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OPTIMUM DESIGN OF COMPRESSION COLUMNS OF WELDED I-SECTION AND COMPARISON WITH ROLLED PROFILES

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Abstract

The structural optimization can achieve weight and cost savings by changing the structural characteristics. The characteristics of a structure are as follows: loads, materials, geometry, topology, shapes and dimensions of profiles, connections, fabrication technology, transport, erection, maintenance. A modern structure should be safe and economic. Safety is guaranteed by fulfilling the design constraints, economy is achieved by minimization of a cost function. The welded I-beams subject to compression, bending and to combined action are optimized considering stability constraints according to Eurocode 3. Comparison is made between welded and rolled I-beams.

1 Introduction

The optimum design process has three main phases as follows:

(a) preparation: selection of candidate structural versions defining the main characteristics to be changed, formulation of design constraints and cost function,
(b) solution of the constrained function minimization problem by using efficient mathematical methods,
(c) evaluation of results by designers, comparison of optimized versions, formulation of design rules, incorporation in expert systems. These phases show that the structural optimization has three main parts as follows: cost function, design constraints, and mathematical method.

The structural characteristics of compression columns are as follows:
--load: static or variable axial compression force,
--geometry: column length, end restraints (pinned, fixed or free),
--material: steel of different grade (yield strength of 235, 275, 355 MPa), high strength steels, Al-alloys, stainless steel, fiber reinforced plastics,
--profile: rolled I, hollow sections (circular, square, rectangular), welded I- or box, cold-formed channel or other profile, Al-alloy profiles with bulbs, profiles constructed from two or more sections,
--fabrication: rolling, welding (different welding technologies), cold-forming, hot finishing of hollow sections.
The Steel Construction Institute (UK) has worked out a design guide [1] containing tables of load-carrying capacities of rolled and hollow sections in the case of axial compression and bending for two different steel grades (yield strength of 275 and 355 MPa). The aim of the present paper is to compare the rolled I-sections of axially compressed rods with optimized welded I-section struts. From the above mentioned characteristics the following are selected.
--load: axial compressive static force
--geometry: pinned ends, constant cross-section, column fabricated with prescribed initial imperfections and residual welding stresses
--material: steel of yield strength 355 MPa
--profile: doubly symmetric welded I-section with two double fillet welds
--variables: four plate dimensions: h – web height, t_W – web thickness, b – flange width, t_F – flange thickness
--objective function: cost function with material, fabrication and painting costs
--design constraints: overall and local buckling according to Eurocode 3 (Part 1.1 1992, Part 1.3 1996), [2,3].
--mathematical methods: Rosenbrock's hillclimb method with an additional discretization for rounded plate dimensions: thicknesses rounded to 1 mm, plate widths rounded to 10 mm

2 Design of compressed I-beam for overall buckling

Local buckling constraints are formulated according to Eurocode 3 Part 1.1 [2].. According to Trahair [4], for doubly symmetric sections, the buckling occurs either by flexure about the weakest z axis, either by flexure about the strongest y axis, or by torsion. According to Eurocode 3 Part 1.3 (1996) the classical critical buckling stresses can be used for buckling checks, calculating a reduced slenderness $\bar{\lambda} = \sqrt{f_y / \sigma_{cr}}$.

Explicit design constraints
These constraints express the upper and lower limits of the design variables. Design variables are web height (h) and thickness (t_w), flange width (b) and thickness (t_f).

$$200 \text{ mm} \le h \le 1000 \text{ mm}.$$
$$6 \text{ mm} \le t_w \le 30 \text{ mm}.$$
$$200 \text{ mm} \le b \le 1000 \text{ mm}.$$
$$6 \text{ mm} \le t_f \le 40 \text{ mm}.$$

Implicit design constraints
Local buckling constraints

for webplate $\frac{h}{t_w} \le 42 \cdot \varepsilon$ (1)

for flange $\frac{b}{t_f} \le 28\,\varepsilon$ (2)

where $\varepsilon = \sqrt{\frac{235}{f_y}}$.

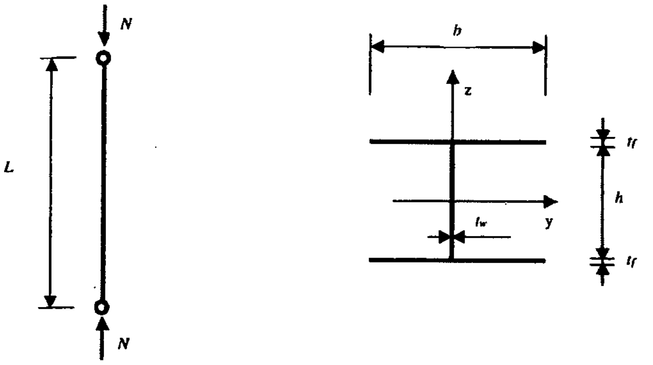

Figure 1. Model of the compressed beam

Overall buckling constraint (around z axis)

$$\frac{N}{A} \le \chi_Z \cdot \frac{f_y}{\gamma_{M1}} \tag{3}$$

where

$$A = h \cdot t_w + 2 \cdot b \cdot t_f \tag{4}$$

is the cross section area of I-beam

$f_y = 355$ MPa the yield stress

$\gamma_{M1} = 1.1$ partial safety factor defined by Eurocode 3

$$\chi_Z = \frac{1}{\Phi_Z + \sqrt{\Phi_Z^2 - \overline{\lambda_Z}^2}} \tag{5}$$

is the buckling factor,

$$\Phi_Z = 0.5 \cdot \left[1 + 0.49\left(\overline{\lambda_Z} - 0.2\right) + \overline{\lambda_Z}^2\right] \tag{6}$$

