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Comparison of rectangular and square box columns composed from cellular plates with welded and rolled stiffeners

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1. Abstract
A cantilever column is loaded by a compression force and a bending moment caused by a horizontal force. It can be derived that, in the case of uniaxial bending, the rectangular cross section is more economic than the square one. In the given numerical case, the plate thicknesses are too large for enabling fabrication. Therefore stiffened plates should be used. Thus, the aim of the present study is to elaborate the minimum cost design of a column with rectangular cross-section and cellular plate walls. Cellular plates are constructed from two plates and longitudinal stiffeners welded between them. Previous studies have shown that welded T-stiffeners are more economic than the halved rolled I-section stiffeners, thus, welded T-stiffeners are used.
Stress and horizontal deformation constraints are formulated. In the stress constraint, face plate buckling is taken into account by using effective widths. Local buckling constraint is used for the web of T-stiffeners.
Variables are as follows: heights of welded T-sections, thicknesses of stiffener webs and flanges, number of stiffeners in both directions, main dimensions of the rectangular box section, thicknesses of outer and inner face plates in smaller and larger walls.
The cost function is formulated according to the fabrication sequence and consists of cost of material, welding and painting. The constrained function minimization is performed by using an effective mathematical optimization method.
2. Keywords: structural optimization, minimum cost design, cellular plates, columns.

3. Introduction

Steel columns are widely used for buildings, bridges, as supports of highways etc. The optimum design of such columns has been treated, which constructed from various structural types, such as circular cylindrical unstiffened and stiffened shells and square box sections with walls from stiffened and cellular plates [1]. Bending caused by horizontal force plays an important role in seismic design. A detailed literature survey concerning the cellular plates can be found in [1].
Steinhardt [2] has proposed a design method for box beams with stiffened flange plates using formulae for effective plate width. Nakai et al. [3] have worked out empirical formulae for stiffened box stub-columns subject to combined actions of compression and bending.
Ge et al. [4] and Usami et al. [5] have studied the cyclic behaviour and ductility of stiffened steel box columns used as bridge piers. Longitudinal flat plate stiffeners and diaphragms as well as constant compressive axial force and cyclic lateral loading have been considered. Empirical formulae have been proposed for ultimate strength and ductility capacity.
Other papers about bridge piers can be found in conference proceedings as follows: Yamao,T. et al. [6], Ohga,M. et al. [7] and Hirota,T. et al. [8].

In our previous studies it has been shown that, in the case of uniaxial compression, cellular plates are more economic than a longitudinally stiffened ones (Farkas & Jármai [9]). In a study we have elaborated a minimum cost design of a cellular plate subject to uniaxial compression (Farkas & Jármai [10]).This method is used in the present paper for a square box column constructed from four equal cellular plates.

A column is loaded by a compression force N_F and a bending moment caused by a horizontal force $H_F = 0.1N_F$ shown in Figure 1. Firstly, the unstiffened rectangular cross section is optimized. It will be derived that, in the case of uniaxial bending, the rectangular cross section is more economic than the square one.
It will be shown that, in the given numerical case, the plate thicknesses are too large for fabrication. Therefore stiffened plates should be used.

Results obtained for square box columns have shown that the cellular plate elements are more economic than the

plates stiffened on one side [1].
The stiffeners can be made of halved rolled I-sections (UB profiles are used) or by welded T-sections. Advantages of welded T-sections are that their dimensions (mainly the web thickness) can be freely varied. The economy of welded T-stiffeners depends on local buckling strength caused by the stress state (compression or bending).
Thus, the aim of the present study is to elaborate the minimum cost design of a column with rectangular and square cross-sections and cellular plate walls. We have considered the welded structure with initial imperfections according to the standards [11, 12]. Dented structures have not been considered [13]. That is another problem.

4. Numerical data

The factored compression force is $N_F = 10^8$ [N], the height of the column is a_0 = 15 m, the steel yield stress is f_y = 355 MPa, the Young-modulus is $E = 2.1\text{x}10^5$ MPa.

