Welded cellular shells can solve fabrication problems

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Abstract

Cellular shells consist of two concentric circular cylindrical shells and longitudinal stiffeners welded between them. It is advantageous to use for stiffeners halved CHS (circular hollow section) profiles, since they enable the easy welding of outer shell parts to the stiffeners. Cellular shells have more advantages over single shells. They produce large bending and torsional stiffness with small diameter and small thickness. In the case of a steel cantilever column structure with limited diameter, loaded by compression, bending and torsion, a single shell should have a large thickness unsuitable for fabrication. This problem can be solved by a cellular shell with thicknesses ensuring easy plastic forming and welding. The cellular shell satisfies the constraint on reduced stress, constraint on panel buckling of the parts of outer shell, constraint on reduced stress in stiffeners as well as constraint on overall buckling of the column.

Keywords: *cellular shells, welded structures, shell buckling, column buckling, tubular structures, circular hollow section profiles*

1. Introduction

Cellular shells consist of two concentric circular cylindrical shells and longitudinal stiffeners welded between them. It is advantageous to use for stiffeners halved CHS (circular hollow section) profiles, since they enable the easy welding of outer shell parts to the stiffeners (Fig.2).

It should be mentioned that other applications of cellular shells have been treated in our previous papers. In [1,2] a cantilever column is optimized for minimum cost, when the horizontal displacement of the column top and the diameter is limited.

2. Problem description

A cantilever column of height L is loaded by a compression (N) and a horizontal (H) force, bending moment caused by the horizontal force $M_b = HL$ and a torsional moment (M_t)) (Fig.1). A steel of yield stress f_y is used. The outer diameter R is limited.

Given numerical data: L = 18 m, R = 1000 mm, f_y = 355 MPa, factored loads N = 2.0x107 [N], H = 0.1N, $M_b = HL$ = 3.6×10^{10} Nmm, M_t = 3.2×10^{10} Nmm, the elastic modulus E = 2.1×10^{5} MPa, Poisson ratio ν = 0.3.

3. Solution with a single circular cylindrical shell

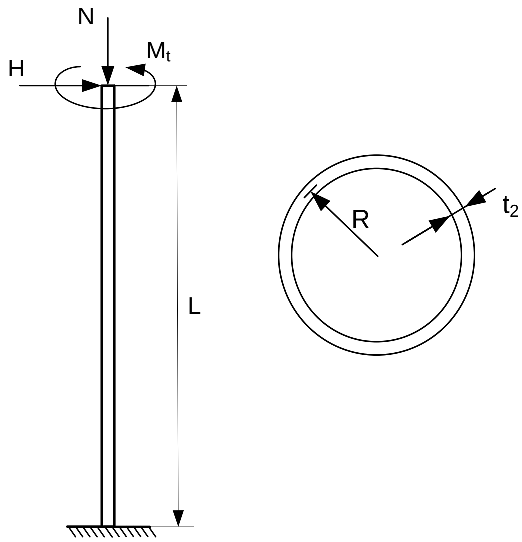

Figure 1. A cantilever column. The cross-section of a single circular cylindrical shell

Thickness t_2 = 49 mm, the cross-sectional area and section modulus (Fig.1)

$$A = 2\pi R t_2, W = R^2 \pi t_2 \quad (1)$$

stresses in MPa

$$\sigma_a = \frac{N}{A} = 64.96,\ \sigma_b = \frac{M_b}{W} = 233.86,\ \tau = \frac{M_t}{2R^2\pi t_2} = 103.938 \quad (2)$$

The reduced stress

$$\sigma_{red} = \sqrt{(\sigma_a + \sigma_b)^2 + 3\tau^2} = 348.86 < f_y, \text{ fulfilled.} \quad (3)$$

It can be seen that a single shell should have a thickness of 49 mm, unsuitable for fabrication.

4. Solution with cellular shell

Dimensions and geometry of a cellular shell are given in Figures 2.

