9th World Congress on Structural and Multidisciplinary Optimization
June 13 -17, 2011, Shizuoka, Japan

Minimum cost design and comparison of tubular trusses with N- and cross-(rhombic)-bracing

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Abstract
Two similar simply supported optimized tubular trusses with parallel chords and N- and rhombic-type bracing are compared to each other. In the optimization process the truss height and cross-sectional areas of circular hollow section (CHS) struts are sought which minimize the structural volume or cost and fulfil the stress and buckling or deflection constraint. The required cross-sectional area of compression rods are calculated using closed formulae to approximate the Eurocode 3 buckling curve. A special method is developed for the optimization of trusses in the case of a deflection constraint. The cost function includes the cost of material, cutting and grinding of CHS strut ends, assembly, welding and painting. The comparison shows that the rhombic-type truss is more advantageous than the N-type one, since its structural volume and cost is smaller.

Keywords: tubular trusses, welded structures, fabrication cost calculation, structural optimization, deflection constraint

1 Introduction
It is useful for designers to compare different structural types to achieve development of competitive structures. For the realistic comparison the different structural types should be optimized. The optimization can be performed according to different aspects. In the present study the volume (mass) and cost serve as objective function to be minimized and the stress, buckling and deflection constraints are considered as main requirements.
Trusses of parallel chords can be constructed using different bracings, such as K-, N- and cross-type ones. The aim of the present study is to compare trusses with N- and cross-type trusses. Cross-(rhombic)-type trusses are often used, but their advantages are not investigated. Adeli and Balasubramanyan [1] have optimized X- (Pratt) type trusses. Simos et al. [2] have compared N- and X-type trusses regarding their resistance against progressive failure.
For the struts of trusses the hollow sections are the most economic profiles because of their large buckling resistance. Optimum design of tubular trusses are treated in books [3,4,5,6]. The speciality of tubular trusses is the geometric constraint, which prescribes the minimum angle between rods to enable the welding of joints.
Compression rods should be designed against overall buckling. In order to minimize the structural volume, it is necessary to have explicit formulae for the cross-sectional areas. Since the buckling formulae of Eurocode 3 are too complicate, approximate expressions are used for hollow section rods.
In the case of optimum design considering the deflection constraint a special method is used developed by the authors. This method enables to calculate the cross-sectional areas required for a prescribed deflection.
In the cost function the costs of material, cutting and grinding of circular hollow section strut ends, assembly, welding and painting are taken into account.
The effect of self mass in this comparative study is neglected.
These problems are complicated, thus only numerical studies can be performed, but the conclusions can be useful for designers.

2 The optimization process
The optimum design procedure for both structural versions can be summarized as follows.
(a) Formulation of the problem: find the optimum height of the simply supported truss with parallel chords, which minimizes the structural volume and cost as well as fulfil the constraints on stress, stability, geometry and deflection.
(b) Selection of design variables: the truss height h and (in steps k1-k6) the factors μ_i determining the ratio between the cross-sectional areas of rod groups.
(c) Determination of rod forces in function of h.
(d) Formulation of constraints on stress, overall and local buckling of tubular rods, on deflection of the mid-span point and on geometry (angle between rods $\alpha_i \geq 30^0$).
(e) Creation of the formulae for cross-sectional areas A_i required for tension and compression rods.

(f) Creation of the formulae for structural volume and cost in function of h and the cross-sectional areas.
(g) Search the optimum h and A_i for minimum volume and cost using a mathematical constrained function minimization method.
(k) In order to fulfil the deflection constraint the following steps are needed:
  (k1) Determination of rod forces from the unique force acting on the mid-span in function of h.
  (k2) Selection of rod groups of equal cross-sectional area based on required A_i (step (e)).
  (k3) Creation of the formulae for v_1 and v_2 (see below).
  (k4) Search the optimal values for h and μ_i to minimize $V_1 = v_1 v_2$ and fulfil the constraint on geometry using a mathematical method.
  (k5) Calculation of the required cross-sectional areas $A = v_2/(Ew_{\mathrm{adm}})$ and $A_i = \mu_i A$, w_{adm} is the admissible deflection.
  (k6) Determination of the final A_i, which are larger from those obtained in steps (g) and (k5).