$\overline{\lambda_Z} = \frac{L}{i_Z \cdot \lambda_E}$ reduced slenderness,

L the column length,

$i_Z^2 = \frac{I_Z}{A}$ radius of inertia,

$I_Z = \frac{b^3 \cdot t_f}{6}$ moment of inertia about z axis,

$\lambda_E = \pi \cdot \sqrt{\frac{E}{f_y}}$ Euler slenderness,

$E = 2.1 \times 10^5$ MPa is the Young modulus.

Torsional-flexural buckling:

$$\frac{N}{A} \le \chi_{TF} \cdot \frac{f_y}{\gamma_{M1}} \tag{7}$$

where

$\gamma_{M1} = 1.1$ is partial safety factor defined by Eurocode 3,

$$\chi_{TF} = \frac{1}{\Phi_{TF} + \sqrt{\Phi_{TF}^2 - \overline{\lambda}^2}} \tag{8}$$

is the buckling factor.

$$\Phi_{TF} = 0.5 \cdot \left[1 + 0.34\left(\overline{\lambda} - 0.2\right) + \overline{\lambda}^2\right] \tag{9}$$

$$\overline{\lambda} = \sqrt{\frac{f_y}{\sigma_{cr}}} \tag{10}$$

is the reduced slenderness,

$$\sigma_{cr} = \sigma_{crTF} \quad \text{but} \quad \sigma_{cr} \le \sigma_{crT} \tag{11}$$

$$\sigma_{crT} = \frac{1}{A \cdot i_0^2} \cdot \left[G \cdot I_t + \frac{\pi^2 \cdot E \cdot I_\omega}{L^2}\right] \tag{12}$$

is the critical torsional stress,

$i_0^2 = i_y^2 + i_Z^2$ is the reduced radius of inertia,

$i_y^2 = \frac{I_y}{A}$ is the radius of inertia,

$$I_y = \frac{h^3 \cdot t_w}{12} + 2 \cdot b \cdot t_f \cdot \left(\frac{h}{2} + \frac{t_f}{2}\right)^2 \tag{13}$$

is the moment of inertia about y axis,

$$I_t = \frac{1.5}{3} \cdot \left(2 \cdot b \cdot t_f^3 + h \cdot t_w^3\right) \tag{14}$$

is the torsional inertia,

$$I_\omega = \frac{h^2 \cdot b^3 \cdot t_f}{24} \tag{15}$$

is the warping constant,

$$\sigma_{cry} = \frac{\pi^2 \cdot E}{\lambda_y^2}, \tag{16}$$

$$\lambda_y^2 = \frac{L^2}{i_y^2}, \tag{17}$$

$$\sigma_{crTF} = \frac{1}{2} \cdot \left[\sigma_{cry} + \sigma_{crT} - \sqrt{\left(\sigma_{cry} + \sigma_{crT}\right)^2 - 4 \cdot \sigma_{cry} \cdot \sigma_{crT}}\right]$$

is the critical stress for torsional-flexural buckling, (18)

$G = \frac{E}{2.6} = 0.8077 \times 10^5$ MPa is the shear modulus.

3 Design of beams for bending

Explicit design constraints

200 mm $\le h \le$ 1500 mm.
6 mm $\le t_w \le$ 40 mm.
200 mm $\le b \le$ 1000 mm.
6 mm $\le t_f \le$ 40 mm.

Implicit design constraints

Local buckling constraints

for webplate $\frac{h}{t_w} \le 124\,\varepsilon$ (19)

for flange $\frac{b}{t_f} \le 28\,\varepsilon$ (20)

where $\varepsilon = \sqrt{\frac{235}{f_y}}$.

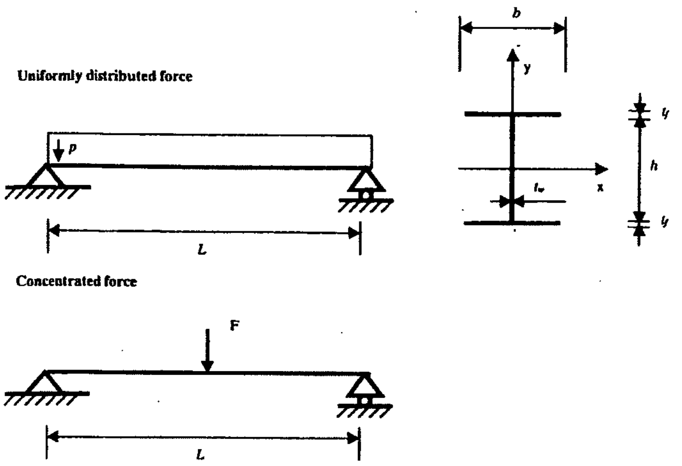

Figure 2. Beam model for bending

Lateral torsional buckling constraint

$$W_{Xel} \geq \frac{W_0}{\chi_{LT}} \tag{21}$$

where

$$W_0 = \frac{M_{max}}{f_y/\gamma_{M1}} \tag{22}$$

is the required section modulus.