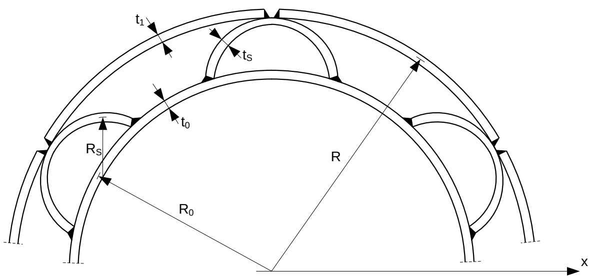

Figure 2. Dimensions of a cellular shell

Thicknesses $t_1 = t_0 = 30$ mm, CHS stiffeners $D_s = 219.1$, $t_s = 20$, $A_s/2 = 12500/2$, $R_s = 99{,}55$ mm, number of longitudinal stiffeners $n_s = 6$.
Inner radius

$$R_0 = \sqrt{\left(R - \frac{2R_s}{\pi} - R_s - \frac{t_s}{2} - \frac{t_1}{2}\right)^2 + R_s^2} - \frac{t_0}{2} = 803.154 \text{ mm} \quad (4)$$

The cross-sectional area

$$A = 2\pi(Rt_1 + R_0t_0) + \frac{n_sA_s}{2} = 3.586x10^5 \text{ mm}^2 \quad (5)$$

Moment of inertia of a halved CHS

$$I_{xs} = \frac{R_s^3\pi t_s}{2}\left(1 - \frac{8}{\pi^2}\right) = 5.871x10^6 \quad \text{mm}^4 \quad (6)$$

Moment of inertia of the stiffeners

$$I_{xs0} = (I_{xs} + A_s s^2)\sum_{i=1}^{n_s}\left[sin\left(\frac{2\pi i}{n_s}\right)\right]^2 = 1.39x10^{10} \text{ mm}^4 \quad (7)$$

Moment of inertia of the whole cross-section of the cellular shell

$$I_x = I_{xs0} + \pi(R^3t_1 + R_0^3t_0) = 1.57x10^{11} \text{ mm}^4 \quad (8)$$

The cellular shell should fulfil the following constraints: constraint on reduced stress, on panel buckling of the parts of outer shell, on reduced stress in stiffeners as well as constraint on overall buckling of the cantilever column.

4.1 Constraint on reduced stress

In order to calculate the shear stress from torsional moment, a reduced thickness is defined as

$$t_{red} = \frac{A}{2\pi(R+R_0)/2} \quad (9)$$

The shear stress

$$\tau = \frac{M_t}{2\pi\left(\frac{R+R_0}{2}\right)^2 t_{red}} = 98.97 \text{ MPa} \quad (10)$$

The normal stresses due compression and bending

$$\sigma_a = \frac{N}{A} = 55.77, \ \sigma_b = \frac{M_b}{I_x/R} = 229.34 \text{ MPa} \quad (11)$$

The reduced stress

$$\sigma_{red} = \sqrt{(\sigma_a + \sigma_b)^2 + 3\tau^2} = 332.67 < f_y \text{, fulfilled.} \quad (12)$$

4.2 Constraint on panel buckling of the outer shell parts

This constraint is formulated according to design rules of the Det Norske Veritas [3]

$$\sigma_{red} \le \frac{f_y}{\sqrt{1+\lambda_p^4}} \quad (13)$$

$$\lambda_p = \sqrt{\frac{f_y}{\sigma_{red}}\left(\frac{\sigma_a+\sigma_b}{f_E} + \frac{\tau}{f_{Et}}\right)} \quad (14)$$

The Euler buckling stress of compression and bending

$$f_E = C\pi^2\frac{E}{12(1-\nu^2)}\left(\frac{t_1}{s_0}\right)^2, \ s_0 = \frac{2R}{n_s}, \ C = \psi\sqrt{1 + \left(\frac{\rho_0\xi}{\psi}\right)^2} \quad (15)$$

$$\psi = 4, \ \xi = 0.702\,Z, \ Z = s_0^2\frac{\sqrt{1-\nu^2}}{Rt_1}, \ \rho_0 = 0.5\left(1 + \frac{R}{150t_1}\right)^{-0.5} \quad (16)$$

The Euler buckling stress for torsion

$$f_{Et} = C_t\pi^2\frac{E}{12(1-\nu^2)}\left(\frac{t_1}{s_0}\right)^2, \ C_t = \psi_t\sqrt{1 + \left(\frac{\rho_{0t}\xi_t}{\psi_t}\right)^2} \quad (17)$$

$$\psi_t = 5.34 + 4\left(\frac{s_0}{L}\right)^2, \quad \xi_t = 0.856Z_t^{0.75}\sqrt{\frac{s_0}{L}}, \quad Z_t = Z, \ \rho_{0t} = 0.6 \quad (18)$$

The constraint is fulfilled, since

$\sigma_{red} = 344.20 < 354.53$ MPa.