2 Optimum design of an N-type planar tubular truss

2.1 Optimum height and cross-sectional areas for stress and overall buckling constraints

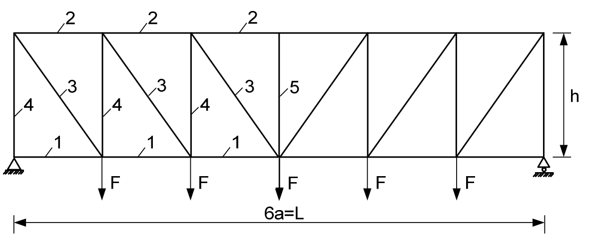

Figure 1. N-type truss with parallel chords, numbering of rod groups

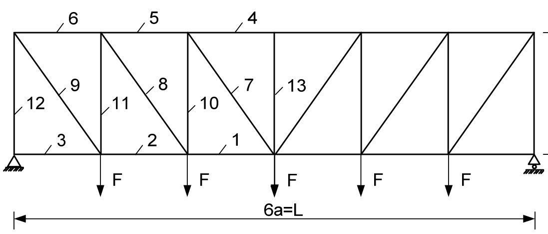

Figure 2. Numbering of rods in Fig.1

As it can be seen on Figure 1, cross-sectional area is the same for all the tension rods of the lower chord (marked by 1), for all the compression rods of the upper chord (mark 2), all the diagonals (3) and verticals (4).

Rod groups of equal cross-sectional areas:

Chords: 1-2-3-4-5-6 (governing A_4), diagonals 7-8-9 (A_9), columns 10-11-12 (A_{12}), central column 13 (A_{13})

(1) tension rods of the lower chord in which the maximum rod force is

$$S_1 = 4aF/h \tag{1}$$

with a required cross-sectional parameters

$$A_1 = S_1/f_{y1}, \quad f_{y1} = f_y/1.1, \quad D_1 = \sqrt{A_1\delta/\pi}, \quad t_1 = D_1/\delta \tag{2}$$

f_y is the steel yield stress, $\delta = D/t$ is the circular hollow section slenderness, we use here the limiting slenderness of $\delta = 50$, prescribed by Wardenier et al. [7]. Note that the available profiles have generally smaller slenderness.

(2) compression rods of the lower chord in which the maximum force is

$$S_2 = 4.5aF/h \tag{3}$$

These rods should be designed against overall buckling. The required cross-sectional area cannot be expressed explicitly using the complicate verification formula of Eurocode 3 [8], therefore we use here the approximate formulae of the Japan Railroad Association [9]

$$\frac{S}{A} \le \chi f_{y1} \tag{4}$$

$$\chi = 1 \qquad \text{for} \qquad \bar{\lambda} \le 0.2 \tag{5a}$$

$$\chi = 1.109 - 0.545\bar{\lambda} \qquad \text{for} \qquad 0.2 \le \bar{\lambda} \le 1 \tag{5b}$$

$$\chi = \frac{1}{0.773 + \bar{\lambda}^2} \qquad \text{for} \qquad \bar{\lambda} \ge 1 \tag{5c}$$

$$\bar{\lambda} = \frac{\lambda}{\lambda_E}, \quad \lambda = \frac{kL}{r}, \quad r = \sqrt{\frac{I_x}{A}}, \quad \lambda_E = \pi\sqrt{\frac{E}{f_y}} \tag{5d}$$

