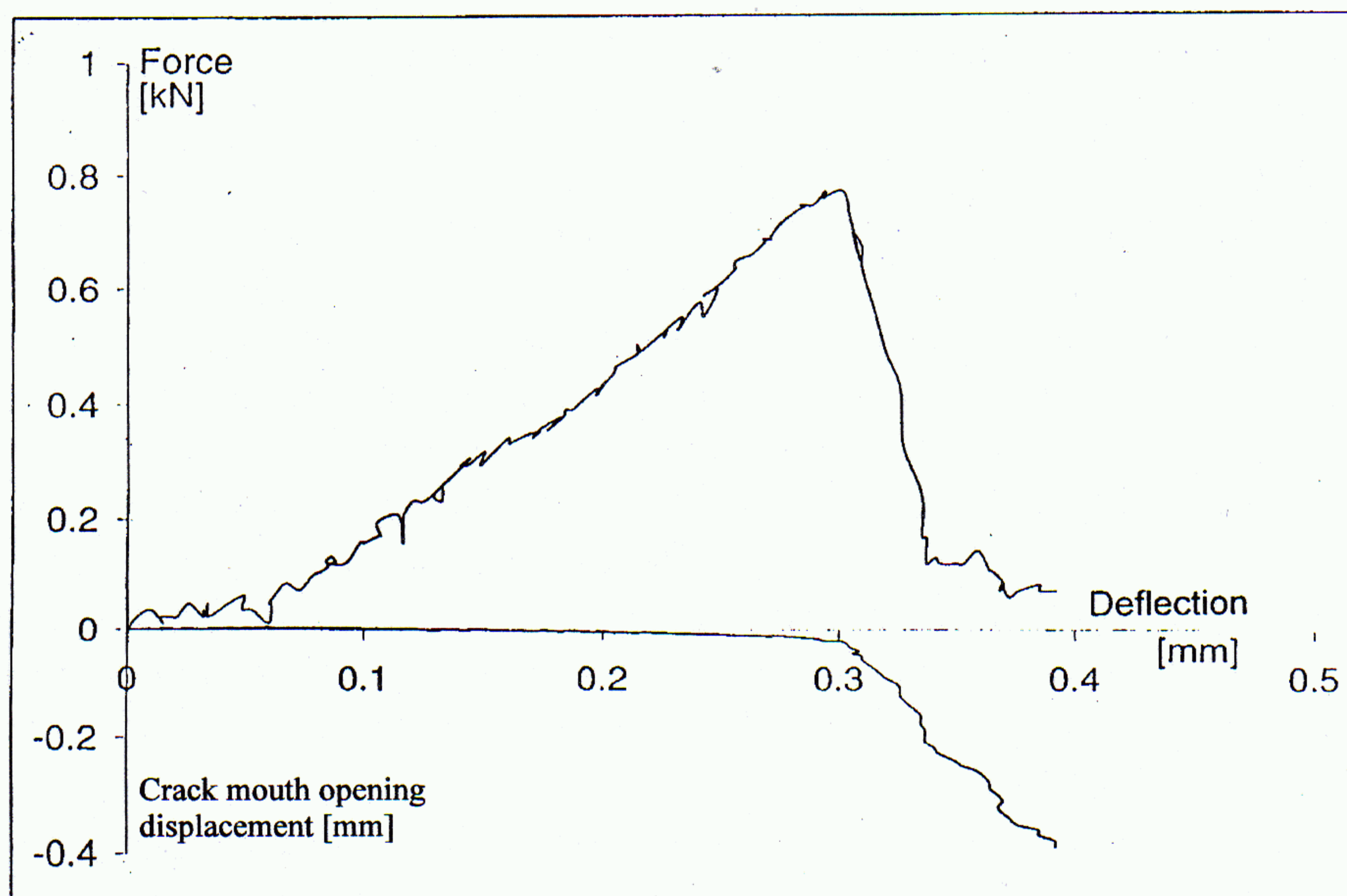


Figure 4. Force vs. deflection and crack mouth opening displacement vs. deflection relation at 0 MPa hydrostatic pressure