5. Minimum cross-sectional area design of a rectangular unstiffened box section

The cross-sectional area is expressed as

$$A = ht_w + 2bt_f \tag{1}$$

h is the height of the web, b is the width of the flange, t_w and t_f the thicknesses of the box section.
Local buckling of plate elements can be avoid by using the constraints on plate slendernesses, where β and δ are the limit slenderness values for the web and the flange.

$$\frac{h}{t_w/2} \le \beta, \quad \frac{b}{t_f} \le \delta \tag{2}$$

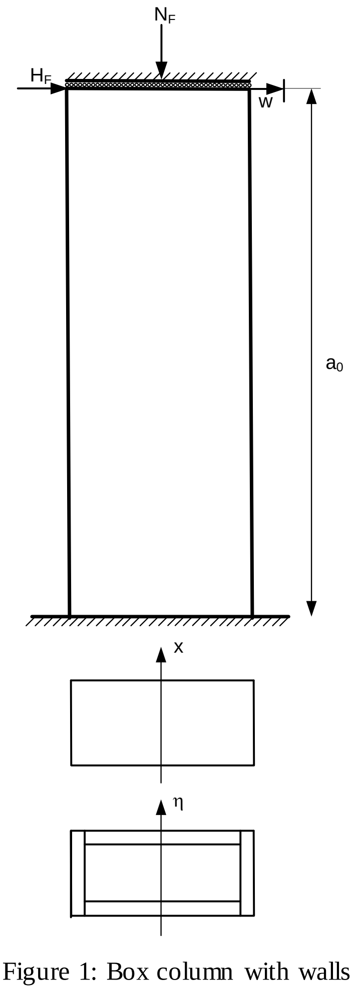

Figure 1: Box column with walls of unstiffened and cellular plates, the two ends are built-in

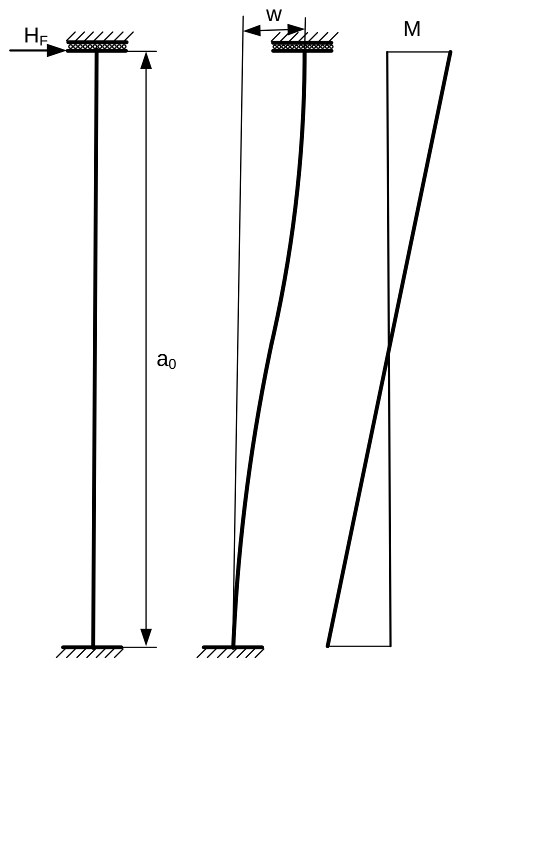

Figure 2: Deformation and bending moment distribution of the column caused by the horizontal force

According to Eurocode 3 [12]

$$1/\delta = 42\varepsilon, \varepsilon = \sqrt{235/f_y}, 1/\delta = 34 \tag{3}$$

The value of β depends on the stress distribution (Fig. 2). The stress constraint is formulated as

$$\sigma = \frac{N_F}{A} + \frac{H_F a_0}{2W_x} = \sigma_1 + \sigma_2 \le f_y \tag{4}$$

Taking the constraints on limiting plate slenderness as active from Eq. 2, since the largest slendernesses give the smallest objective function of the column, the moment of inertia is as follows:

$$I_x = \frac{h^3 t_w}{12} + 2bt_f \left(\frac{h}{2}\right)^2 = \frac{\beta h^4}{6} + \frac{\delta b^2 h}{2} \tag{5}$$

The section modulus is the following:

$$W_x = \frac{I_x}{h/2} = \frac{\beta h^3}{3} + \delta b^2 h \tag{6}$$

For

$$\psi = \frac{\sigma_1 - \sigma_2}{\sigma_1 + \sigma_2} \ge -1 \tag{7}$$

$$\frac{1}{\beta} = \frac{42\varepsilon}{0.67 + 0.33\psi} \tag{8}$$

Eqs (7) and (8) give the limiting plate slenderness β in the case of different edge stresses according to Eurocode 3 [12] Table 5.2. σ_1 σ_2 are defined in Eq (4).
The constraint on horizontal displacement of the top (Fig. 1) is formulated for a column which is built-in at both ends (Fig. 2), thus

$$w = \frac{H_F a_0^3}{12E\gamma_M I_x} \le \frac{a_0}{\phi} = 15 \text{ mm} \tag{9}$$

γ_M = 1.5 is the safety factor used for factored forces in the case of displacement calculation.

$\phi = 1000$.