For rods of circular hollow section (CHS) with a symbol of $\delta = D/t$

$$A = \frac{\pi D^2}{\delta}, \quad I_x = \frac{\pi D^4}{8\delta} \tag{6}$$

In order to design rods of CHS we introduce notations

$$\vartheta = \frac{100D}{L}, \quad c = \frac{100k\sqrt{8}}{\lambda_E}, \quad \nu = \frac{10^4 S\delta}{L^2 \pi f_{y1}} \tag{7}$$

with these notations

$$\bar{\lambda} = \frac{c}{\vartheta} \tag{8}$$

and one obtains closed formulae

for $0.2\vartheta \le c \le \vartheta$

$$\vartheta = 0.24572c\left(1 + \sqrt{1 + \frac{14.93475\nu}{c^2}}\right) \tag{9a}$$

for $\vartheta \le c$

$$\vartheta = \left[0.3865\nu\left(1 + \sqrt{1 + \frac{6.69424c^2}{\nu}}\right)\right]^{1/2} \tag{9b}$$

k is the effective buckling length factor, according to Rondal et al [9] for chords 0.9 and for bracings 0.75, L is the rod length between joints.
Knowing ϑ, the cross-sectional characteristics are

$$D = \frac{\vartheta L}{100}, \quad t = \frac{D}{\delta}, \quad A = \frac{\pi D^2}{\delta} \tag{10}$$

In order to obtain comparable optima the calculated rod diameters and thicknesses are not modified according to fabricated available profiles.

Using notation $b = \sqrt{a^2 + h^2}$

the rod forces for rods 3 (compression) and 4 (tension) are as follows:

$$S_3 = 2.5bF/h, \quad S_4 = 2.5F \tag{11}$$

Since the middle vertical rod is loaded only by a secondary force, its cross-sectional area, diameter and thickness are taken as

$$A_5 = 0.5A_4, \quad D_5 = \sqrt{A_5\delta / \pi}, \quad t_5 = D_5 / \delta \tag{12}$$

The volume of the truss is given by

$$V = (A_1 + A_2)L + 6A_3b + 6A_4h + A_5h \tag{13}$$

The cost function contents the cost of material, cutting and grinding of CHS strut ends, assembly, welding and painting.
The cost of material is given by

$$K_M = k_M \rho V \tag{14}$$

where an average specific cost of k_M = 1.0 \$/kg is considered, $\rho = 7.85 \times 10^{-6}$ kg/mm^3 for steel.
The cost of cutting and grinding of CHS strut ends is calculated with a formula proposed by Glijnis [11]

$$K_{CG}(\$) = k_F \Theta_{CG} \frac{2.5\pi D}{(350 - 2t)0.3\sin\alpha} \tag{15}$$

where k_F = 1.0 \$/min is the specific fabrication cost, $\Theta_{CG} = 3$ is a factor for work complexity, 350mm/min is the cutting speed, 0.3 is the efficiency factor, diameter D and thickness t are in mm, α is the inclination angle of diagonal braces, in our case

$$\sin\alpha = \frac{h}{\sqrt{a^2 + h^2}} \tag{16}$$

In our case the KCG formula should be multiplied for diagonals (3) and verticals (4) by 12, for vertical (5) by 2.
The general formula for the welding cost is as follows [4,5,6]

$$K_w = k_w \left(C_1 \Theta \sqrt{\kappa \rho V} + 1.3 \sum_i C_{wi} a_{wi}^n C_{pi} L_{wi} \right) \tag{17}$$

where k_w [\$/min] is the welding cost factor, C_1 is the factor for the assembly usually taken as C_1 = 1 min/kg$^{0.5}$, Θ is the factor expressing the complexity of assembly, the first member calculates the time of the assembly, κ is the number of structural parts to be assembled, ρV is the mass of the assembled structure, the second member estimates the time of welding, C_w and n are the constants given for the specified welding technology and weld type.
Furthermore C_{pi} is the factor for the welding position (download 1, vertical 2, overhead 3), L_w is the weld length, the multiplier 1.3 takes into account the additional welding times (deslagging, chipping, changing the electrode).

In our case k_w = 1.0 \$/min, $\kappa = 15$, $\Theta = 3$,

the cost of assembly and welding using SMAW (shielded metal arc welding) fillet welds is given by

$$K_W = k_W \left[\Theta \sqrt{\kappa \rho V} + 1.3x0.7889x10^{-3} \left(12\pi D_4 t_4^2 + \frac{12\pi D_3 t_3^2}{\sin\alpha} + 2\pi D_5 t_5^2 \right) \right] \tag{18}$$

k_W = 1.0 \$/min, $\kappa = 7$.