For uniformly distributed force p the bending moment is as follows: $M_{max} = \frac{p \cdot L^2}{8}$, (23)

For concentrated force F the bending moment is as follows: $M_{max} = \frac{F \cdot L}{4}$, (24)

$f_y = 355$ MPa is the yield stress,

$\gamma_{M1} = 1.1$ is the partial safety factor according to Eurocode 3,

$$W_{Xel} = \frac{2 \cdot I_X}{h + t_f} \tag{25}$$

is the elastic section modulus,

$$I_X = \frac{h^3 \cdot t_w}{12} + 2 \cdot b \cdot t_f \cdot \left(\frac{h}{2} + \frac{t_f}{2} \right)^2 \tag{26}$$

is the moment of inertia about x axis,

$$\chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \overline{\lambda_{LT}}^2}} \tag{27}$$

is lateral-torsional buckling factor,

$$\Phi_{LT} = 0.5 \cdot \left[1 + 0.49 \cdot \left(\overline{\lambda_{LT}} - 0.2 \right) + \overline{\lambda_{LT}}^2 \right] \tag{28}$$

$$I_y = 2 \cdot \frac{b^3 \cdot t_f}{12} \tag{29}$$

is the moment of inertia about y axis,

$$I_\omega = \frac{b^3 \cdot h^2 \cdot t_f}{24} \tag{30}$$

is the warping moment of inertia,

$$I_t = 0.5 \cdot \left(h \cdot t_w^3 + 2 \cdot b \cdot t_f^3 \right) \tag{31}$$

is the torsional moment of inertia,

$$\overline{\lambda_{LT}} = \frac{\lambda_{LT}}{\lambda_1} \cdot \sqrt{\beta_W} \tag{32}$$

is the reduced slenderness,

$$W_{pl} = \frac{h^3 \cdot t_w}{4} + b \cdot t_f \cdot h \tag{33}$$

is the plastic section modulus,

$$\beta_W = \frac{W_{xel}}{W_{xpl}} \tag{34}$$

is the reduction parameter,

$$\lambda_1 = \pi \cdot \sqrt{\frac{E}{f_y}} \tag{35}$$

$$\lambda_{LT} = \frac{L \cdot \sqrt[4]{\frac{W_{pl}^2}{I_y \cdot I_\omega}}}{\sqrt{C_1} \cdot \sqrt[4]{1 + \frac{L^2 \cdot G \cdot I_t}{\pi^2 \cdot E \cdot I_\omega}}} \tag{36}$$

is the slenderness.

For uniformly distributed force: $C_1 = 1.132$

For concentrated force: $C_1 = 1.365$

4 Design of beams for combined bending and compression

Explicit design constraints

200 mm $\leq h \leq$ 1500 mm.
6 mm $\leq t_w \leq$ 40 mm.
200 mm $\leq b \leq$ 1000 mm.
6 mm $\leq t_f \leq$ 40 mm.

Implicit design constraints

Local buckling constraints for webplate:

$$\psi = -\frac{\frac{M}{W_X} - \frac{N}{A}}{\frac{M}{W_X} + \frac{N}{A}} \tag{37}$$

if $\psi > -1$, then $\frac{h}{t_w} \leq \frac{42 \cdot \varepsilon}{0.67 + 0.33\psi}$

if $\psi \leq -1$, then $\frac{h}{t_w} \leq 62 \cdot \varepsilon \cdot (1 - \psi) \cdot \sqrt{-\psi}$

Overall buckling and lateral-torsional buckling:

$$\frac{N}{\chi_y \cdot A \cdot f_y} + \frac{k_{LT} \cdot M}{\chi_{LT} \cdot W_X \cdot f_y} \leq 1 \tag{38}$$

where

$M = \frac{F \cdot L}{4}$ is the maximal bending moment,

$$I_X = \frac{h^3 \cdot t_w}{12} + 2 \cdot b \cdot t_f \cdot \left(\frac{h}{2} + \frac{t_f}{2} \right)^2 \tag{39}$$

is the moment of inertia about x axis,

$$W_X = \frac{2 \cdot I_X}{h + t_f} \tag{40}$$

is the section modulus,

$$\chi_y = \frac{1}{\Phi_y + \sqrt{\Phi_y^2 - \overline{\lambda}_y^2}} \quad (41)$$

is the flexural buckling factor,

$$\Phi_y = 0.5 \cdot \left[1 + 0.49 \cdot \left(\overline{\lambda}_y - 0.2\right) + \overline{\lambda}_y^2\right],$$

$$\overline{\lambda_y} = \frac{K_y \cdot L}{r \cdot \lambda_E}$$ is the reduced slenderness,

where $K_y = 1$,

$$r = \sqrt{\frac{I_y}{A}}, \quad (42)$$

$$A = h \cdot t_w + 2 \cdot b \cdot t_f \quad (43)$$

is the cross section area,

$$I_y = \frac{b^3 \cdot t_f}{6} \quad (44)$$

is the moment of inertia about y axis,

$$k_{LT} = 1 - \frac{\mu_{LT} \cdot N}{\chi_y \cdot A \cdot f_y}, \quad (45)$$

$$\mu_{LT} = 0.15 \overline{\lambda_y} \cdot \beta_{MLT} - 0.15 \quad (46)$$

where $\beta_{MLT} = 1.4$

$$\chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \overline{\lambda_{LT}}^2}} \quad (47)$$