The optimum is found by using a MathCAD program. It makes a systematic search to find the minimum of the cross-sectional area (A). Table 1 shows the search results. The minimal area means the minimum mass of the column. Note that the actual value of β is determined by iteration.

Table 1: Cross-sectional area (A) in the function of dimensions of the rectangular box section. Dimensions in mm, stresses in MPa, cross-sectional area in mm^2. The optimum values are marked by bold letters.

h	b	w	σ_1	σ_2	σ	$1/\beta$	Ax10^{-5}
2500	2250	14.9	180	157	337	49.00	5.529
2600	2110	14.8	186	163	349	49.15	5.370
2700	**2000**	**14.7**	**188**	**166**	**354**	**49.30**	**5.310**
2800	1910	14.2	188	167	355	49.33	5.325
2900	1820	13.8	187	168	355	49.44	5.351
3000	1730	13.4	185	168	353	49.56	5.392

It can be seen that the displacement constraint is active for smaller h-values and the stress constraint is active for

larger h-values.
The optimum plate thicknesses are

$$t_w / 2 = 2700 / 49.30 = 54.8, t_f = 2000 / 34 = 58.8 \text{ mm.}$$

For the optimum the δ value is calculated by Eq (3). For the square box a similar systematic search is performed considering that $h = b$.
The optimum dimensions of the unstiffened *square* box section one can find in a similar way

$h = b = 2400$ mm, $\sigma_1 = 174, \sigma_2 = 150, 1/\beta = 49, t_w / 2 = 49, t_f = 70.6$ mm, $A = 5.739\text{x}10^{-5}$ mm^2.

From the above calculation one can conclude that

(a) The rectangular cross-section is more economic than the square one since

$$\frac{A_s - A_r}{A_s} 100 = 7.5\%$$

where A_s and A_r are the square and the rectangular columns cross sections.

(b) The plate thicknesses are very large, unsuitable for fabrication, thus, stiffened plate walls should be used.
(c) The optimum ratio of b/t for a rectangular box section is 2700/2000 = 1.35.

Based on the above conclusions, in the present study the optimum cost design of a rectangular box column with cellular plate walls is derived.

6. Minimum cost design of column of rectangular box section with cellular plate walls

Cellular plates are constructed from two plates and longitudinal stiffeners welded between them. Welded and rolled T-sections are selected for stiffeners. Figures 3 and 4 show the dimensions of cellular plate walls. Variables are as follows: height of welded T-sections $h_1/2 = h/2 - t_f$, $h_{11}/2 = h_2/2 - t_{f1}$, thickness of stiffener webs t_w and t_{w1}, number of stiffeners in both directions n and n_1, main dimensions of the rectangular box section b_0 and b_{01}, thicknesses of outer and inner face plates in smaller and larger walls t and t_1. Ranges of variables are as follows: t = 4 – 40 mm, h = 300 – 1000 mm, b = 30 – 300 mm. Ranges for number of stiffeners are n = 2 – 20 mm.

For the rolled stiffener cross sections we chose universal beam UB profiles. The sizes of the section t_f, t_w, b are calculated by the catalogue of ArcelorMittal [14] with curve fitting calculation in the function of the height of the section h.

$$t_f = a + ba*h + c*h^2 + d*h^3 + e*h^4 + f*h^5 + g*h^6 + ha*h^7 + p*h^8 \quad (10)$$

$$t_w = a + ba*h + c*h^2 + d*h^3 + e*h^4 + f*h^5 + g*h^6 + ha*h^7 + p*h^8 \quad (11)$$

$$b = a + ba*h + c/h + d*h^2 + e/h^2 + f*h^3 + g/h^3 + ha*h^4 + p/h^4 + r*h^5 + o/h^5 \quad (12)$$

A T-stiffener has four dimensions. Using Eqs (10), (11) and (12) the number of unknown dimensions is reduced to one (h). Table 2 shows the approximation functions. For a good approximation so many decimal numbers are needed. The curve fitting is made by TableCurve2D program.