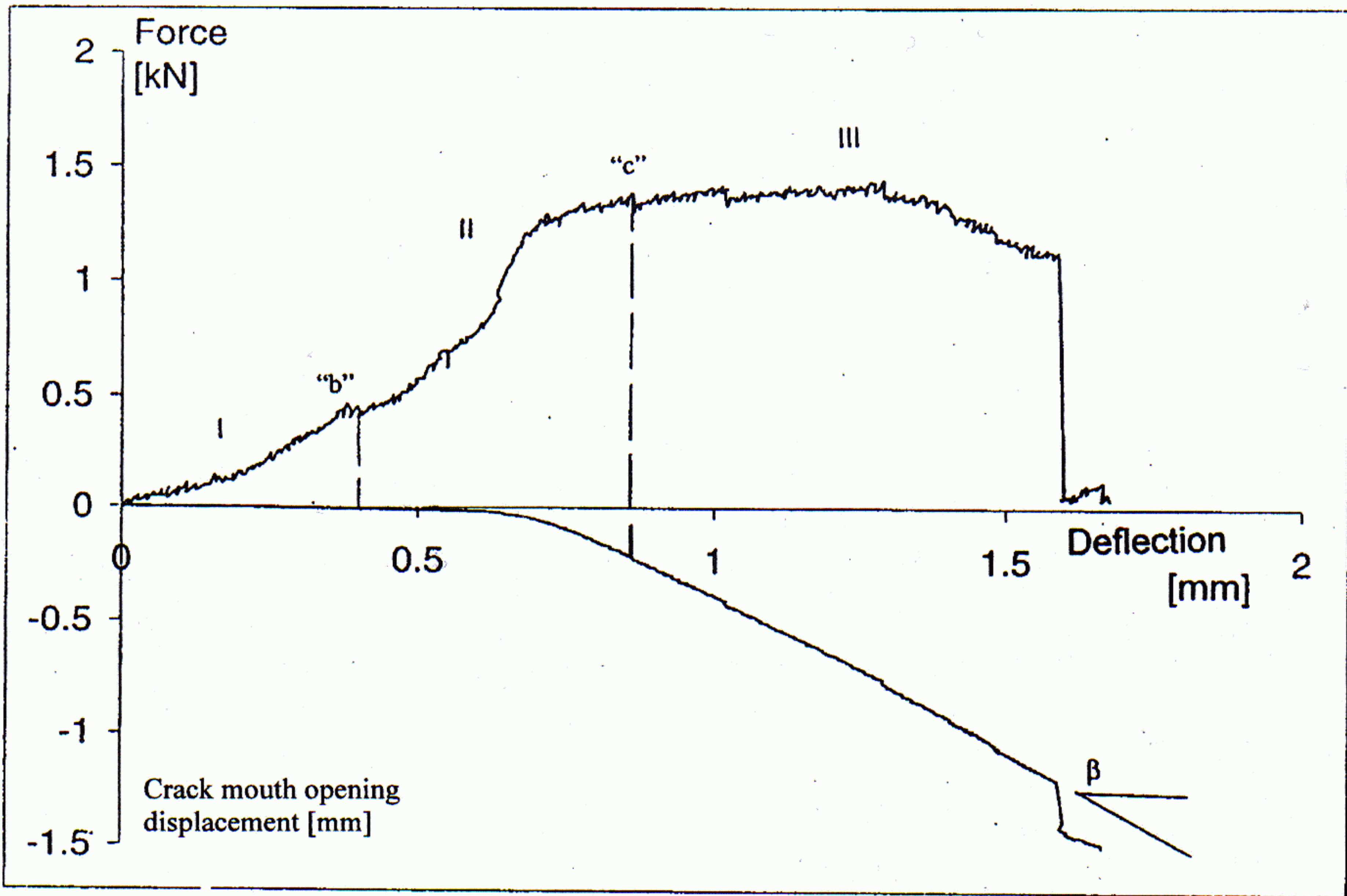


Figure 5. Force vs. deflection and crack mouth opening displacement vs. deflection relation at 10 MPa hydrostatic pressure