$$\Phi_{LT} = 0.5 \cdot \left[1 + 0.49 \cdot \left(\overline{\lambda}_{LT} - 0.2\right) + \overline{\lambda}_{LT}^2\right],$$

$$\overline{\lambda_{LT}} = \sqrt{\frac{W_X \cdot f_y}{M_{cr}}}, \quad (48)$$

$$M_{cr} = C_1 \cdot \frac{\pi^2 \cdot E \cdot I_y}{L_2} \cdot \sqrt{\frac{I_\omega}{I_y} + \frac{L^2 \cdot G \cdot I_t}{\pi^2 \cdot E \cdot I_y}}$$

is the elastic critical bending moment, where $C_1 = 1.365$,

$$I_t = 0.5 \cdot \left(h \cdot t_w^3 + 2 \cdot b \cdot t_f^3\right) \quad (49)$$

is the torsional moment of inertia,

$$I_\omega = \frac{b^3 \cdot h^2 \cdot t_f}{24} \quad (50)$$

is the warping constant.

5 Cost calculation

The objective function is the cost of the structure. Total cost consists of material, welding and painting costs.
The cost function is as follows:

$$K = K_m + K_w + K_p \quad (51)$$

where
K_m – material cost,
K_w – welding cost,
K_p – painting cost.

Material cost:
According to the Japanese price list (Price list 1999) k_M = 91.5 yen/kg = 0.832 \$/kg

$$K_m = k_m \cdot \rho \cdot A \cdot L \quad (52)$$

where

k_m – specific material cost
ρ - material density,
A – cross-section area,
L – length.

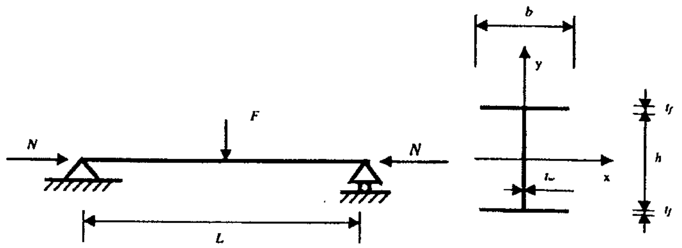

Figure 3. Model of beam under combined bending and compression

The fabrication cost contents the assembly, tacking, welding and additional works and can be calculated according to Farkas & Jármai [6], Jármai & Farkas [7].
The fabrication cost factor can be taken, according to Tizani et al [8], as k_F = 40 \$/h = 0.6667 \$/min, the constant for assembly is C_A = 1.0 min/kg$^{0.5}$, the difficulty factor expressing the complexity of the structure is taken as $\Theta_F = 2$, the number of the assembled structural elements is $\kappa = 3$, $L_{wi} = 4L$ is the weld length in mm, $a_w = 0.4t_w$ is the weld size, the welding time component is, according to COSTCOMP [9], Bodt [10], Farkas & Jármai [6] and Jármai & Farkas [7] for GMAW-C (gas metal arc welding with CO_2) $C_w a_w^n = 0.3394 10^{-3} a_w^2$.

Welding cost

$$K_w = k_f \left[2(3\rho AL)^{0.5} + 1.3x0.3394x10^{-3}x4L(0.4t_w)^2\right] \quad (53)$$

where
k_f – specific fabrication cost,
t_w – thickness of webplate.

Painting cost

The painting cost factor is according to Tizani et al.[8] k_P = 14.4\$/m^2, where the surface is $S = (2h + 4b)L$ in m^2.

$$K_p = k_p \cdot (2 \cdot h + 4 \cdot b) \cdot L \quad (54)$$

where
k_p – specific painting cost,
h – height of webplate,
b – width of flange.

The specific costs are as follows
k_m=0.832 \$/kg, k_f=0.6667 \$/min, k_p=14.4x10^{-6} \$/mm^2, ρ=7.85*10^{-6} kg/mm^3.

6 Numerical examples

6.1 Optimum design of compressed columns for overall buckling

The I-sections are loaded by different compression forces N between 1000 and 16000 kN. The step length between loads is 1000 kN. The length of the columns L varies between 3 and 15 m. The step length is 1 m. Model of the column is according to Figure 1. with two pinned supports at the ends [11].

Explicit constraints

$$200 \text{ mm} \leq h \leq 1000 \text{ mm}.$$
$$6 \text{ mm} \leq t_w \leq 30 \text{ mm}.$$
$$200 \text{ mm} \leq b \leq 1000 \text{ mm}.$$
$$6 \text{ mm} \leq t_f \leq 40 \text{ mm}.$$

Implicit constraints

Local buckling,

Overall buckling around the z axis,

Lateral-torsional buckling.

Optimization is performed using Rosenbrock's Hillclimb procedure [6]. Discrete values are according to the ARBED production data.

Table 1 shows the optimized column sizes for $L = 3$ m length.

First column is N, the compression force, next four columns are the sizes of the cross-section (h, t_w, b, t_f), last column is the cost of the column K/k_m in kg. Table 2 shows the optimized column sizes for $L = 4$ m length.

6.2 Optimum design of beams under bending for lateral-torsional buckling

There are two different kinds of bending: bending caused by uniformly distributed load and concentrated force. In the case of concentrated force (F) the lower and upper limits are 1000 and 16000 kN. The step length for the force is 1000 kN.