Table 2. Curve fitting approximation of the sizes of the rolled I-beam

	t_f	t_w	b
a	-26.93815960004096	4.598131596507252	-1108926.658794802
ba	0.7030053163805572	-0.1667245080692302	2054.96457373585
c	-0.00569333794408951	0.002662252638593643	394347552.4221416
d	2.383106250400329D-05	-1.662919423768273D-05	-2.475920494568994
e	-5.605511588090933D-08	5.42570607199179D-08	-91315532919.66857
f	7.662794270183799D-11	-1.003562930723944D-10	0.001858445891156483
g	-5.902409057606285D-14	1.063362616433473D-13	13189053888762.85

ha	2.267417890058806D-17	-6.028516559742138D-17	-7.856977790442618D-07
p	-2.999371273581411D-21	1.419727612597333D-20	-1073670362507492
r			1.422535840934241D-10
o			3.744384150518803D16

7. Geometric characteristics for displacement constraint

Cross-sectional area for both cellular plate walls

$$A = \frac{h_1 t_w}{2} + b t_f + 2 s_y t, s_y = \frac{b_0}{n}, h_1 = h - 2t_f \tag{13}$$

$$A_1 = \frac{h_{11} t_{w1}}{2} + b_1 t_{f1} + 2 s_z t_1, s_z = \frac{b_{01} - h - 3t}{n_1}, h_{11} = h_2 - 2t_{f1} \tag{14}$$

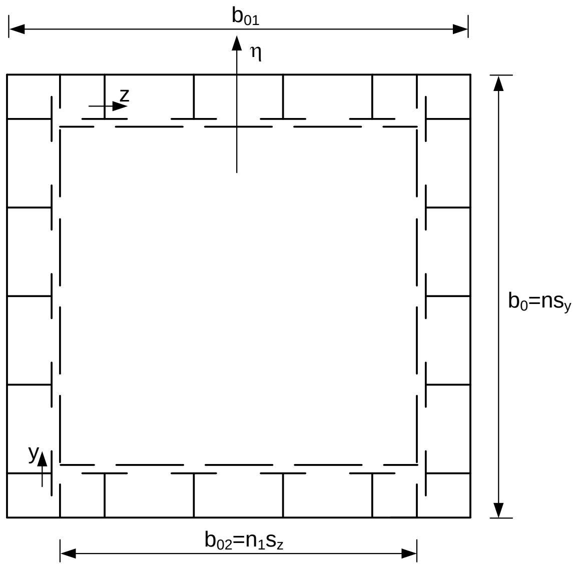

Figure 3: Cross-section of the rectangular box column with cellular plate walls (see also Fig. 1). The T-stiffeners and the plate parts are marked by lines only.

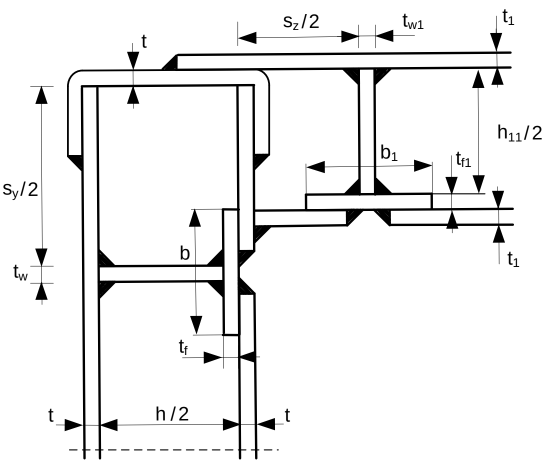

Figure 4: Details of the corner for the box section with cellular plate walls

Distance of the gravity centre

$$z_G = \frac{1}{A}\left[\frac{h_1 t_w}{2}\left(\frac{h_1 + t}{2}\right) + b t_f \frac{h_1 + t + t_f}{2} + s_y t\left(\frac{h_1}{2} + t + t_f\right)\right] \tag{15}$$

Moment of inertia

$$I_y = s_y t z_G^2 + s_y t\left(\frac{h_1}{2} + t + t_f - z_G\right)^2 + \frac{h_1^3 t_w}{96} + I_{y1} \tag{16}$$

$$I_{y1} = \frac{h_1 t_w}{2}\left(\frac{h_1 + t}{2} - z_G\right)^2 + b t_f\left(\frac{h_1 + t + t_f}{2} - z_G\right)^2 \tag{17}$$

Moment of inertia of the whole rectangular box section for axis η

$$I_\eta = 2nI_y + 2nA\left(\frac{b_{01}}{2} - z_G\right)^2 + 2\frac{b_{01}^3 t_1}{12} + 2\frac{t_1}{12}\left(b_{01} - \frac{h_1}{2} - t - t_f\right)^3 + 2I_{\eta 1} \quad (18)$$

$$I_{\eta 1} = 2\left(\frac{h_{11}t_{w1}}{2} + b_1 t_{f1}\right)s_z^2 \frac{n_1(n_1+2)(n_1+1)}{24} \quad (19)$$

The displacement constraint is given as

$$w = \frac{H_F a_0^{\ 3}}{12E\gamma_M I_x} \le \frac{a_0}{\phi} = 15 \text{ mm} \quad (20)$$

or

$$I_\eta \ge I_0 = \frac{H_F L^2 \phi}{12E\gamma_M} \quad (21)$$

Numerical data are given in Section 4. With these data the moment of inertia is as follow:

$I_0 = 5.9524x10^{11}$ mm^4.

