

Volume and cost minimization of a displacement-constrained tubular truss

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ABSTRACT: A simply supported planar truss with N-type bracing is optimized for minimum volume and cost. The lower chord of the truss is horizontal, but the symmetric upper chord parts are non-parallel and their inclination angle as well as the cross-sectional area of CHS (circular hollow section) rods are optimized. For the calculation of required cross-sectional area of compression struts closed formulae are used as a good approximation of Eurocode 3 buckling curve. A special method is developed for the minimum volume design considering the deflection constraint. In the case of a strong displacement constraint the cross-sectional areas required for the allowed deflection are larger than those required for stress and buckling constraints. The cost function includes the cost of material, cutting and grinding of CHS strut ends, assembly, welding and painting. Special mathematical methods are used to find the optima in the case of a numerical problem.

Keywords: tubular truss, structural optimization, overall buckling, displacement constraint, minimum cost design

1. INTRODUCTION

The aim of the present study is to solve the optimum design problem of a truss subject to a strong displacement constraint. In the case of stress constraints the tension rods are designed for yield stress by using a safety factor for loading and the compression rods are designed for overall buckling. In the case of a strong displacement constraint the required cross-sectional areas are larger than those required for stress constraints.

In the optimum design process of a truss the optimal value of the cross-sectional areas of struts and the geometric characteristics of the truss are sought which minimize an objective function and fulfil the design and fabrication constraints. The objective function can be the volume (weight) or cost, the design constraints are the limitation of stress and displacement, the fabrication constraints ease the manufacture (welding) process.

In the case of an active displacement constraint a special method is developed to calculate the required cross-sectional areas and the truss geometry.

It is shown that the non-parallel chords are more economic than the beam with parallel chords. Thus, in our case the angle of the upper chord (unknowns h_9 and h_{13} in Figure 1) is optimized.

Another problem is the grouping of rods having the same cross-sectional area. The design of all the rods having different cross-sectional areas can cause difficulties in fabrication, but the design of all the

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rods with the same cross-sectional area would be uneconomic. Thus, the economy depends on grouping of rods. In our case four groups are used.

For the minimization of structural volume or cost the cross-sectional areas of rods is needed. The cross-sectional area of compression rods cannot be calculated from the Eurocode 3 buckling formulae. Therefore approximate formulae of Japan Road Association are used. Stress and buckling constraints are calculated using factored forces, whilst the deflection is calculated with forces without a safety factor.

To obtain comparable optima the required cross-sectional areas are not rounded to available profiles and the most economic $\delta = D/t = 50$ slenderness of CHS is used.

The limitation of the angle between CHS struts (minimum 30°) is taken into account as a fabrication constraint. Another fabrication constraint is that the diameters of the chords should be larger than those of verticals and diagonals of the bracing.

The effect of self mass in this comparative study is neglected.

2. SURVEY OF SELECTED LITERATURE

In order to illustrate the literature of the optimum design of trusses, the characteristics of some articles are summarized in Table 1.

Table 1. Literature survey of selected journal articles about the optimization of trusses.

Abbreviations: AISC American Institute of Steel Construction, CHS Circular Hollow Section, AASHTO American Assoc. of State and Highway Transportation Officials, EC3 Eurocode 3 [11], W – American wide flange beam, PSO particle swarm optimizer, ACO ant colony strategy, HS harmony search, MINLP mixed-integer nonlinear programming, alum – aluminium.