In the case of uniformly distributed force (p) the lower and upper limits are calculated from the concentrated force, divided the minimum force by the maximum length and the maximum value of force by the minimum length.

Table 1. Optimized column sizes for $L = 3$ m

N [kN]	h [mm]	t_w [mm]	b [mm]	t_f [mm]	K/k_m [kg]
1000	200	6	200	9	190.7766
2000	200	6	270	12	265.807
3000	200	6	320	15	341.3516
4000	200	6	340	19	415.1408
5000	200	6	380	21	485.9301
6000	200	7	380	25	560.9492
7000	200	6	400	28	623.5882
8000	200	7	400	32	701.3389
9000	210	7	420	34	768.2083
10000	200	7	450	35	831.6299
11000	200	6	470	37	891.0337
12000	200	6	490	39	952.71
13000	200	6	520	40	1039.397
14000	200	6	560	40	1111.7
15000	200	6	600	40	1183.911
16000	200	6	640	40	1256.039

Table 2. Optimized column sizes for $L = 4$ m length

N [kN]	h [mm]	t_w [mm]	b [mm]	t_f [mm]	K/k_m [kg]
1000	200	6	220	10	277.7621
2000	200	6	270	14	379.5146
3000	210	7	340	15	488.5141
4000	200	6	380	18	576.3604
5000	200	6	420	20	672.0706
6000	200	6	440	23	771.5653
7000	200	6	440	27	868.2568
8000	200	6	460	29	952.8267
9000	200	6	480	31	1041.575
10000	200	6	480	35	1146.229
11000	220	7	510	35	1231.61
12000	200	6	510	39	1321.094
13000	220	7	530	40	1418.065
14000	230	7	580	39	1509.556
15000	230	7	610	40	1612.776
16000	230	7	650	40	1708.463

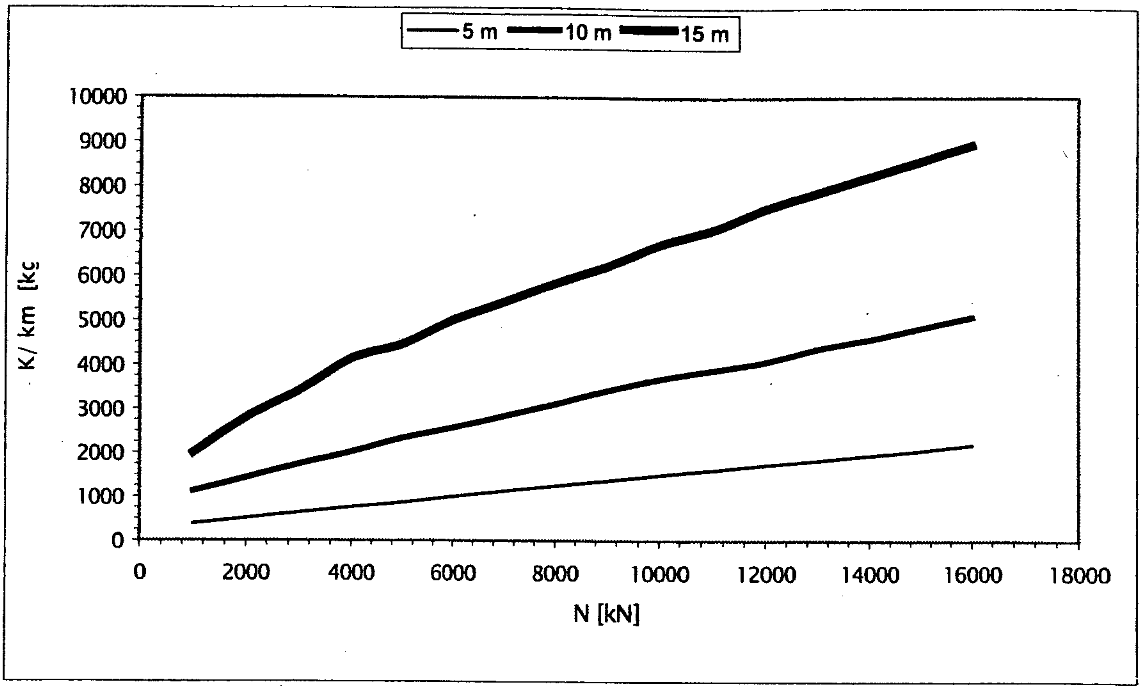

Figure 4. Optimum costs of columns with different length in the function of compression force

