STABILITY AND DUCTILITY OF STEEL STRUCTURES

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COST MINIMIZATION OF LONGITUDINALLY STIFFENED PLATES LOADED BY UNIAXIAL COMPRESSION AND LATERAL PRESSURE

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ABSTRACT

The elastic secondary deflection due to compression and lateral pressure is calculated using the Paik's solution of the differential equation for orthotropic plates. Besides this deflection deformations arise due to lateral pressure and the shrinkage of longitudinal welds. The stress constraint includes three effects as follows: the average compression stress, the stress due to bending from lateral pressure and the stress due to bending moment, which is calculated as the average compression stress multiplied by the sum of deflections. Trapezoidal stiffeners are used to avoid the tripping of open section ribs. The constraint on local buckling of the base plate strips is also included considering the effect of initial imperfections and residual welding stresses with formulae proposed by Mikami and Niwa. The unknowns are the thickness of the base plate as well as the dimensions and number of stiffeners. The cost function to be minimized includes material and welding costs.

KEYWORDS

Stiffened plates, welded structures, stability, cost calculation, structural optimization, residual welding distortions

INTRODUCTION

Welded stiffened plates are widely used in many structures such as bridges, ships, roofs, bunkers, etc. Many studies have been published in the field of strength, stability, design, fabrication and optimization of such structures. Various types of plate geometry, loadings, stiffener shapes have been investigated. The strength and design in the case of compression and lateral pressure has been dealt with e.g. by Mansour (1971), Smith et al (1992), Davidson et al (1992), Bonello et al (1993), Mikami & Niwa (1996), Paik et al (2001) and Paik & Kim (2002).

Structural optimization of stiffened plates has been worked out by Farkas (1984), Farkas & Jármai (1997) and applied to bridge decks (Jármai et. al. 1998), bunkers (Farkas & Jármai 2001), uniaxially compressed plates with stiffeners of various shapes (Farkas & Jármai 2000), biaxially compressed plates (Farkas et. al. 2001).

In the present paper the minimum cost design is dealt with for longitudinally stiffened plates using the strength calculation methods of Mikami & Niwa (1996) and that of Paik et al (2001). Deflections due to lateral pressure, compression stress and shrinkage of longitudinal welds are taken into account in the stress constraint. Trapezoidal stiffeners are used to avoid the tripping of open section ribs. The local buckling constraint of the base plate strips is formulated as well. The cost function includes material and welding costs. The unknowns are the thickness of the base plate as well as the dimensions and number of stiffeners.

GEOMETRIC CHARACTERISTICS OF THE STIFFENED PLATE

The stiffened plate is shown in Figure 1. A part of the plate with a trapezoidal stiffener can be seen in Figure 2. The geometric characteristics of this cross-section are as follows.

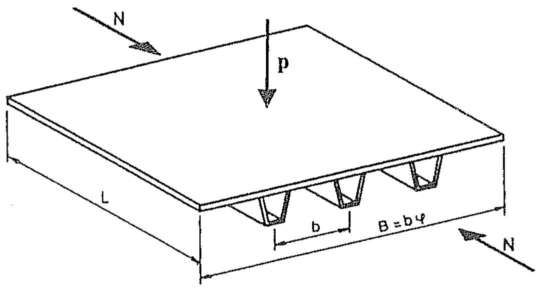

Figure 1. Longitudinally stiffened plate loaded by uniaxial compression and lateral pressure

$$A_S = (a_1 + 2a_2)t_S \ ; \qquad I_S = a_1 h_S^3 t_S + \frac{2}{3} a_2^3 t_S \sin^2\alpha \tag{1}$$

$a_1 = 90$, $a_3 = 300$ mm, thus

$$h_S = \left(a_2^2 - 105^2\right)^{1/2} \ ; \qquad \sin^2\alpha = 1 - \left(\frac{105}{a_2}\right)^2 \tag{2}$$

$$y_G = \frac{a_1 t_S (h_S + t_F/2) + 2a_2 t_S (h_S + t_F)/2}{b t_F + A_S} \tag{3}$$

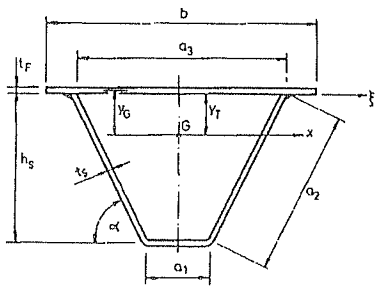

Figure 2. Dimensions of a trapezoidal stiffener

$$I_x = \frac{bt_F^3}{12} + bt_F y_G^2 + a_1 t_S \left(h_S + \frac{t_F}{2} - y_G \right)^2 + \frac{1}{6} a_2^3 t_S \sin^2 \alpha + 2a_2 t_S \left(\frac{h_S + t_F}{2} - y_G \right)^2 \tag{4}$$

The fillet weld size is

$a_W = 0.5t_S$, but $a_{Wmin} = 4$ mm.

Local buckling of a trapezoidal stiffener is defined as

$$a_2 / t_S \leq 38\varepsilon ; \qquad \varepsilon = \sqrt{235 / f_y} \tag{5}$$

f_y is the yield stress.
This constraint is treated as active, thus, the only unknown dimension of stiffener is the thickness t_S.