Author(s)	Examples	Math. Method	Material	Buckling calculation	Cross-section	Constraint
Gil 2001 [1]	non-parallel chords	conjugates gradient	Steel	EC3		stress and geometrical
Tong 2001 [2]	10-,25-bar	combinatorial	alum.			stress and fundamental frequency
Makris 2002 [3]	3-,10-,25-,60-and 132-bar	strain-energy-density	alum.	no buckling		Displacement
Hasancebi 2002 [4]	224-bar pyramid, 3D simply supported	simulated annealing	steel	AISC	CHS, W-section	layout optimization
Kripakaran 2007 [5]	10-,18-,21-bar	new algorithm	steel, alum.	AASHTO, Euler	CHS	minimum cost
Lamberti 2008 [6]	18-bar cantilever, 25-bar 3D, 45-,72- and 200-bar	simulated annealing	steel, alum.	Euler		stress, nodal displacement
Silih 2008 [7]	non-parallel chords	MINLP	steel	EC3	CHS	minimum mass or cost
Kaveh 2009 [8]	10-,25-,120-,200-, and 244-bar transmission tower	PSO, ACO,HS	steel, alum.	AISC		stress, nodal displacements
Jármai 2004 [9]	Simply supported, parallel chords, 5, 8 spacing	Leap-frog, dynamic-Q	steel	EC3	CHS	optimum height, effect of loads, min. volume

Remarks. (1) In trusses the compression members should be designed against overall buckling. The use of Euler-formula gives unsafe design, since it does not take into account the effect of initial imperfections and residual stresses. Therefore buckling formulae of Eurocode 3 or another up-to-date improved buckling formulae should be used.

(2) The type of the investigated cross section should be given, since it has been shown [10] that the cross-sectional form affects the optima significantly.

3. MINIMUM VOLUME DESIGN OF THE TUBULAR TRUSS WITH NON-PARALLEL CHORDS

Relatively simple formulae can be derived for trusses to minimize the structural volume and fulfil a displacement constraint.

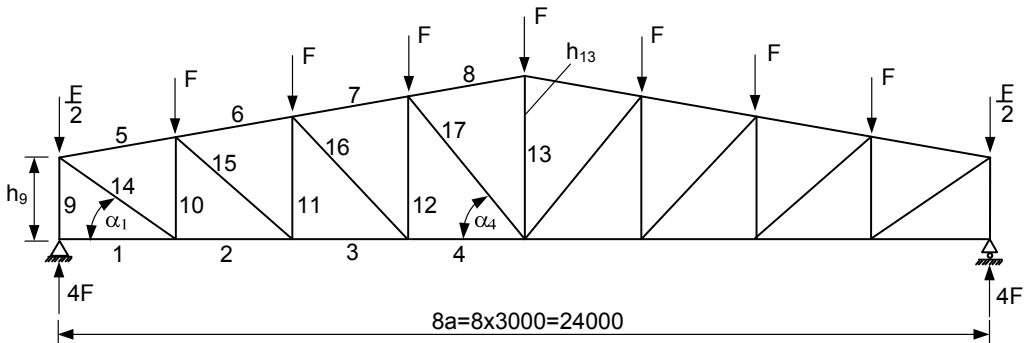


Figure 1. The simply supported truss with non-parallel chords.

The truss rods are divided into n -groups having the same cross-sectional areas, so

$$A_i = \mu_i A \quad i = 1 \dots n \tag{1}$$

and the displacement constraint is given by

$$w = \frac{1}{EA} \sum_i \frac{S_i s_i L_i}{\mu_i} \leq w_0 \tag{2}$$

where E is the elastic modulus, S_i is the rod force, s_i is the rod force from the unit force acting at the midspan, L_i is the rod length, w_0 is the admissible deflection.

From equation (2) one obtains

$$A \geq \frac{1}{E w_0} \sum_i \frac{S_i s_i L_i}{\mu_i} \tag{3}$$

The structural volume is calculated as

$$V = \sum_i A_i L_i = \frac{1}{E w_0} \sum_i \mu_i L_i \sum_i \frac{S_i s_i L_i}{\mu_i} = \frac{v_1 v_2}{E w_0} \tag{4}$$

where

$$v_1 = \sum_i \mu_i L_i, v_2 = \sum_i \frac{S_i s_i L_i}{\mu_i} \tag{5}$$

In the minimum volume design the truss geometry is sought, which minimizes

$$V_1 = v_1 v_2 \tag{6}$$

In the case of the simply supported truss shown in Figure 1 the spacing is constant, the non-parallel upper chord is determined by variable heights h_9 and h_{13} . The truss is subject to a set of vertical static forces F acting on the upper nodes. The displacement of the central lower node is prescribed. It is supposed that all the truss nodes are restrained against transverse deformation.