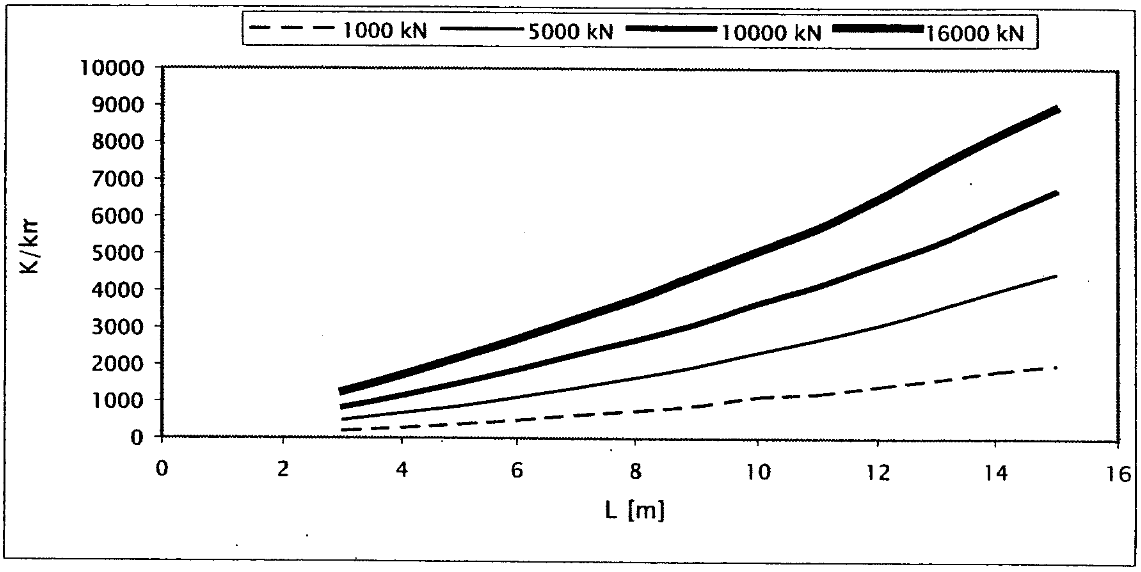

Figure 5. Optimum costs of columns with different compression forces in the function of length

$$p_{lower} = \frac{F_{lower}}{L_{upper}}, \quad p_{upper} = \frac{F_{upper}}{L_{lower}} \tag{55}$$

The lower limit for the uniformly distributed force is 70 N/mm, the upper limit is 5000 N/mm. Discrete values of the uniformly distributed force are: 70, 250, 500, 1000 N/mm. From 1000 N/mm the step length is 500 N/mm. Length of the beam varies between 3 and 16 meters. Step length is 1 m.. The model can be seen on Figure 2.

Explicit constraints

$200 \text{ mm} \le h \le 1500 \text{ mm}.$

$6 \text{ mm} \le t_w \le 40 \text{ mm}.$

$200 \text{ mm} \le b \le 1000 \text{ mm}.$

$6 \text{ mm} \le t_f \le 40 \text{ mm}.$

Implicit constraints

Local buckling,

Lateral-torsional buckling

Table 3 shows the optimized beam sizes under bending due to concentrated forces. Beam length is $L = 3$ m. First column is F, the concentrated force, next four columns are the sizes of the cross-section (h, t_w, b, t_f), last column is the cost of the column K/k_m in kg.

Table 4 shows the optimized beam sizes for $L = 4$ m length. Table 4 contains results up to 13000 kN, because for 5 m beams length the upper limits of the cross-sectional sizes could not satisfy the constraints for larger forces.

Table 3. Optimized beam sizes under bending due to concentrated force for $L = 3$ m length

F [kN]	h [mm]	t_w [mm]	b [mm]	t_f [mm]	K/k_m [kg]
1000	470	6	200	37	910.6053
2000	810	7	220	39	1244.048
3000	960	8	260	40	1501.136
4000	1140	10	280	40	1756.982
5000	1200	10	330	40	1942.989
6000	1290	11	360	40	2134.678
7000	1340	11	400	40	2285.704
8000	1410	12	430	40	2466.511
9000	1390	12	490	40	2616.589
10000	1400	12	540	40	2763.312
11000	1410	12	590	40	2909.933
12000	1500	13	600	40	3055.578
13000	1500	13	650	40	3193.882
14000	1500	13	700	40	3332.111
15000	1460	12	780	40	3476.093
16000	1410	12	960	36	3777.215

Table 4. Optimized beam sizes under bending due to concentrated force for $L = 4$ m length

F [kN]	h [mm]	t_w [mm]	b [mm]	t_f [mm]	K/k_m [kg]
1000	670	6	240	40	1958.982
2000	830	7	360	40	2740.171
3000	1110	9	390	40	3317.269
4000	1300	11	430	40	3865.859
5000	1360	11	520	39	4324.372
6000	1380	12	610	38	4785.437
7000	1430	12	650	40	5118.787
8000	1430	12	740	40	5531.738
9000	1480	12	800	40	5874.655
10000	1500	13	870	40	6296.27
11000	1500	13	960	40	6708.524
12000	1500	21	1000	40	7548.049
13000	1500	38	1000	40	9341.489

Table 5 Optimum sizes of L=3 m beam due to uniformly distributed force

p [N/mm]	h [mm]	t_w [mm]	b [mm]	t_f [mm]	K/k_m [kg]
70	200	6	200	10	491.21
250	360	6	200	19	681.6
500	440	6	200	31	839.3
1000	680	6	200	40	1082.186
1500	780	7	250	40	1315.751
2000	870	8	290	40	1518.62
2500	980	8	320	40	1683.456
3000	950	8	390	40	1856.443
3500	990	8	430	40	1997.198
4000	1090	9	440	40	2130.476
4500	1420	12	420	34	2344.105
5000	1350	11	430	40	2376.98

Optimum sizes due to uniformly distributed force for L=3 m and L=6 m beam lengths can be found in Table 5 and 6.