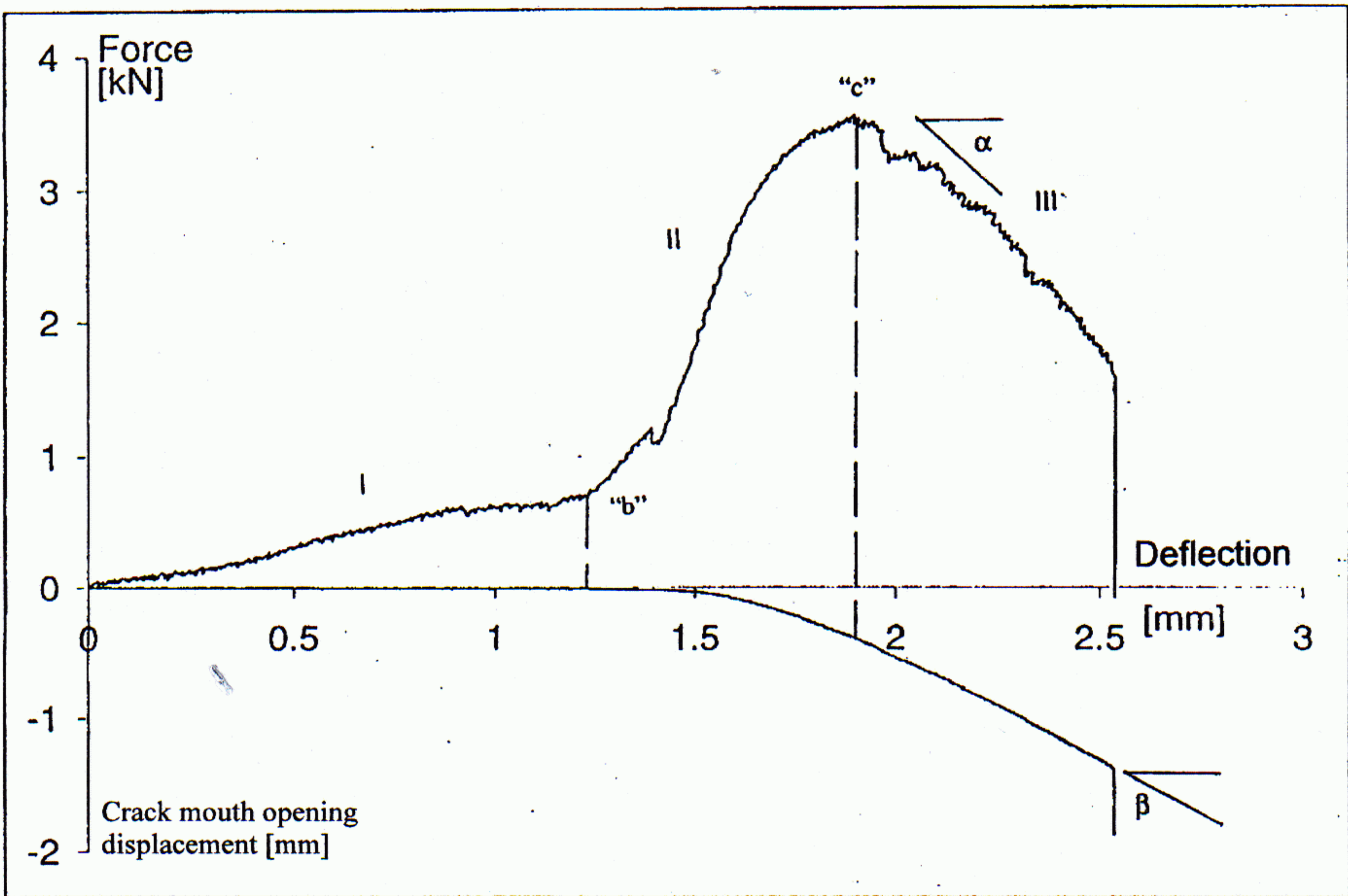


Figure 6. Force vs. deflection and crack mouth opening displacement vs. deflection relation at 30 MPa hydrostatic pressure