The variables to be optimized are the heights h_9 and h_{13} as well as the cross sectional areas of rods (A and μ_i).

The calculations show that, in the case of a strong displacement constraint the necessary rod cross-sectional areas are so large that the stress constraints on tension and overall buckling are fulfilled. In spite of this fact these constraints should be checked.

To facilitate the welding of nodes for tubular trusses a geometric fabrication constraint should be considered that the minimal angle between rods should be 30° , in our case (Figure 1)

$$\tan \alpha_1 = \frac{h_9}{a} \geq \tan 30^\circ \quad (7)$$

from which

$$h_9 \geq a \tan 30^\circ = 1732 \text{ mm} \quad (8)$$

and

$$\tan \alpha_4 \leq 60^\circ = \frac{\pi}{3} \quad (9)$$

These constraints are in our case always active.

The rod forces and lengths (S_i , s_i , L_i) are expressed in function of h_9 and the inclination angle of the upper chords α .

$$\tan \alpha = \frac{h_{13} - h_9}{4a}, \cos \alpha = \frac{1}{\sqrt{(\tan \alpha)^2 + 1}}, \sin \alpha = \sqrt{1 - (\cos \alpha)^2} \quad (10)$$

The formulae for S_i , s_i and L_i are given in Tables 2, 3, 4 and 5.

Table 2. Characteristics of rods in the lower chord.

i	S_i	s_i	L_i
1	0	0	a
2	$3.5Fa/h_{10}$	$0.5a/h_{10}$	a
3	$6Fa/h_{11}$	a/h_{11}	a
4	$7.5Fa/h_{12}$	$1.5a/h_{12}$	a

Table 3. Characteristics of rods in the upper chord.

i	S_i	s_i	L_i
5	$\frac{3.5Fa}{h_{10} \cos \alpha}$	$\frac{0.5a}{h_{10} \cos \alpha}$	$\frac{a}{\cos \alpha}$
6	$\frac{6Fa}{h_{11} \cos \alpha}$	$\frac{a}{h_{11} \cos \alpha}$	$\frac{a}{\cos \alpha}$
7	$\frac{7.5Fa}{h_{12} \cos \alpha}$	$\frac{1.5a}{h_{12} \cos \alpha}$	$\frac{a}{\cos \alpha}$
8	$\frac{8Fa}{h_{13} \cos \alpha}$	$\frac{2a}{h_{13} \cos \alpha}$	$\frac{a}{\cos \alpha}$

Table 4. Characteristics of verticals.

i	S_i	s_i	L_i
9	$4F$	0.5	h_9
10	$-3.5F+S_5\sin\alpha$	$-0.5+s_5\sin\alpha$	$h_{10}=h_9+atana$
11	$-2.5F+S_6\sin\alpha$	$-0.5+s_6\sin\alpha$	$h_{11}=h_9+2atana$
12	$-1.5F+S_7\sin\alpha$	$-0.5+s_7\sin\alpha$	$h_{12}=h_9+3atana$
13	$-F+2S_8\sin\alpha$	$2s_8\sin\alpha$	h_{13}

Table 5. Characteristics of diagonals.

i	S_i	s_i	L_i
14	$S_3 L_{14} \cos \alpha / a$	$s_3 L_{14} \cos \alpha / a$	$\sqrt{h_9^2 + a^2}$
15	$(2.5F - S_6 \sin \alpha) L_{15} / h_{10}$	$(0.5 - s_6 \sin \alpha) L_{15} / h_{10}$	$\sqrt{h_{10}^2 + a^2}$
16	$(1.5F - S_7 \sin \alpha) L_{16} / h_{11}$	$(0.5 - s_7 \sin \alpha) L_{16} / h_{11}$	$\sqrt{h_{11}^2 + a^2}$
17	$(0.5F - S_8 \sin \alpha) L_{17} / h_{12}$	$(0.5 - s_8 \sin \alpha) L_{17} / h_{12}$	$\sqrt{h_{12}^2 + a^2}$

The rods are divided to four groups having the same cross-section: lower chord (1, 2, 3, 4), upper chord (5, 6, 7, 8), verticals (9, 10, 11, 12, 13) and diagonals (14, 15, 16, 17).