Table 6 Optimum sizes of $L = 6$ m beam due to uniformly distributed force

p [N/mm]	h [mm]	t_w [mm]	b [mm]	t_f [mm]	K/k_m [kg]
70	240	6	200	33	1407.683
250	590	6	260	40	2343.407
500	810	7	360	40	3250.734
1000	1110	9	490	40	4522.912
1500	1220	10	630	40	5530.587
2000	1490	13	680	39	6425.999
2500	1460	12	840	40	7223.161
3000	1470	12	990	40	8063.084
3500	1500	37	1000	40	11047.56

Table 7. Optimized beam sizes under bending and compression ($N = 1000$ kN) due to concentrated force for $L = 3$ m length

F [kN]	h [mm]	t_w [mm]	b [mm]	t_f [mm]	K/k_m [kg]
1000	500	6	220	39	1003.718
2000	700	8	270	40	1337.321
3000	820	9	320	40	1592.013
4000	1100	12	300	40	1851.649
5000	1120	11	360	40	1999.114
6000	1240	12	380	40	2188.745
7000	1270	13	420	40	2364.386
8000	1420	14	420	40	2534.15
9000	1470	14	450	40	2660.197
10000	1470	14	500	40	2798.74
11000	1400	13	580	40	2916.621
12000	1310	12	680	40	3077.392
13000	1320	12	730	40	3223.799
14000	1320	12	780	40	3361.981
15000	1480	14	810	36	3533.458
16000	1260	12	930	40	3727.278

Table 8. Optimized beam sizes under bending and compression ($N = 3000$ kN) due to concentrated force for $L = 4$ m length

F [kN]	h [mm]	t_w [mm]	b [mm]	t_f [mm]	K/k_m [kg]
1000	520	8	360	40	1929.935
2000	920	13	360	39	2506.841
3000	1140	15	380	40	2949.638
4000	1450	18	400	34	3448.123
5000	1390	17	440	40	3586.281
6000	1420	17	500	40	3843.866
7000	1480	17	580	37	4117.838
8000	1450	16	630	40	4294.865
9000	1490	16	690	39	4526.065
10000	1470	16	750	40	4759.701
11000	1500	16	800	40	4979.086
12000	1410	15	920	40	5251.134
13000	1410	15	990	40	5507.95
14000	1500	22	950	40	5942.075
15000	1500	24	1000	40	6274.82
16000	1500	34	1000	40	7112.201

6.3 Optimum design of beams compressed and bent simultaneously for overall and lateral-torsional buckling

The I-beams are under compression and bending. The compression force (N) is between 1000 and 16000 kN, the step length is 1000 kN. The bending force (F) is between 1000 and 16000 kN, the step length is 1000 kN. The length of the beams is between 3 and 15 m. The step length is 1 m.

Table 7 shows the optimized beam sizes under bending and compression due to concentrated forces. Beam length is $L = 3$ m. First column is F, the concentrated force, next four columns are the sizes of the cross-section (h, t_w, b, t_f), last column is the cost of the column K/k_m in kg. Table 8 shows the optimized beam sizes for $L = 4$ m length.

7 Comparison of welded and rolled I-sections

The comparison of rolled and welded I-beams is performed for compression and bending. Rolled cross section sizes are selected from the catalogue of ARBED.

The compression force is as follows:

$$F \le A \cdot \chi \cdot \frac{f_y}{\gamma_{M1}} \tag{56}$$

where A is the cross-section of I-beam.

$$\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \bar{\lambda}^2}} \tag{57}$$

is the factor for overall buckling,

$$\Phi = 0.5 \cdot \left[1 + 0.49\left(\bar{\lambda} - 0.2\right) + \bar{\lambda}^2\right] \tag{58}$$

$\overline{\lambda_Z} = \frac{L}{i_Z \cdot \lambda_E}$ is the reduced slenderness,

$$\lambda_E = \pi \cdot \sqrt{\frac{E}{f_y}}, \tag{59}$$

$f_y = 235$ MPa is the yield stress,

$\gamma_{M1} = 1.1$ is the partial safety factor according to Eurocode 3.

L is the length.

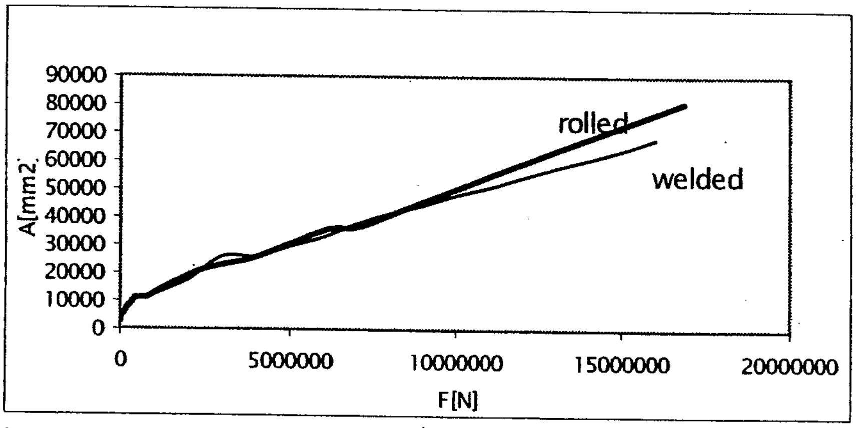

Figure 6. Optimum cross-section areas of 3 m long welded and rolled compressed I-sections in the function of concentrated force