The cost of painting is calculated as

$$K_P = k_P S_P, k_P = 28.8x10^{-6} \text{ \$/mm}^2. \tag{19}$$

The superficies to be painted is

$$S_P = L\pi D_1 + L\pi D_2 + 6h\pi D_4 + 6\pi D_3 b + h\pi D_5 \tag{20}$$

The total cost is given by

$$K = K_M + K_{CG} + K_W + K_P \tag{21}$$

Numerical data: factored forces F = 500 kN, a = 6 mm f_y = 355 MPa, E = 2.1x10^5 MPa.

The search for optimum h is performed by using a MathCAD and a PSO algorithm [6]. The results are given in Table 1.

Table 1. Volume and cost in function of h. Optima are marked by bolt letters

h mm	V x10^{-8} mm^3	K \$
7100	10.58	17040
7200	10.57	17033
7300	10.56	**17031**
7400	10.5546	17032
7500	10.5517	17040
7600	**10.5506**	17040
7700	10.5524	17050
7800	10.56	17070

It can be seen that h_{opt} = 7600 mm for V_{min} and h_{opt} = 7300 mm for K_{min}. It can be seen that h_{opt} = 7400-7700 mm for V_{min} and h_{opt} = 7200-7400 mm for K_{min}. This means that the optimum for volume and for cost are different. Note that the change in volume and in cost in the optimum domain is very small.
The cross-sectional areas for h = 7400 mm are as follows: A_4 = 7185, A_9 = 4986, A_{12} = 5342, A_{13} = 2155 mm^2.

2.2 Optimum height and cross-sectional areas for deflection constraint
The deflection constraint is formulated as

$$w = \sum \frac{S_i s_i L_i}{EA_i} \le w_0 \tag{22}$$

where E is the elastic modulus, S_i is the force acting in a rod, s_i is the rod force for F = 1, L_i is the rod length, A_i is the cross-sectional area, w_0 is the allowable displacement.
In the calculation the cross-sectional areas are taken into account with different multipliers as

$$A_i = \mu_i A \tag{23}$$

so the displacement constraint is given by

$$w = \frac{1}{EA}\sum_i \frac{S_i s_i L_i}{\mu_i} \le w_0 \tag{24}$$

from which one obtains

$$A \ge \frac{1}{Ew_0}\sum_i \frac{S_i s_i L_i}{\mu_i} = \frac{v_2}{Ew_0} \tag{25}$$

The structural volume is calculated as

$$V = \sum_i A_i L_i = A\sum_i \mu_i L_i = Av_1$$

$$V = \sum_i A_i L_i = \frac{1}{EA}\sum_i \frac{S_i s_i L_i}{\mu_i}\sum_i \mu_i L_i = \frac{v_1 v_2}{Ew_0} \tag{26}$$

In the optimum design h_{opt} is sought, which minimizes the structural volume or the value of

$$V_1 = v_1 v_2. \tag{27}$$

In our case the deflection is calculated with forces without safety factor 1.5, thus F = 333333 N. The effect of self mass is neglected.

$$v_1 = 2L + 6\mu_4 h + 6\mu_3 b + \mu_5 h \tag{28}$$

$$v_2 = S_1 s_1 L + S_2 s_2 L + \frac{6S_3 s_3 b}{\mu_3} + \frac{6S_4 s_4 h}{\mu_4} \tag{29}$$

$$s_1 = a/h, s_2 = 1.5a/h, s_3 = 0.5b/h, s_4 = 0.5 \tag{30}$$

The values of μ_i are selected as $\mu_1 = \mu_2 = 1$, $\mu_3 = \mu_4 = 0.75$, $\mu_5 = 0.4$ taking into account the fabrication of tubular joints. The results of the search are given in Table 2.