4.3 Stress constraint for stiffeners

In addition to the compression, bending and shear stresses the stiffeners are loaded by local bending due to difference of torsional forces acting on the outer and inner shells. This force difference can be calculated as

$$F = 2\pi\tau\frac{Rt_1 - R_0 t_0}{n_s} \tag{19}$$

This force acts on a stiffener, which is a shell of the halved circular arch of thickness t_s and length L. In order to determine the local bending moments on this arch, displacement equations should be solved for two quarter arches (Figure 3-5).

Pairs of the vertical force V, horizontal forces F-N as well as N, and bending moment M are acting on the two quarter arches. They produce angle deformations

$$\varphi_V = \frac{VR_S{}^2}{EI}\ ,\ \varphi_{F-N} = \frac{(F-N)R_S{}^2}{EI}\left(\frac{\pi}{4}-1\right)\ ,\ \varphi_M = \frac{\pi MR_S}{4EI} \tag{20}$$

vertical deformations

$$y_V = \frac{VR_S^3}{EI}\ , y_{F-N} = \frac{(F-N)R_S^3}{EI}\left(\frac{3\pi}{8}-2\right), y_M = \frac{MR_S^2}{EI}\left(\frac{\pi}{4}-1\right) \tag{21}$$

and horizontal deformations

$$u_V = \frac{\pi VR_S^3}{8EI}\ ,\ u_{F-N} = \frac{(F-N)R_S^3}{2EI}\ ,\ u_M = \frac{MR_S^2}{EI} \tag{22}$$

The equations of deformations express the fact that the deformations of the left quarter shell equal to the ones of the right quarter shell.
Equation for φ:

$$\frac{VR_S{}^2}{EI} - \frac{(F-N)R_S{}^2}{EI}\left(\frac{\pi}{4}-1\right) + \frac{\pi MR_S}{4EI} = \frac{VR_S{}^2}{EI} - \frac{NR_S{}^2}{EI}\left(\frac{\pi}{4}-1\right) - \frac{\pi MR_S}{4EI} \tag{23}$$

Equation for y:

$$\frac{VR_S^3}{EI} - \frac{(F-N)R_S^3}{EI}\left(\frac{3\pi}{8}-2\right) + \frac{MR_S^2}{EI}\left(\frac{\pi}{4}-1\right) = -\frac{VR_S^3}{EI} + \frac{NR_S^3}{EI}\left(\frac{3\pi}{8}-2\right) - \frac{MR_S^2}{EI}\left(\frac{\pi}{4}-1\right) \tag{24}$$

Equation for u:

$$-\frac{\pi VR_S^3}{8EI} + \frac{(F-N)R_S^3}{2EI} - \frac{MR_S^2}{EI} = -\frac{\pi VR_S^3}{8EI} + \frac{FN}{2EI} + \frac{MR_S^2}{EI} \tag{25}$$

The solution of the three deformation equations is

$$N = 0.5F,\ M = 0,\ V = F\left(\frac{3\pi}{16}-1\right) = -0.41096F \tag{26}$$

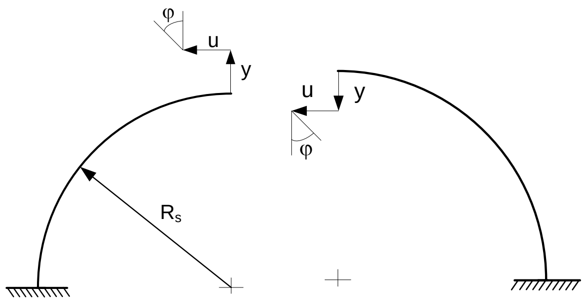

Figure 4. Displacements of the two half arches

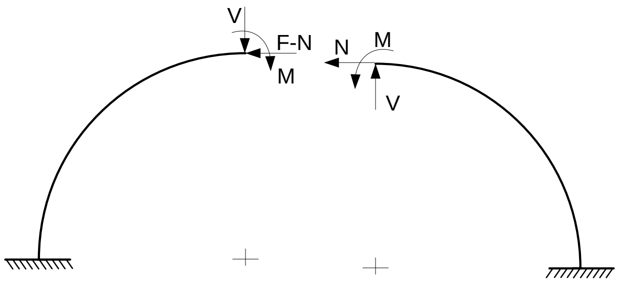

Figure 5. Forces and bending moments acting on the two half arches

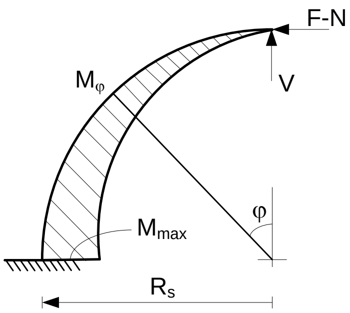

Figure 6. Bending moment diagram of the half stiffener

The negative sign means that the force V acts on the opposite direction.
The bending moment