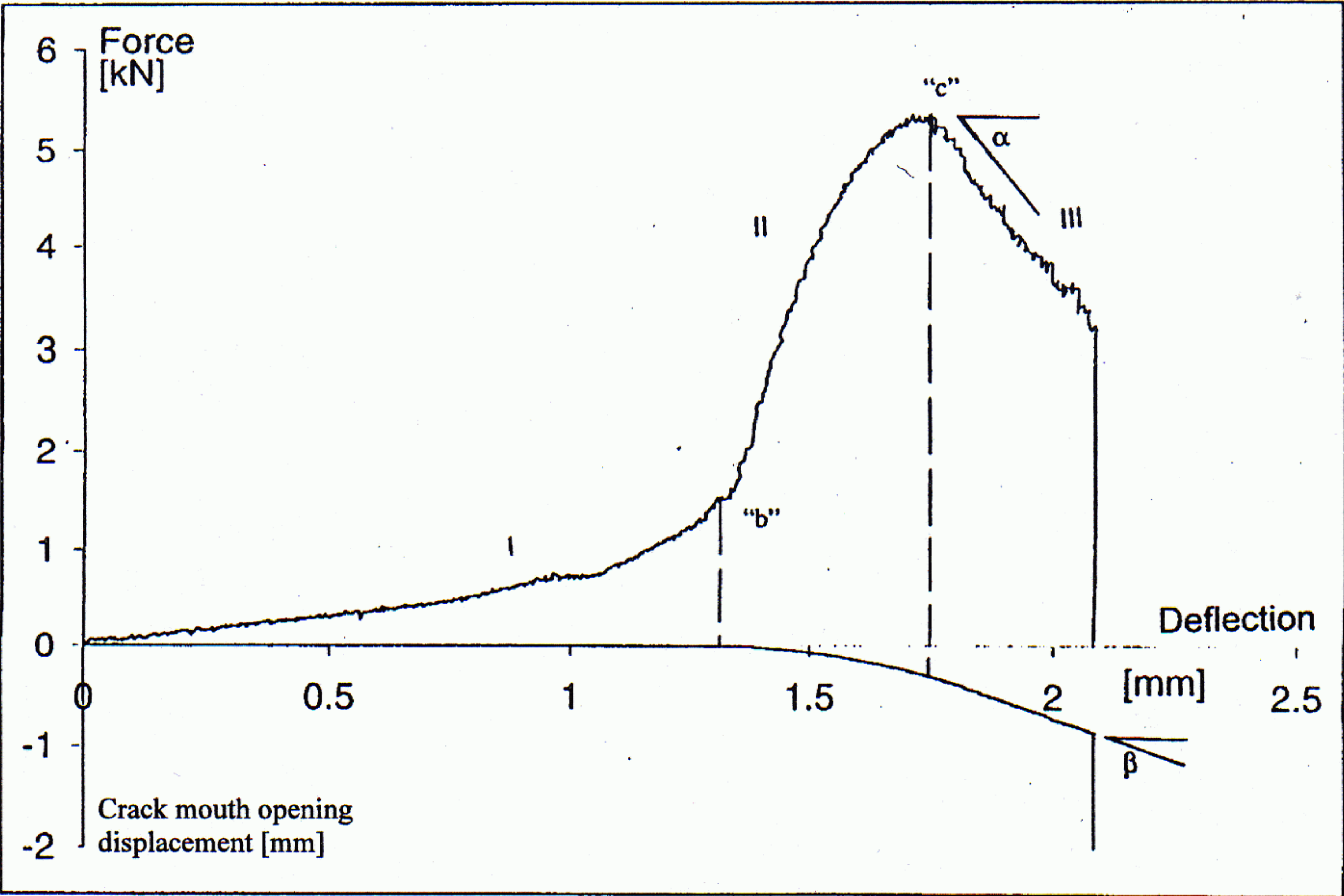


Figure 7. Force vs. deflection and crack mouth opening displacement vs. deflection relation at 60 MPa hydrostatic pressure