Table 2. Search for h_{opt} in the case of a deflection constraint. Optimum is marked by bolt letters

h mm	V_1x10^{-15} mm^3
8900	6.588
9000	6.584
9100	**6.582**
9200	**6.582**
9300	6.584
9400	6.587

For an allowed deflection of $w_0 = L/1500$ = 24 mm the required cross-sectional areas are as follows: A_4 =7975, A_9 = 0.75x7975 =5981, A_{13} = 0.4x7975 = 3190 mm^2.
It can be seen that the cross-sectional areas required for the allowed deflection are larger than those required for stress and buckling constraints.
The corresponding structural volume and cost for these cross-sectional areas is V = 1.321x10^9 mm^3 and K = 20410 \$.

3 Optimum design of a rhombic-type planar tubular truss

3.1 Optimum height and cross-sectional areas for stress and overall buckling constraints
According to Figure 3, four rod groups of equal cross-sectional area are selected as follows: chords marked by 1,2,3, 4,5,6,7 tension diagonals 8,9,10, compression diagonals 11,12, column 13.

(1) tension rods of the lower chord in which the maximum rod force is

$$S_1 = 4{,}25aF/h \tag{31}$$

with a required cross-sectional parameters

$$A_1 = S_1 / f_{y1}, \quad f_{y1} = f_y / 1.1, \quad D_1 = \sqrt{A_1\delta/\pi}, \quad t_1 = D_1/\delta \tag{32}$$

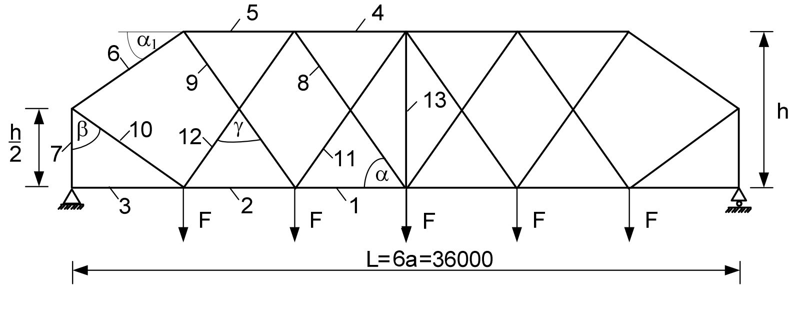

Figure 3. Rhombic-type truss with parallel chords

(2) compression rods of the upper chord (cross-sectional area A_2) in which the maximum force is

$$S_4 = 4.25aF / h \tag{33}$$

(3) tension diagonals (cross-sectional area A_3) with rod force

$$S_9 = 1.25qF / h, \quad q = \sqrt{h^2 + a^2} \tag{34}$$

(4) compression diagonals (cross-sectional area A_4) with rod force

$$S_{11} = 0.25qF / h \tag{35}$$

According to Eurocode 3, Part 3-1 [12] the effective buckling length of these diagonals is $0.5q$ (36)
tension column (cross-sectional area A_5) with rod force

$$S_{13} = 0.5F \tag{37}$$

The structural volume is given by

$$V = 3aA_1 + (2a + q_1 + h/2)A_2 + (2q + q_1)A_3 + 2qA_4 + hA_5 \tag{38}$$

The cost function contents the cost of material, cutting and grinding of CHS strut ends, assembly, welding and painting.
The cost of material is given by Eq. (14), the cost of cutting and grinding of CHS strut ends is calculated with a formula Eq.(15).
In our case the diagonals (11,12) should be interrupted in the middle of rods. Thus

$$K_{CG1} = \Theta_{CG} \frac{2.5\pi}{0.3}\left[\frac{8D_{10}}{(350 - t_{10})\sin\alpha} + \frac{2D_4}{(350 - t_4)\sin\alpha_1} + \frac{2D_4}{(350 - t_4)\sin\beta}\right] + K_{CG2} + K_{CG3} \tag{39}$$

$$\sin\alpha = h/q, \quad \tan\alpha_1 = h/2a, \quad \sin\beta = a/q_1, \quad q_1 = \sqrt{\frac{h^2}{4} + a^2} \tag{40}$$