In order to facilitate the fabrication, the lower and upper chords have the same cross-section ($\mu_l = \mu_u = 1$) and the optimal values of μ_3 (multiplier for verticals) and μ_4 (multiplier for diagonals) are sought, which should be smaller than μ_l .

The components of $V_1 = v_1 v_2$ to be minimized are as follows.

$$v_1 = 8(a + L_7) + 2\mu_3 \sum_{i=9}^{12} h_i + \mu_3 h_{13} + 2\mu_4 \sum_{i=14}^{17} L_i \quad (11)$$

$$v_2 = 2a \sum_{i=2}^4 S_i s_i + 2L_7 \sum_{i=5}^8 S_i s_i + \frac{2}{\mu_3} \sum_{i=9}^{12} S_i s_i h_i + \frac{S_{13} s_{13} h_{13}}{\mu_3} + \frac{2}{\mu_4} \sum_{i=14}^{17} S_i s_i L_i \quad (12)$$

With the optimum values of h_9 , h_{13} , μ_3 and μ_4

$$A_1 = A_2 = \frac{v_{2opt}}{EW_{adm}}, A_3 = \mu_{3opt} A_1, A_4 = \mu_{4opt} A_1 \quad (13)$$

The minimum structural volume is

$$V_{min} = v_1 A_1 \quad (14)$$

For a circular hollow section (CHS)

$$A = \pi D t = \pi D^2 / \delta, \delta = D / t \quad (15)$$

from which

$$D = \sqrt{\frac{A \delta}{\pi}}, t = \frac{D}{\delta} \quad (16)$$

In the design we should use the maximum value of δ , but it is limited to 50 [12]. In the case of available CHS profiles according to [13] δ is varied between 10-50. In order to obtain realistic optima in all cases the optimum $\delta = 50$ is used.

4. CHECK OF THE COMPRESSION RODS FOR OVERALL BUCKLING

For check of overall buckling the approximate formulae of the Japan Road Association (JRA) [14] can be used instead of EC3 curve (b). In this case closed formulae can be given for cross-sectional sizes.

$$N / A \leq \chi f_y \quad (17)$$

$$\chi = 1 \quad \text{for} \quad 0 \leq \bar{\lambda} \leq 0.2 \quad (18a)$$

$$\chi = 1.109 - 0.545 \bar{\lambda} \quad \text{for} \quad 0.2 \leq \bar{\lambda} \leq 1 \quad (18b)$$

$$\chi = \frac{1}{0.773 + \bar{\lambda}^2} \quad \text{for} \quad \bar{\lambda} \geq 1 \quad (18c)$$

Introducing the symbol

$$\mathcal{G} = 100D/L \quad (19)$$

and using $\bar{\lambda} = c/\mathcal{G}$ the closed formulae are as follows.

For $0.2\mathcal{G} \leq c \leq \mathcal{G}$

$$\mathcal{G} = 0.24572c \left[1 + \sqrt{1 + \frac{14.93475\nu}{c^2}} \right] \quad (20a)$$

and for $\mathcal{G} \leq c$

$$\mathcal{G} = \left\{ 0.3865\nu \left[1 + \sqrt{1 + \frac{6.69424c^2}{\nu}} \right] \right\}^{1/2} \quad (20b)$$

for CHS

$$c = \frac{100K\sqrt{8}}{\lambda_E}, \nu = \frac{10^4 S}{L^2} \cdot \frac{\delta}{\pi f_y} \quad (21)$$

where the limiting value of $\delta = D/t = 50$ is used.

$$D = \frac{\mathcal{G}L}{100} \quad (22)$$

In the case of very long struts with small compressive force, the limitation of the strut slenderness can be governing. From the limitation of

$$\lambda = K_R L / r \leq \lambda_{\max} \quad (23)$$

the required radius of gyration is

$$r \geq K_R L / \lambda_{\max} \quad (24)$$

According to [15] $\lambda_{\max} = 180$.