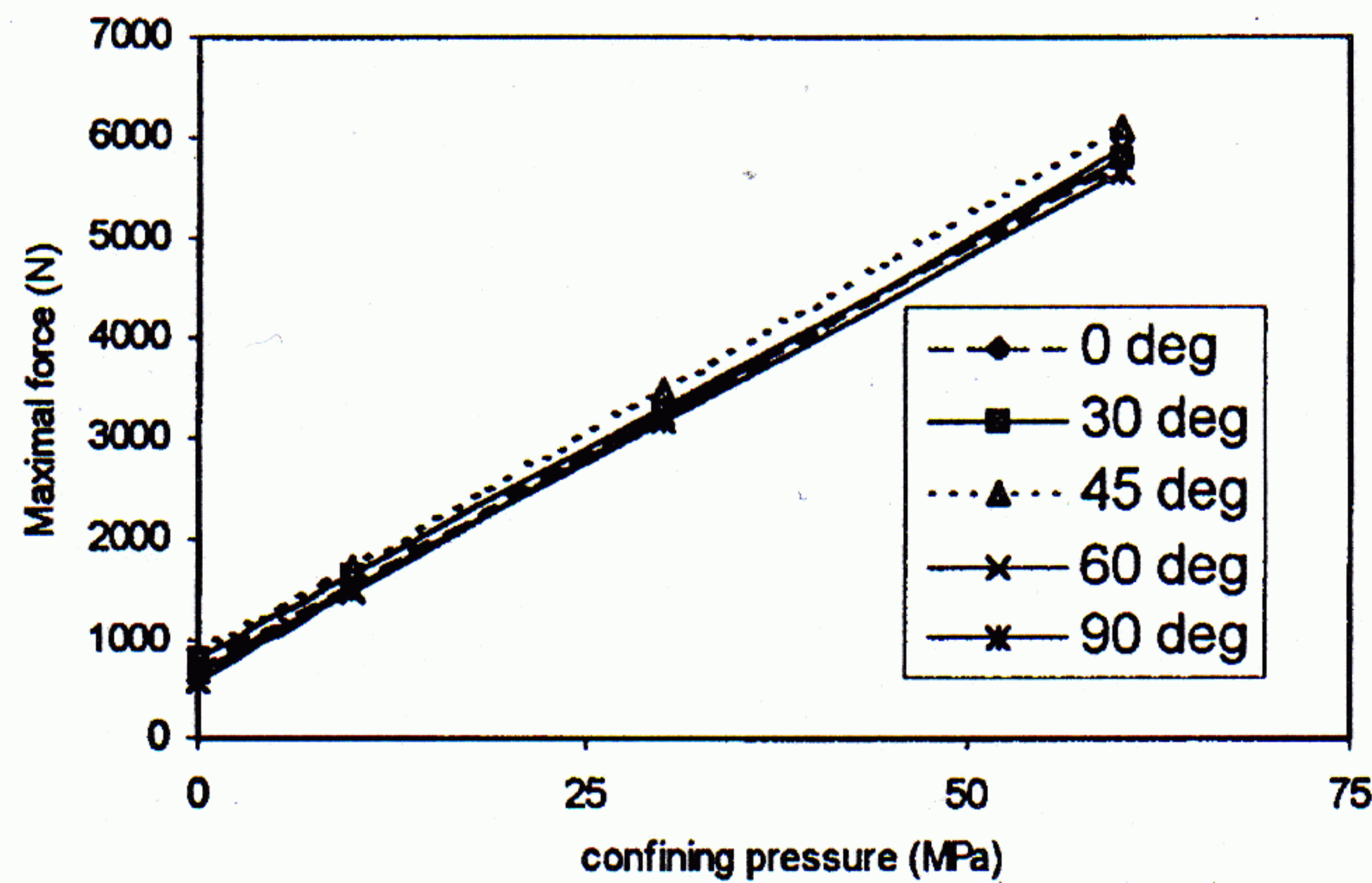


Figure 8. Force vs. confining pressure in case of different anisotropy directions