$$K_{CG2} = \Theta \frac{2{,}5\pi}{0.3}\left[\frac{4D_4}{(350 - t_4)} + \frac{2D_{10}}{(350 - t_{10})\sin\alpha_1} + \frac{2D_{10}}{(350 - t_{10})\sin\beta}\right] \tag{41}$$

$$K_{CG3} = \Theta_{CG} \frac{2.5\pi}{0.3}\left[\frac{8D_{11}}{(350 - t_{11})\sin\alpha} + \frac{8D_{11}}{(350 - t_{11})\sin\gamma} + \frac{8D_{13}}{350 - t_{13}}\right] \tag{41a}$$

The cost of assembly and welding using SMAW (shielded metal arc welding) fillet welds is given by

$$K_W = \Theta\sqrt{\kappa\rho V} + 1.3x0.7889x10^{-3}\pi\left((T_1 + T_2 + T_3)\right) \tag{42}$$

$$T_1 = \frac{8D_{10}t_{10}^2}{\sin\alpha} + \frac{2D_4t_4^2}{\sin\alpha_1} + \frac{2D_4t_4^2}{\sin\beta} \tag{42a}$$

$$T_2 = 2D_4t_4^2 + \frac{2D_{10}t_{10}^2}{\sin\alpha_1} + \frac{2D_{10}t_{10}^2}{\sin\beta} \tag{42b}$$

$$T_3 = \frac{8D_{11}t_{11}^2}{\sin\alpha} + \frac{8D_{11}t_{11}^2}{\sin\gamma} + 2D_{13}t_{13}^2 \tag{42c}$$

$k_W = 1.0$ \$/min, $\kappa = 21$.

The cost of painting is calculated with Eq.(19). The superficies to be painted is

$$S_P = \pi\left(10aD_4 + 4qD_{10} + 4qD_{11} + 2q_1D_4 + 2qD_{10} + hD_{13}\right) \quad (43)$$

The total cost is given by

$$K = K_M + K_{CG} + K_{CG1} + K_{CG2} + K_W + K_P \quad (44)$$

In the optimization process a fabrication constraint should be taken into account, namely the prescription for tubular truss nodes that the angle between rods should be larger than 30^0 to guarantee the easy welding of nodes. In our case this constraint is formulated as

$$\alpha \leq 30^0 \quad (45)$$

The search for optimum h is performed by using a MathCAD and the PSO algorithm [6]. The results are given in Table 3.

Table 3. Volume and cost in function of h. Optima are marked by bold letters

h mm	V x10^{-8} mm^3	Kx10^{-4} \$	(90-α)^0
9000	7.294	1.414	56.3
10000	7.048	1.378	59.0
10300	6.991	1.370	59.8
10400	**6.973**	**1.368**	**60.0**
10500	6.957	1.366	60.2
11000	6.883	1.357	61.4

3.2 Optimum height and cross-sectional areas for deflection constraint

The structural volume is calculated as

$$V = \sum_i A_i L_i = A\sum_i \mu_i L_i = Av_1 \quad (46)$$

$$V = \sum_i A_i L_i = \frac{1}{EA}\sum_i \frac{S_i s_i L_i}{\mu_i}\sum_i \mu_i L_i = \frac{v_1 v_2}{Ew_0} \quad (47)$$

In the optimum design h_{opt} is sought, which minimizes the structural volume or the value of

$$V_1 = v_1 v_2. \quad (48)$$

μ-factors are taken considering the cross-sectional areas corresponding to the average h_{opt} = 10400 mm as follows: $A_4 = 5201$, $A_{10} = 2957$, $A_{11} = 1073$, $A_{13} = 773$ mm^2, thus, $\mu_1 = \mu_2 = 1$, $\mu_3 = 0.6$, $\mu_4 = 0.2$, $\mu_5 = 0.15$.