8. Geometric characteristics for stress constraint

The local buckling of face plates is avoided by considering effective plate widths according to Eurocode 3 [11]:
The cross sectional area A_e is

$$A_e = \frac{h_1 t_w}{2} + bt_f + 2s_{ye}t, s_y = \frac{b_0}{n}, h_1 = h - 2t_f, s_{ye} = \rho_y s_y \quad (22)$$

$$\rho_y = \frac{\lambda_{py} - 0.22}{\lambda_{py}^2} \quad \text{if} \quad \lambda_{py} = \frac{s_y}{56.8\varepsilon t} \ge 0.673, \ \varepsilon = \sqrt{\frac{235}{f_y}} \quad (23a)$$

$$\rho_y = 1 \quad \text{if} \quad \lambda_{py} < 0.673 \quad (23b)$$

For the displacement constraint I_η is used (calculated with z_G for plates with full width), for the stress constraint $I_{\eta e}$ is used with z_{Ge} considering the effective plate widths.

$$z_{Ge} = \frac{1}{A_e}\left[\frac{h_1 t_w}{2}\left(\frac{h_1 + t}{2}\right) + bt_f \frac{h_1 + t + t_f}{2} + s_{ye}t\left(\frac{h_1}{2} + t + t_f\right)\right] \quad (24)$$

$$A_{1e} = \frac{h_{11}t_{w1}}{2} + b_1 t_{f1} + 2s_{ze}t_1, s_z = \frac{b_{01} - h - 3t}{n}, h_{11} = h_2 - 2t_{f1}, s_{ze} = \rho_z s_z \quad (25)$$

$$\rho_z = \frac{\lambda_{pz} - 0.22}{\lambda_{pz}^2} \quad \text{if} \quad \lambda_{pz} = \frac{s_z}{56.8\varepsilon t_1} \ge 0.673 \quad (26)$$

$$\rho_z = 1 \quad \text{if} \quad \lambda_{pz} < 0.673 \quad (27)$$

$$I_{ye} = s_{ye}tz_{Ge}^2 + s_{ye}t\left(\frac{h_1}{2} + t + t_f - z_{Ge}\right)^2 + \frac{h_1^3 t_w}{96} + I_{y1e} \quad (28)$$

$$I_{y1e} = \frac{h_1 t_w}{2}\left(\frac{h_1 + t}{2} - z_{Ge}\right)^2 + bt_f\left(\frac{h_1 + t + t_f}{2} - z_{Ge}\right)^2 \quad (29)$$

$$I_{\eta e} = 2nI_{ye} + 2nA_e\left(\frac{b_{01}}{2} - z_{Ge}\right)^2 + 2\frac{b_{01}^3 t_1}{12} + 2\frac{t_1}{12}\left(b_{01} - \frac{h_1}{2} - t - t_f\right)^3 + 2I_{\eta 1e} \quad (30)$$

$$I_{\eta 1e} = 2\left(\frac{h_{11}t_{w1}}{2} + b_1 t_{f1}\right)s_{ze}^2 \frac{n_1(n_1+2)(n_1+1)}{24} \quad (31)$$

The stress constraint is given by

$$\sigma = \frac{N_F}{A_0} + \frac{H_F a_0}{2W_0} \le f_y \quad (32)$$

where

$$A_0 = 2nA_e + 2n_1 A_{1e} + 4A_U \quad (33)$$

U-profiles are used to strengthen the corners (Fig. 4) with a cross-sectional area

$$A_U = \left(\frac{h_1}{2} + t_f + 2t + 80\right)t \quad (34)$$

and

$$W_0 = \frac{I_{\eta e}}{\frac{b_{01}}{2} - z_{Ge}} \quad (35)$$

It should be noted that the effect of the global buckling of box column walls can be neglected, since the cellular plates have very large torsional stiffness. Calculations show that all the optimized column structures satisfy the constraint on flexural buckling about the weak axis.

9. Constraint on local buckling of welded stiffener webs

The webs are subject to uniform compression. According to Eurocode 3 [12]

$$\frac{h_1}{2t_w} \le 42\varepsilon_1, \varepsilon_1 = \sqrt{\frac{235}{\sigma}} \quad (36)$$

and

$$\frac{h_{11}}{2t_{w1}} \le 42\varepsilon_1 \quad (37)$$

10. Fabrication constraints

In order to guarantee the welding of stiffeners web to the base plates, to have enough space, the following constraints should be considered

$$n \le \frac{b_0}{300+b}, n_1 \le \frac{b_{01}}{300+b_1} \quad (38)$$

300 mm space is needed to ease the welding process.