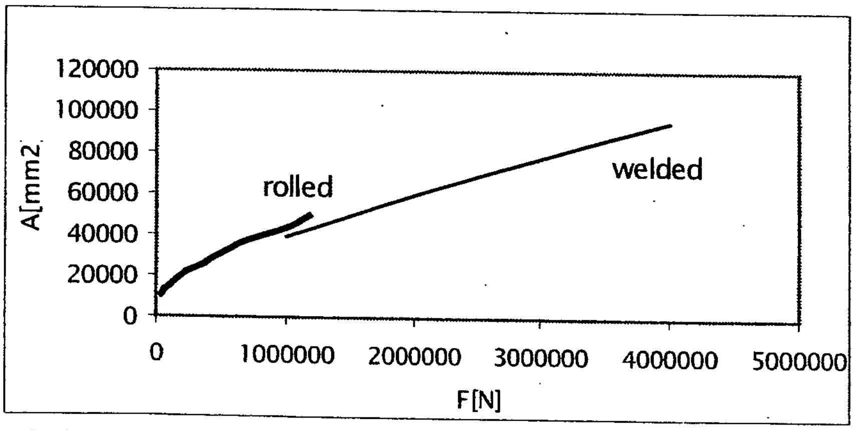

Figure 7. Optimum cross-section areas of 12 m long welded and rolled compressed I-sections in the function of concentrated force

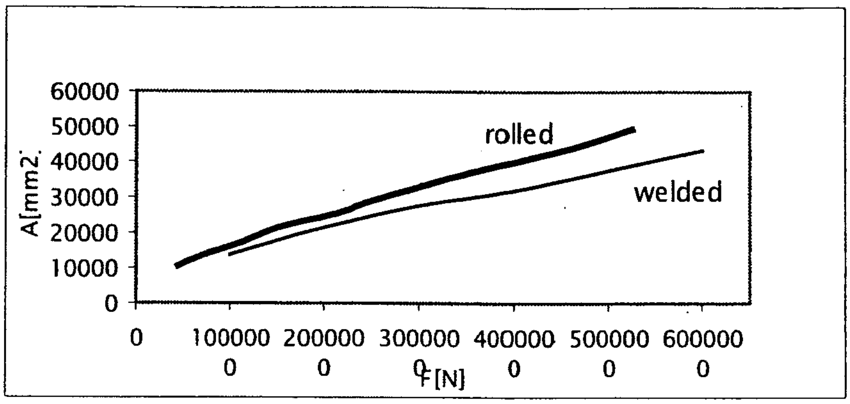

Figure 8. Optimum cross-section areas of 3 m long welded and rolled I-sections under bending in the function of concentrated force

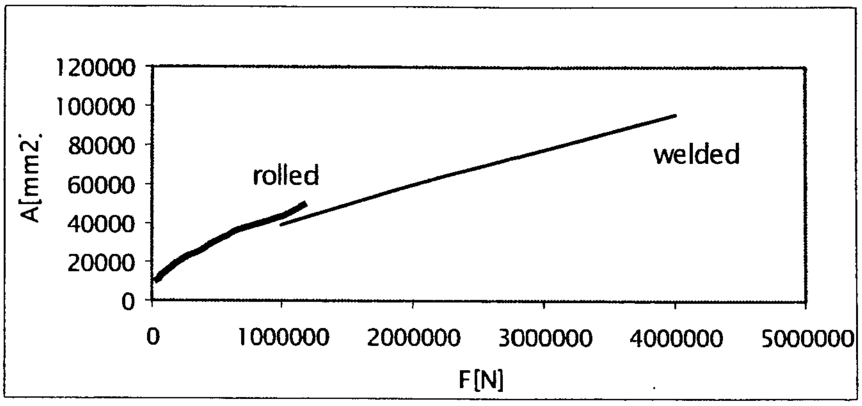

Figure 9. Optimum cross-section areas of 12 m long welded and rolled I-sections under bending in the function of concentrated force

In the case of beams under bending the force is as follows:

for concentrated force: $$F \leq \frac{Wxel \cdot \chi_{LT} \cdot f_y \cdot 4}{L \cdot \gamma_{M1}} \qquad (60)$$

for uniformly distributed force: $$p \leq \frac{Wxel \cdot \chi_{LT} \cdot f_y \cdot 8}{L^2 \cdot \gamma_{M1}}$$

8 Conclusions

The optimization technique many times gave local minima. Using other starting points we can avoid this problem.

For the compressed columns the active constraints at 1000 kN compression force were the local buckling constraints, the overall buckling and the lateral-torsional buckling was inactive. Increasing the compression force local buckling constraint of flange became passive, because of the increase of flange thickness. At 4000 kN compression force the overall buckling became active up to 14000 kN. From 15000 kN the lateral-torsional buckling constraint is active.

In the optimization of beams under bending, for smaller loads (uniformly distributed, or concentrated) the lateral-torsional constraint is active, for larger loads the buckling constraints are active. In the case of beams optimized for combined bending and compression the overall and lateral-torsional buckling constraints were active.

In the case of compressed columns the cost difference at 1000 kN and 2000 kN is about 16-20%. Increasing the force the difference becomes smaller: for 15000 kN and 16000 kN forces the cost difference is only 5-6%. Increasing the forces to double the difference in cost is about 25-30% between 3000 kN-6000 kN, and 33-42% between 8000 kN-16000 kN.

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