The other rod forces are as follows:

$$S_2 = 2.75aF/h = S_5,\quad S_6 = 2.5Fq_1/h,\quad S_7 = 2.5F,\quad S_8 = S_{11} = 0.25Fq/h = S_{12} \quad (49a)$$

$$S_9 = 1.25Fq/h,\quad S_{10} = 2.5Fq_1/h \quad (49b)$$

$$s_1 = 1.25a/h = s_4,\quad s_2 = 0.75a/h = s_5,\quad s_3 = 0,\quad s_7 = 0.5 \quad (50a)$$

$$s_8 = s_9 = 0.25q/h,\quad s_6 = -0.5q_1/h,\quad s_{10} = 0.25q_1/h,\quad s_{11} = s_{12} = -0.25q/h,\quad s_{13} = 0.5 \quad (50b)$$

$$v_1 = 5 + q_1 + \frac{h}{2} + \mu_3\left(q_1 + 2q\right) + \mu_4 2q + \mu_5 h \quad (51)$$

$$v_{21} = \left(S_1s_1 + S_2s_2 + S_4s_4 + S_5s_5\right)a + S_6s_6q_1 + S_7s_7h/2 \quad (52a)$$

$$v_{22} = \frac{(S_8 s_8 + S_9 s_9)q + S_{10} s_{10} q_1}{\mu_3} \tag{52b}$$

$$v_{23} = \frac{(S_{11} s_{11} + S_{12} s_{12})q}{\mu_4} + \frac{S_{13} s_{13} h}{\mu_5} \tag{52c}$$

$$v_2 = v_{21} + v_{22} + v_{23} \tag{53}$$

$$s_1 = 1.25a/h = s_4, \quad s_2 = 0.75a/h = s_5, \quad s_3 = 0, \quad s_7 = 0.5 \tag{54a}$$

$$s_8 = s_9 = 0.25q/h, \quad s_6 = -0.5q_1/h, \quad s_{10} = 0.25q_1/h, \quad s_{11} = s_{12} = -0.25q/h, \quad s_{13} = 0.5 \tag{54b}$$

The results of the search are given in Table 4.

Table 4. Search for h_{opt} in the case of a deflection constraint. Optimum is marked by bolt letters

h mm	V_Ix10^{-16} mm^3	$(90\text{-}\alpha)^0$
10200	1.924	59.5
10300	1.922	59.8
10400	**1.921**	**60.0**
10500	1.920	60.2

It can be seen that V_I decreases with the increase of h, but the inclination angle of diagonals shall be smaller than 30^0, therefore h_{opt} = 10400 mm.
For h = 10400 mm truss height for a force F = 333 kN the deflection is w = 35 mm. To allowed deflection of 24 mm correspond the following cross-sectional areas: A_4 = 5549>5201, A_{10} = 3329>2957, A_{11} = 1110>1073, A_{13} = 832>773 mm^2.
The corresponding structural volume and cost for these cross-sectional areas is V = 7.535x10^8 mm^3 and K = 14500 \$.

4 Comparison of the two bracing types

The data for the comparison are summarized in Tables 5 and 6.

Table 5. Comparison of the minima of the volume and cost for stress and buckling constraints

Truss type	Stress and buckling constraints, F = 500 kN	Deflection constraint F = 333 kN
N	h_{opt} = 7400 mm V = 10.55x10^8, K = 17030 \$	h_{opt} = 9100 mm V = 13.21x10^8 , K = 20410 \$
rhombic	h_{opt} = 10400 mm V = 6.973x10^8, K = 13680 \$	h_{opt} = 10400 mm V = 7.535x10^8 , K = 14500 \$

Table 6. Cost components in Table 5. (in \$)

	K_M	K_{CG}	K_W	K_P	K
N-type	8285	1889	1903	4955	17030
Rhombic	5474	1969	1507	3902	13680

The volume and cost minima are smaller for rhombic-type truss both in the case of stress and deflection constraint. In the case of stress constraint this difference is 100(10.55-6.973)/10.55 = 34% in volume and 20% in cost. In the case of deflection constraint this difference is 37% in volume and 29% in cost.
The analysis of cost components (Table 6) shows that the material, welding and painting cost for rhombic-type truss is smaller, the cutting and grinding cost is larger than that for N-type truss.
It can be concluded that, in this numerical problem, the rhombic-type truss is more advantageous than the N-type one. The greatest difference occurs in volumes for deflection constraint.