3. Constitutive relations

As the experiments have shown, tensile force at the crack-tip (F) and crack mouth opening displacement (h) are functions of the vertical displacement of the loaded point of the specimen (s) and the hydrostatic pressure (P). Suppose that the force (F) and the crack mouth opening displacement (h) depend only on the vertical displacement (s) and the hydrostatic pressure (P):

$$F = \hat{F}(s, P) \quad (1)$$

and

$$h = \hat{h}(s, P). \quad (2)$$

Equations (3) and (4) give the total differential of both functions:

$$dF = \frac{\partial \hat{F}}{\partial s} ds + \frac{\partial \hat{F}}{\partial P} dP, \quad (3)$$

$$dh = \frac{\partial \hat{h}}{\partial s} ds + \frac{\partial \hat{h}}{\partial P} dP. \quad (4)$$

The partial differentials of tensile force and hydrostatic pressure are abbreviated by μ^* , δ^* , ν^* and $1/\kappa^*$. These abbreviations are known as stiffness functions. *Figure 9* illustrates their physical meaning.

δ^* shows the change in tensile force with tensile deformation at constant hydrostatic pressure, i.e., $\delta^* = \frac{\partial \hat{F}}{\partial s}$;

μ^* shows the change in tensile force with hydrostatic pressure at constant tensile deformation; thus $\mu^* = \frac{\partial \hat{F}}{\partial P}$;

κ^* corresponds to the hydrostatic stiffness, $\frac{1}{\kappa^*} = \frac{\partial \hat{h}}{\partial P}$;

ν^* describes an angle of dilatation, i.e., $-\nu^* = \frac{\partial \hat{h}}{\partial s}$.