K_R is the strut end restraint factor, for chords $K_R = 0.9$, for verticals and diagonals $K_R = 0.75$ [16].

For the check of overall buckling the following constraint should be fulfilled for all compression rods

$$A_i \geq \frac{\pi D_i}{\delta} \quad (25)$$

where A_i is the optimum cross-sectional area for displacement constraint and D_i is the required diameter from overall buckling calculation.

5. THE COST FUNCTION

The cost function contents the cost of material, cutting and grinding of CHS strut ends, assembly, welding and painting.

The cost of material is given by

$$K_M = k_M \rho V_2 \quad (26)$$

where an average specific cost of $k_M = 1.0$ \$/kg is considered, $\rho = 7.85 \times 10^{-6}$ kg/mm³ for steel. V_2 is the actual structural volume (see equation (35)).

The cost of cutting and grinding of CHS strut ends is calculated with a formula proposed by Glijnjs [17]

$$K_{CG} (\$) = k_F \Theta_{CG} \frac{2.5\pi D}{(350 - 2t)0.3 \sin \alpha} \quad (27)$$

where $k_F = 1.0$ \$/min is the specific fabrication cost, $\Theta_{CG} = 3$ is a factor for work complexity, 350mm/min is the cutting speed, 0.3 is the efficiency factor, diameter D and thickness t are in mm, α is the inclination angle of diagonal braces.

In our case for verticals

$$K_{CG} = \Theta_{CG} 2.5\pi 9D_3 \frac{1 + \frac{1}{\cos \alpha}}{(350 - 2t_3)0.3} \quad (28)$$

For diagonals at the lower strut ends

$$K_{CG1} = \Theta_{CG} 2.5\pi 2D_4 \frac{\sum_{i=1}^4 \frac{1}{\cos \alpha_i}}{(350 - 2t_4)0.3} \quad (29)$$

where

$$\tan \alpha_1 = h_9 / a, \tan \alpha_2 = h_{10} / a, \tan \alpha_3 = h_{11} / a, \tan \alpha_4 = h_{12} / a \quad (30)$$

For diagonals at the upper strut ends

$$K_{CG2} = \Theta_{CG} 2.5\pi 2D_4 \frac{\sum_{i=1}^4 \frac{1}{\cos \beta_i}}{(350 - 2t_4)0.3} \quad (31)$$

where

$$\beta_i = 90^\circ - \alpha - \alpha_i, i = 1, 2, 3, 4 \quad (32)$$

The general formula for the welding cost is as follows [10,17,18]]

$$K_w = k_w \left(C_1 \Theta \sqrt{\kappa \rho V} + 1.3 \sum_i C_{wi} a_{wi}^n C_{pi} L_{wi} \right) \quad (33)$$

where k_w [\$/min] is the welding cost factor, C_1 is the factor for the assembly usually taken as $C_1 = 1$ min/kg^{0.5}, Θ is the factor expressing the complexity of assembly, the first member calculates the time of the assembly, κ is the number of structural parts to be assembled, ρV is the mass of the assembled structure, the second member estimates the time of welding, C_w and n are the constants given for the specified welding technology and weld type.

Furthermore C_{pi} is the factor for the welding position (download 1, vertical 2, overhead 3), L_w is the weld length, the multiplier 1.3 takes into account the additional welding times (deslagging, chipping, changing the electrode).

In our case $k_w = 1.0$ \$/min, $\Theta = 3$, the cost of assembly and welding using SMAW (shielded metal arc welding) fillet welds is given by for verticals

$$K_w = k_w \left[\Theta \sqrt{21\rho V_2} + 1.3 \times 0.7889 \times 10^{-3} \times 9\pi D_3 \left(1 + \frac{1}{\cos \alpha} \right) t_3^2 \right] \quad (34)$$

$$V_2 = 8aA_1 + 8L_7A_2 + 2A_3 \sum_{i=9}^{12} h_i + A_3 h_{13} + 2A_4 \sum_{i=14}^{17} L_i \quad (35)$$