11. Cost function

The cost function is formulated according to the fabrication sequence.
(1) Welding of outer face plates with butt welds (SAW – submerged arc welding). A plate element has sizes of 6000x1500 mm or less.
Plate of sizes $a_0 x b_0$: volume $V_0 = a_0 b_0 t$, weld length $L_{W0} = 2b_0 + (q-1)a_0$,

$$K_{W0} = k_W\left(\Theta\sqrt{3q\rho V_0} + 1.3C_W t^{n_0} L_{W0}\right), \; k_W = 1.0 \text{ \$/min} \quad (39)$$

where k_W is the specific welding cost, we used 60 \$/hour which can be valid in Europe.

q is the number of plate elements in the direction of b_0 so that $b_0 / q \le 1500$ mm,

Θ is the factor of complexity,

ρ is the density of the steel,

t is the thickness of the plate,

L_{Wi} is the weld length.

The factor of complexity of the assembly is taken as $\Theta = 2$.

$$\text{For} \quad t < 11 \quad C_W = 0.1346x10^{-3}, n_0 = 2 \tag{40}$$

$$\text{for} \quad t \ge 11 \quad C_W = 0.1033x10^{-3}, n_0 = 1.904 \tag{41}$$

Plate of sizes

$$a_0 x b_{01} : V_{01} = a_0 b_{01} t_1 ; L_{W01} = 2b_{01} + (q_1 - 1)a_0 \tag{42}$$

$$K_{W01} = k_W \left(\Theta \sqrt{3 q_1 \rho V_{01}} + 1.3 C_W t_1^{n_0} L_{W01} \right) \tag{43}$$

q and q_1 are the numbers of plate strips of width smaller than 1500 mm.

(2) Welding of stiffeners' webs to outer face plates and to flange with double fillet welds (GMAW-C gas metal arc welding with $C0_2$).
Plate of sizes $a_0 x b_0$.
For welded stiffeners:

$$V_1 = \left(\frac{h_1}{2} t_w + b t_f \right) a_0 n + V_0 , L_{W1} = 4 a_0 n \tag{44a}$$

For rolled stiffeners:

$$V_1 = \left(\frac{h_1}{2} t_w + b t_f \right) a_0 n + V_0 , L_{W1} = 2 a_0 n \tag{44b}$$

For welded stiffeners:

$$K_{W1} = k_W \left(\Theta \sqrt{(2n+1) \rho V_1} + 1.3x0.3394x10^{-3} a_W^2 L_{W1} \right) \tag{45a}$$

For rolled stiffeners:

$$K_{W1} = k_W \left(\Theta \sqrt{(n+1) \rho V_1} + 1.3x0.3394x10^{-3} a_W^2 L_{W1} \right) \tag{45b}$$

$a_W = 0.4 t_w$ but $a_{W\min} = 4$ mm

Plate of sizes $a_0 x b_{01}$.
For welded stiffeners:

$$V_{11} = \left(\frac{h_{11}}{2} t_{w1} + b_1 t_{f1} \right) a_0 n_1 + V_{01}, L_{W11} = 4 a_0 n_1 \tag{46a}$$

For rolled stiffeners:

$$V_{11} = \left(\frac{h_{11}}{2} t_{w1} + b_1 t_{f1} \right) a_0 n_1 + V_{01}, L_{W11} = 2 a_0 n_1 \tag{46b}$$

For welded stiffeners:

$$K_{W11} = k_W \left(\Theta \sqrt{(2n_1 + 1) \rho V_{11}} + 1.3x0.3394x10^{-3} a_{W1}^2 L_{W11} \right) \tag{47a}$$

For rolled stiffeners:

$$K_{W11} = k_W \left(\Theta \sqrt{(n_1 + 1) \rho V_{11}} + 1.3x0.3394x10^{-3} a_{W1}^2 L_{W11} \right) \tag{47b}$$

$a_{W1} = 0.4 t_{w1}$ but $a_{W1\min} = 4$ mm

(3) Welding of inner plate strips of width s_y and s_z from 3-3 parts with butt welds excluding the outside strips:

$$V_2 = a_0 s_y t \tag{48}$$

$$K_{W2} = (n-1)k_W\left(\Theta\sqrt{3\rho V_2} + 1.3C_W t^{n_0} 2s_y\right) \tag{49}$$

$$V_{21} = a_0 s_z t_1 \tag{50}$$

$$K_{W21} = (n_1-1)k_W\left(\Theta\sqrt{3\rho V_{21}} + 1.3C_W t_1^{n_0} 2s_z\right) \tag{51}$$