5 Conclusions

A comparison is carried out for a numerical problem of simply supported trusses with parallel chords with the same number of joint spacing and with the same loading.
The comparison of the optimized versions of planar N- and rhombic-type tubular trusses shows that the rhombic-type truss has smaller volume and cost in the case of stress and deflection constraint.
In the case of stress constraint the compression rods are designed against overall buckling using an approximate buckling curve instead of the Eurocode 3 curve. In the case of the deflection constraint a special method is worked out to obtain the required cross-sectional areas of struts. These areas are always larger than those required for overall buckling.
Stress and buckling constraints are calculated using factored forces, the deflection is calculated with forces without a safety factor. To obtain comparable optima the required cross-sectional areas are not rounded to available profiles and the most economic $\delta = D/t = 50$ slenderness of CHS is used.
Special fabrication constraints are taken into account that the diameters of chords should be larger than those of bracing and the angle between rods should be larger than 30^0 to ease the welding of nodes.
The cost function includes the cost of material, cutting and grinding of CHS rod ends, assembly and welding as well as painting. In the case of rhombic-type truss the compression diagonals should be interrupted in the middle joints and additive costs of cutting and grinding as well as assembly and welding are taken into account. Despite of these additive costs the rhombic-type truss has smaller volume and total cost than the N-type one.
The calculations also show that the optimum truss height and cross-sectional areas are approximately the same for minimum volume and minimum cost. Thus, the cost for minimum volume is a good approximation for the minimum cost.

Acknowledgements

The research was supported by the Hungarian Scientific Research Fund OTKA T 75678 and by the TÁMOP 4.2.1.B-10/2/KONV entitled *Increasing the quality of higher education through the development of research - development and innovation* program at the University of Miskolc.

References

[1] H.Adeli and K.V.Balasubramanyan, *Expert systems for structural design.* Englewood, Cliffs, N.J. Prentice Hall. 1988.
[2] G.Simos, M.Christos, K.Konstantinos and Ch.C.Baniotopoulos, Trusses classification according to robustness criteria. *Proc. Conf. Eurosteel 2008.* Graz, Austria. Ofner et al. eds. Brussels, ECCS, Volume B, 1849-1854, 2008.
[3] J.Farkas, *Optimum design of metal structures*. Chichester: Ellis Horwood 1984.
[4] J.Farkas, and K.Jármai, *Analysis and optimum design of metal structures.* Rotterdam: Balkema 1997.
[5] J.Farkas, and K.Jármai, *Economic design of metal structures.* Rotterdam: Millpress 2003.
[6] J.Farkas, and K.Jármai, *Design and optimization of metal structures*. Chichester: Horwood Publishing Ltd 2008.
[7] J.Wardenier, Y.Kurobane, J.A.Packer, D.Dutta and N.Yeomans, *Design guide for circular hollow section (CHS) joints under predominantly static loading.* Köln, TÜV Rheinland, 1991.
[8] Eurocode 3. *Design of steel structures. Part 1-1. General structural rules*. Brussels, CEN, 2002.
[9] Hasegawa,A.,H. Abo et al. Optimum cross-sectional shapes of steel compression members with local buckling. *Proc. JSCE Structural Engineering/Earthquake Engineering*, Vol. 2, pp. 121-129. 1985.
[10] J.Rondal, K.G.Würker, D.Dutta, J.Wardenier and N.Yeomans, *Structural stability of hollow sections*. Köln, TÜV Rheinland, 1992.
[11] Glijnis, P.C. Southern African Institute of Steel Construction, Johannesburg, South Africa, Personal communications, 1999.
[12] Eurocode 3. *Design of steel structures. Part 3-1: 2006. Towers, masts and chimneys. Towers and masts.*