For diagonals at the lower strut ends

$$K_{w1} = 1.3 \times 0.7889 \times 10^{-3} \times 2\pi D_4 t_4^2 \sum_{i=1}^4 \frac{1}{\cos \alpha_i} \quad (36)$$

For diagonals at the upper strut ends

$$K_{w1} = 1.3 \times 0.7889 \times 10^{-3} \times 2\pi D_4 t_4^2 \sum_{i=1}^4 \frac{1}{\cos \beta_i} \quad (37)$$

The cost of painting is calculated as

$$K_p = k_p S_p, k_p = 28.8 \times 10^{-6} \text{ \$/mm}^2 \quad (38)$$

The superficies to be painted is

$$S_p = 8a\pi D_1 + 8L_7\pi D_2 + 2\pi D_3 \sum_{i=9}^{12} h_i + \pi D_3 h_{13} + 2\pi D_4 \sum_{i=14}^{17} L_i \quad (39)$$

The total cost is given by

$$K = K_M + K_{CG} + K_{CG1} + K_{CG2} + K_W + K_{w1} + K_{w2} + K_p \quad (40)$$

6. NUMERICAL DATA

Loads for displacement calculation (without safety factor) $F = 120,000$ N, for stress and buckling constraints $F_0 = 1.5$, $F = 180,000$ N (safety factor of 1.5). Yield stress of steel $f_y = 355$ MPa, elastic modulus $E = 2.1 \times 10^5$ MPa, span length $L = 24$ m, allowable displacement at the middle of the span $w_0 = 32$ mm = $L/750$.

7. THE OPTIMIZATION PROCESS

Calculate the optimum values of h_9 , h_{13} , μ_3 and μ_4 to obtain V_{\min} or K_{\min} and fulfil the constraints on displacement, on minimum angle α_1 [equation (8)], maximum angle α_4 [equation (9)] as well as on stress and overall buckling.

The ranges of unknowns are as follows: $1732 < h_9 < 5000$, $4000 < h_{13} < 8000$ and $h_9 < h_{13}$, $0.5 < \mu_3 < 1$, $0.5 < \mu_4 < 1$.

In the case of minimum volume design equations (13) and (14) give the results and equations(25) should be fulfilled. In the case of minimum cost equation (40) should be minimized, for which equations (11), (12), (13), (16) and (35) should be used.

8. RESULTS OF THE OPTIMIZATION

The fabrication constraints [equation (7) and (8)] determine the optimal pair of unknowns h_9 and h_{13} as follows: for a given h_9 a value of h_{13} smaller than h_{13opt} gives larger $v_1 v_2$, larger does not fulfil the fabrication constraint equation (8). Table 6 shows the max h_{13} in function of h_9 .

Table 6. Maximum h_{13} values in function of h_9 . Values in mm.

h_9	1750	1850	1950	2000	2100	2200	2300
h_{13OPT}	6340	6310	6280	6260	6220	6190	6160

Using a MathCAD algorithm the following optima are determined: in the case of $\mu_3 = \mu_4 = 0.6$, $h_{9opt} = 1950$, $h_{13opt} = 6280$, $v_1 v_{2min} = 2.321 \times 10^{15}$, $V_{\min} = 3.454 \times 10^8 \text{ mm}^3$, $K_{\min} = 7825$ \\$, $A_1 = A_2 = 3708$, $A_3 = A_4 = 2225 \text{ mm}^2$.

The cross-sectional areas required for stress and buckling constraints are as follows: $A_1 = A_2 = 2195$, $A_3 = 2084$, $A_4 = 2094$ mm². It can be seen that the cross-sectional areas determined for a strong displacement constraint are larger than those required for stress or buckling constraints.

For comparison the optimum data for the truss of parallel chords: $h_{9\text{opt}} = h_{13\text{opt}} = 5000$ mm, $V_{\text{min}} = 5.852 \times 10^8$ mm³. $K_{\text{min}} = 11350$ \$. It can be seen that the truss of non-parallel chords is much more economic than the truss of parallel chords.