(3a) Welding of the outside strips of width $s_y/2$ and $s_z/2$:

$$V_{2a} = a_0 s_y t/2, V_{21a} = a_0 s_z t_1/2 \tag{52}$$

$$K_{W2a} = 2k_W\left(\Theta\sqrt{3\rho V_{2a}} + 1.3C_W t^{n_0} s_y\right) \tag{53}$$

$$K_{W21a} = 2k_W\left(\Theta\sqrt{3\rho V_{21a}} + 1.3C_W t_1^{n_0} s_z\right) \tag{54}$$

(4) Welding of inner face plate strips to the stiffener flanges with double fillet welds:

$$V_3 = V_1 + a_0 b_0 t, L_{W2} = 2a_0 n \tag{55}$$

$$K_{W3} = k_W\left(\Theta\sqrt{(n+2)\rho V_3} + 1.3x0.3394x10^{-3} a_{W2}^2 L_{W2}\right) \tag{56}$$

$a_{W2} = 0.7t$ but $a_{W2\min} = 3$ mm

$$V_{31} = V_{11} + a_0(b_{01} - h - 3t)t_1, L_{W21} = 2a_0 n_1 \tag{57}$$

$$K_{W31} = k_W\left(\Theta\sqrt{(n_1+2)\rho V_{31}} + 1.3x0.3394x10^{-3} a_{W21}^2 L_{W21}\right) \tag{58}$$

$a_{W21} = 0.7t_1$ but $a_{W21\min} = 3$ mm

(5) Welding of 2 U-elements to the ends of the smaller wall with 2-2 fillet welds

$$A_U = \left(\frac{h_1}{2} + t_f + 2t + 80\right)t, \quad V_4 = 2A_U a_0 + V_3 \tag{59}$$

$$K_{W4} = k_W\left(\Theta\sqrt{3\rho V_4} + 1.3x0.3394x10^{-3} a_{W2}^2 4a_0\right) \tag{60}$$

(6) Welding of larger walls to the smaller ones with fillet welds

$$V_5 = 2V_4 + 2V_{31}, L_{W3} = 8a_0 \tag{61}$$

V_5 contains the whole volume (see also Eqs. (44), (48), (55) and (61).

$$K_{W5} = k_W\left(\Theta\sqrt{4\rho V_5} + 1.3x03394x10^{-3} a_{W21}^2 L_{W3}\right) \tag{62}$$

The material cost

$$K_M = k_M \rho V_5, k_M = 1.0 \text{ \$/kg} \tag{63}$$

The painting cost is calculated as

$$K_P = k_P \Theta S_P, k_P = 14.4x10^{-6} \text{ \$/mm}^2 \tag{64}$$

Surface to be painted

$$S_P = 2a_0\left(2b_0 + 2b_1 - \frac{h_1}{2} - t_f - 2t - \frac{h_{11}}{2} - t_{f1} - 2t_1\right) \tag{65}$$

The total cost

$$K = K_M + 2\left(K_{W0} + K_{W01} + K_{W1} + K_{W11} + K_{W2} + K_{W2a} + K_{W21} + K_{W21a}\right) + \\ + 2\left(K_{W3} + K_{W31}\right) + K_{W4} + K_{W5} + K_P \tag{66}$$

12. Particle swarm optimization

The particle swarm optimization (PSO) is a parallel evolutionary computation technique developed by Kennedy and Eberhart [15] based on the social behaviour metaphor. A standard textbook on PSO, treating both the social and computational paradigms, is Yang [16]. The PSO algorithm is initialized with a population of random candidate solutions, conceptualized as particles. Each particle is assigned a randomized velocity and is iteratively moved through the problem space. It is attracted towards the location of the best fitness achieved so far by the particle itself and by the location of the best fitness achieved so far across the whole population (global version of the algorithm).

Additionally, each member learns from the others, typically from the best performer among them. Every individual of the swarm is considered as a particle in a multidimensional space that has a position and a velocity. These particles fly through hyperspace and remember the best position that they have seen. Members of a swarm communicate good positions to each other and adjust their own position and velocity based on these good positions. The Particle Swarm method of optimization testifies the success of bounded rationality and decentralized decision making in reaching at the global optima. It has been used successfully to optimize extremely difficult multimodal functions.

PSO shares many similarities with evolutionary computation techniques such as Genetic Algorithms (*GA*). The system is initialized with a population of random solutions and searches for optima by updating generations. However, unlike *GA*, PSO has no evolution operators such as crossover and mutation. In *PSO*, the potential solutions, called particles, fly through the problem space by following the current optimum particles.