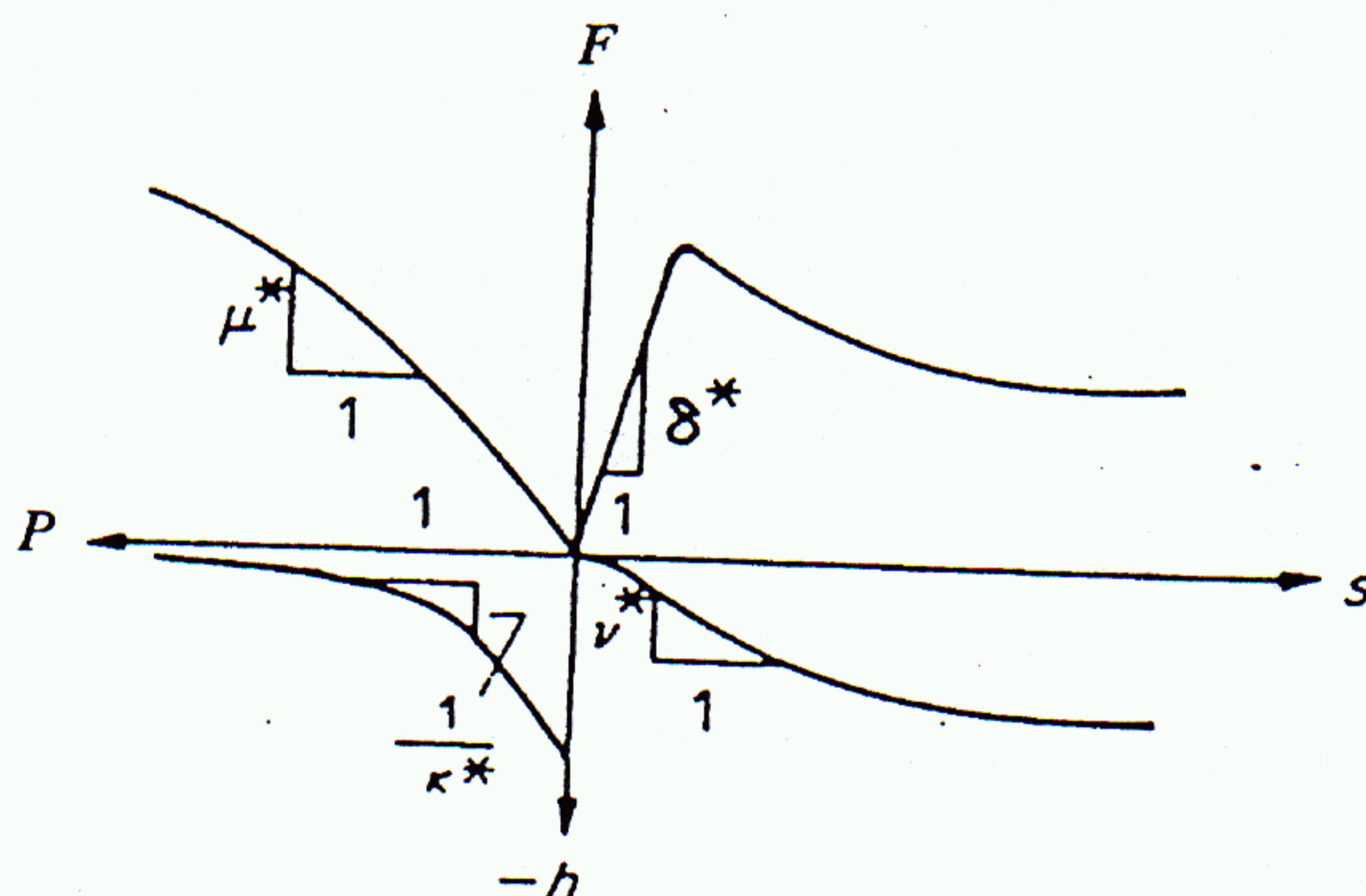


Figure 9. Physical meaning of the abbreviations

After substituting the above-mentioned abbreviations in Equations (3) and (4) and after rearrangement, it is possible to express force increments dF and dP in terms of deflection and crack mouth opening displacement increments.

Thus, the constitutive law is formed as Equation (7).

$$dF = \delta^* ds + \mu^* dP, \quad (5)$$

$$-dh = \nu^* ds - 1/\kappa^* dP, \quad (6)$$

$$\begin{bmatrix} dF \\ dP \end{bmatrix} = \begin{bmatrix} \delta^* + \mu^* \kappa^* \nu^* & \mu^* \kappa^* \\ \nu^* \kappa^* & \kappa^* \end{bmatrix} \begin{bmatrix} ds \\ dh \end{bmatrix}, \quad (7)$$

where

$$\begin{aligned} \begin{bmatrix} dF \\ dP \end{bmatrix} &: \text{force vector;} \\ \begin{bmatrix} \delta^* + \mu^* \kappa^* \nu^* & \mu^* \kappa^* \\ \nu^* \kappa^* & \kappa^* \end{bmatrix} &: \text{stiffness matrix;} \\ \begin{bmatrix} ds \\ dh \end{bmatrix} &: \text{displacement vector.} \end{aligned}$$

4. Conclusions

Three-point bending tests were performed on anisotropic gneiss under four different confining pressure levels, in order to examine the influence of confining pressure on the force-deflection and crack opening displacement-deflection relation. The results show linear relationship between the maximal force and the confining pressure; anisotropy has no considerable effect on it.

Linear constitutive equations were determined to describe the behaviour of material under three-point bending load.

The experimental investigations have shown, however, that the behaviour of material is nonlinear. Especially with a restriction of crack mouth opening displacement, discrepancies are found between the actual and calculated behaviour of the material by using a linear scale. Therefore, attempts have to be made to find scales for the stiffness functions that sufficiently describe the behaviour of the material.

Acknowledgement

The authors are thankful for the help of both Mme F. Homand, professor of the INPL-Nancy and the French Government for supporting the re-

searches. Our acknowledgement is to the Hungarian Scientific Research Foundation (OTKA) for Grant No. F 022 620 and Mr. Thomas Cholnoky, as well.

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