9. CONCLUSIONS

The optimization problem to be solved is the following: found the optimal geometry and cross-sectional areas of rods which minimize the structural volume or cost for a simply supported tubular truss with non-parallel chords for a strong displacement constraint.

For the solution of this problem a developed calculation method is used. Besides the displacement constraint the rods are checked for tension stress and overall buckling. It is shown that, in the case of a strong displacement constraint the cross-sectional areas are larger than those required for constraints on stress and buckling.

The fabrication (welding) constraints on minimal angle between tubular rods (30°) have been also active. In the calculation of overall buckling the Eurocode 3 formulae are approximated by formulae of Japan Road Association enabling the explicit expression of the necessary cross-sectional area.

Special formulae are used for the cost calculation. The cost function expresses the cost of material, cutting and grinding of the tubular (CHS) rod ends, assembly, welding and painting. It is shown that, in this case, the structural optima for minimum volume and minimum cost are the same.

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REFERENCES

- [1] Hasancebi O., Erbatur F.: *Layout optimization of trusses using simulated annealing*. Advances in Engineering Software 33 (2002) 681-696.
- [2] Tong W.H., Liu G.R.: *An optimization procedure for truss structures with discrete variables and dynamic constraints*. Computers and Structures 79(2001) 155-162.
- [3] Kaveh A., Talatahari S.: *Particle swarm optimizer, ant colony strategy and harmony search scheme hybridized for optimization of truss structures*. Computers and Structures 87 (2009) 267-283.
- [4] Lamberti L.: *An efficient simulated annealing algorithm for design optimization of truss structures*. Computers and Structures 86 (2008) 1936-1953.
- [5] Gil L., Andreu A.: *Shape and cross-section optimization of a truss structure*. Computers and Structures 79 (2001) 681-689.
- [6] Kripakaran P., Gupta A., Baugh J.W.jr.: *A novel optimization approach for minimum cost design of trusses*. Computers and Structures 85 (2007) 1782-1794.
- [7] Makris P.A., Provatidis Ch.G.: *Weight minimization of displacement-constrained truss structures using a strain energy criterion*. Computer Methods in Applied Mechanics and Engineering 191 (2002) 2159-2177.
- [8] Šilih S., Kravanja S.: *Topology, shape and standard sizing optimization of trusses using MINLP optimization approach*. In: Design, Fabrication and Economy of Welded Structures. Internat. Conf. Proc. 2008. Miskolc, Hungary. Jármai, K., Farkas, J. (eds.), Horwood Publ., Chichester 2008, 143-150.
- [9] Jármai K., Snyman J.A., Farkas J.: *Application of novel constrained optimization algorithms to the minimum volume design of planar CHS trusses with parallel chords*. Engineering Optimization 36 (2004), 4, 457-471.

- [10] Farkas J., Jármai K.: *Analysis and optimum design of metal structures*. Balkema, Rotterdam – Brookfield 1997.
- [11] EN 1993-1-1: 1992. *Design of steel structures*. Part 1.1. *General rules and rules for buildings*.
- [12] Wardenier J., Kurobane Y. et al.: *Design guide for circular hollow section joints under predominantly static loading*. TÜV Rheinland, Köln 1991.
- [13] prEN 10210-2. 1996. *Hot finished structural hollow sections*.
- [14] Hasegawa A.,H., Abo et al. 1985. *Optimum cross-sectional shapes of steel compression members with local buckling*. Proc. JSCE Structural Engineering/ Earthquake Engineering 2(1985), 121-129.
- [15] BS 5400 1983: *Steel, concrete and composite bridges*. Part 3: *Code of practice for design of steel bridges*. British Standard Institution, London.
- [16] Rondal J., Würker K-G. et al.: *Structural stability of hollow sections*. TÜV Rheinland, Köln 1992.
- [17] Farkas J., Jármai K.: *Economic design of metal structures*. Millpress, Rotterdam 2003.
- [18] Farkas J., Jármai K.: *Design and optimization of metal structures*. Horwood Publishing, Chichester 2008.