$$M(\varphi) = 0.5FR_s(1-cos\varphi) + 0.41096FR_s sin\varphi \tag{27}$$

For the maximum bending moment one obtains

$$M_{max} = 0.91095FR_s \tag{28}$$

Stress from local bending of a stiffener is given by

$$\sigma_s = \frac{6FR_s 0.91095}{Lt_s^2} = 46.25 \text{ MPa} \quad (29)$$

This stress is perpendicular to the other normal stresses. The constraint on reduced stress for a stiffener

$$\sigma_{reds} = \sqrt{(\sigma_a + \sigma_b)^2 + \sigma_s^2 + (\sigma_a + \sigma_b)\sigma_s + 3\tau^2} = 354.96 < f_y \text{ , fulfilled.} \quad (30)$$

4.4 Constraint on overall buckling of the cantilever column

This constraint is formulated according to EN 1993-1-1 [4] for compression and bending with overall buckling

$$\eta = \frac{N}{\chi A f_y} + k_{yy} \frac{M_b}{W_x f_y} \le 1 \quad (31)$$

$$\chi = \frac{1}{\phi + \sqrt{\Phi^2 - \lambda^2}} \text{ , } \phi = 0.5[1 + 0.34(\lambda - 0.2) + \lambda^2], \ \lambda = \sqrt{\frac{f_y}{\sigma_E}}, \sigma_E = \frac{\pi^2 E I_x}{4L^2 A} \quad (32)$$

$$k_{yy} = C_{my}\left(1 + 0.6\lambda \frac{N}{\chi A f_y}\right) \text{ if } \lambda < 1 \quad (33)$$

$$k_{yy} = C_{my}\left(1 + 0.6 \frac{N}{\chi A f_y}\right) \text{ if } \lambda \ge 1 \text{ , } C_{my} = 0.9 \quad (34)$$

$$\eta = 0.785 < 1 \text{ , fulfilled.} \quad (35)$$

It can be seen that, in the case of the given numerical problem, the cellular shell is a much better solution than the single shell.

5. Comparison of the single and the cellular shell

Table 1 shows the characteristics of the two structural solutions. The outer diameter is limited to $D = 2R = 2$ m. Constraint on reduced stress $\sigma_{red} \le f_y = 355$ MPa, constraint on overall buckling $\eta \le 1$.

Table 1. Comparison of the single and the cellular shell characteristics.

Characteristics	Single shell	Cellular shell
thicknesses mm	49	30, 30, 20
Ax10^{-5} mm^2	3.079	3.586
W_xx10^{-8} mm^3	1.539	1.57
σ_a MPa	64.96	55.77
σ_b MPa	233.86	229.2
τ MPa	103.94	98.97
σ_s MPa	--	46.25
σ_{red} MPa	348.86	354.96
overall buckling η	0.875	0.785

It can be seen that, in the case of the given numerical problem, the cellular shell is a much better solution than the single shell.

Conclusions

A cantilever column is loaded by a compression force, a horizontal force (causes a bending moment) and a torsional moment. The column should be constructed with a circular cylindrical shell with a limited diameter. In the case of the given loads and the limited shell diameter a single shell can be designed with a thickness unsuitable for fabrication. In this case the fabrication problem can be solved with a welded cellular shell constructed from two circular shells and halved CHS stiffeners having thicknesses suitable for fabrication.

The designed cellular shell fulfils the constraints on reduced stress, on panel buckling of the outer shell parts, on reduced stress in stiffeners and on overall buckling of the cantilever column. To fulfil these constraints the cellular shell should have a little larger cross-sectional area and longitudinal fillet welds connecting the stiffeners to the inner shell and the outer shell parts to the stiffeners. The main advantage of the cellular shell are the smaller thicknesses, much more suitable for plastic forming and welding than is the very thick single shell.

The study shows the difference between thick unstiffened and thin stiffened structural versions, which is the main problem in the design of welded structures.

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