Each particle keeps track of its coordinates in the problem space which are associated with the best solution (fitness) it has achieved so far. (The fitness value is also stored.) This value is called *pbest* ($\boldsymbol{p}^b$). Another "best" value that is tracked by the particle swarm optimizer is the best value, obtained so far by any particle in the neighbours of the particle. This location is called *lbest*, when a particle takes all the population as its topological neighbours, the best value is a global best and is called *gbest* ($\boldsymbol{g}^b$).

The particle swarm optimization concept consists of, at each time step, changing the velocity of (accelerating) each particle toward its p*best* and *lbest* locations (local version of *PSO*). Acceleration is weighted by a random term, with separate random numbers being generated for acceleration toward *pbest* and *lbest* locations.

In past several years, *PSO* has been successfully applied in many research and application areas. It is demonstrated that *PSO* gets better results in a faster, cheaper way compared with other methods.

One reason that *PSO* is attractive is that there are few parameters to adjust. One version, with slight variations, works well in a wide variety of applications. Particle swarm optimization has been used across a wide range of applications, as well as for specific applications focused on a specific requirement.

The method is derivative free, constrained problems can simply be accommodated using penalty functions. The calculation of the velocity vector and the new position is according to Eqs. 67 and 68.

$$\boldsymbol{v}_i^{k+1} := \boldsymbol{v}_i^k + c_1 r_1 (\boldsymbol{p}_i^b - \boldsymbol{x}_i^k) + c_2 r_2 (\boldsymbol{g}^b - \boldsymbol{x}_i^k)\ , \tag{67}$$

$$\boldsymbol{x}_i^{k+1} := \boldsymbol{x}_i^k + \boldsymbol{v}_i^{k+1} \boldsymbol{t}\ , \tag{68}$$

where r_1 and r_2 are independently generated random numbers in the interval [0,1], and c_1, c_2 are parameters with appropriately chosen values. $\boldsymbol{v}$ is the velocity vector, $\boldsymbol{x}$ is the position vector, t is the time step. We have used crazy birds with the probability of 1.5 %. The particle number was 500. The cognitive learning coefficient is c_1 = 2.0, the social learning coefficient is c_2 = 1.4 [17].

In the PSO process the cost function represents the fitness value to be minimized. This value is used for the selection of the alternatives.

Table 3: Results of optimization with discrete values

	Rectangular, welded	Square, welded	Rectangular, rolled	Square, rolled
b_0	2650	4500	1110	4250
b_{01}	5250	4500	4450	4250
q	2	3	1	3
q_1	4	3	3	3
h	560	670	610	914
h_2	400	460	914	914
t_w	8	10	11.1	15.9
t_{w1}	6	8	15.9	15.9
n	4	8	4	8
n_1	7	3	6	3
t	6	8	5	5
t_1	7	4	10	5
t_f	40	40	17.3	23.9
t_{f1}	30	40	23.9	23.9
b	300	260	228.2	304.1
b_1	300	140	304.1	304.1
σ [MPa]	354.6<355	354.7<355	346.5<355	352.2<355
K [$]	97857.3	102667.4	92226.1	96837.6

13. Results of the optimization

Table 3 shows the optimum sizes of the structure using particle swarm optimization. There are 14 unknowns for welded, 8 unknowns for rolled sections and one constraint on stress (Eq. 32), one on horizontal deformation (Eq. 20), two for stiffener web buckling (Eqs, 36, 37) and two for fabrication (Eq. 38). The stress constraint is usually active. Reliability aspects here in this design has not been considered, but in other cases it has been considered [18].

14. Conclusions

A cantilever column loaded by a compression force and a bending moment caused by a horizontal force is investigated. We found that, in case of uniaxial bending, the rectangular cross section is more economic than the square one. In the given numerical example, the plate thicknesses should be too large for fabrication in the unstiffened case. Therefore stiffened plates should be used. We have elaborated the minimum cost design of a column with rectangular cross-section and cellular plate walls.
Stress and horizontal deformation constraints are formulated. In the stress constraint the face plate buckling is taken into account by using effective widths. Local buckling constraint is used for the web of welded T-stiffeners. The calculation shows, that the rectangular cellular plate is more economic then the square one. The cost saving is around 13 %. Calculations show, that using rolled stiffeners is slightly more economic than welded stiffeners due to the fact that less welding is needed.
The cost function is formulated according to the fabrication sequence and consists of cost of material, welding and painting. The constrained function minimization is performed by using the particle swarm optimization method. The result shows, that using cellular plate for this type of column can be economic, even if the welding is an expensive procedure.

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