CALCULATION OF THE DEFLECTION DUE TO COMPRESSION AND LATERAL PRESSURE

Paik et al (2001) have used the differential equations of large deflection orthotropic plate theory and the Galerkin method to derive the following cubic equation for the elastic deflection A_m of a stiffened plate loaded by uniaxial compression and lateral pressure

$$C_1 A_m^3 + C_2 A_m^2 + C_3 A_m + C_4 = 0 \tag{6}$$

where

$$C_1 = \frac{\pi^2}{16}\left(E_x \frac{m^4 B}{L^3} + E \frac{L}{B^3} \right); \qquad C_2 = \frac{3\pi^2 A_{om}}{16}\left(E_x \frac{m^4 B}{L^3} + E \frac{L}{B^3} \right)$$

$$C_3 = \frac{\pi^2 A_{om}^2}{8}\left(E_x \frac{m^4 B}{L^3} + E \frac{L}{B^3} \right) + \frac{m^2 B}{L}\sigma_{xav} + \frac{\pi^2}{t_F}\left(D_x \frac{m^4 B}{L^3} + 2H \frac{m^2}{LB} + D \frac{L}{B^3} \right)$$

$$C_4 = A_{om} \frac{m^2 B}{L} \sigma_{xav} - \frac{16LB}{\pi^4 t_F} p$$

$$E_x = E\left(1 + \frac{nA_S}{Bt_F} \right); \qquad E_y = E \tag{7}$$

The number of stiffeners is $n = \varphi - 1$.

The flexural and torsional stiffnesses of the orthotropic plate are as follows:

$$D_x = \frac{Et_F^3}{12\left(1 - \nu_{xy}^2\right)} + \frac{Et_F y_G^2}{1 - \nu_{xy}^2} + \frac{EI_x}{b}; D_y = \frac{Et_F^3}{12\left(1 - \nu_{xy}^2\right)} \tag{8}$$

$$\nu_x = \frac{\nu}{0.86}\sqrt{\frac{\frac{E}{E_x}\left(\frac{Et_F^3}{12} + Et_F y_G^2 + \frac{EI_x}{b} \right) - \frac{Et_F^3}{12}}{\frac{EI_x}{b}\left(\frac{E}{E_x} \right)^2}} \tag{9}$$

$$\nu_y = \frac{E}{E_x}\nu_x ; \quad \nu_{xy} = \sqrt{\nu_x \nu_y} \tag{10}$$

$$H = \frac{G_{xy} I_t}{b}; G_{xy} = \frac{E}{2(1+\nu_{xy})}; I_t = \frac{4A_P^2}{\sum b_i / t_i}; A_P = h_S \frac{a_1 + a_3}{2} = 195 h_S \tag{11}$$

$$\sum \frac{b_i}{t_i} = \frac{a_1 + 2a_2}{t_S} + \frac{a_3}{t_F} \tag{12}$$

The deflection due to lateral pressure is

$$A_{om} = \frac{5qL^4}{384EI_x}; \quad q = pb; \quad b = B / \varphi \tag{13}$$

The average compression stress is

$$\sigma_{xav} = \frac{N}{Bt_F + (\varphi - 1)A_S} \tag{14}$$

It is assumed that, for a simply supported plate, the largest value of A_m is given by the smallest number of half buckling length $m = 1$.

The solution of Eqn.6. is

$$A_m = -\frac{C_2}{3C_1} + k_1 + k_2 \tag{15}$$

where

$$k_1 = \sqrt[3]{-\frac{Y}{2} + \sqrt{\frac{Y^2}{4} + \frac{X^3}{27}}}; k_2 = \sqrt[3]{-\frac{Y}{2} - \sqrt{\frac{Y^2}{4} + \frac{X^3}{27}}} \tag{16}$$

$$X = \frac{C_3}{C_1} - \frac{C_2^2}{3C_1^2}; Y = \frac{2C_2^3}{27C_1^3} - \frac{C_2 C_3}{3C_1^2} + \frac{C_4}{C_1} \tag{17}$$

DEFLECTION DUE TO SHRINKAGE OF LONGITUDINAL WELDS

According to Farkas & Jármai (1997) or Jármai & Farkas (1999)

$$f_{\max} = CL^2 / 8 \tag{18}$$

where the curvature for steels is

$$C = 0.844x10^{-3} Q_T y_T / I_x \tag{19}$$

Q_T is the heat input, y_T is the weld eccentricity

$$y_T = y_G - t_F / 2 \tag{20}$$

I_x is the moment of inertia of the cross-section containing a stiffener and the base plate strip of width b.

The heat input for a stiffener is

$$Q_T = 2x59.5a_W^2 \tag{21}$$

THE STRESS CONSTRAINT

$$\sigma_{\max} = \sigma_{xav} + \frac{M}{I_x} y_G \le \sigma_{UP} \tag{22}$$

where

$$M = \sigma_{xav}\left(A_{0m} + A_m + f_{\max}\right) + \frac{qL^2}{8} \tag{23}$$

According to Mikami & Niwa (1996), the calculation of the local buckling strength of a face plate strip of width

$$b_l = \max\,(a_3,\, b - a_3)$$

is performed taking into account the effect of initial imperfections and residual welding stresses

$$\sigma_{UP} = f_y \qquad \text{when} \qquad \lambda_p \le 0.526 \tag{24a}$$

$$\sigma_{UP} = \left(\frac{0.526}{\lambda_p}\right)^{0.7} \qquad \text{when} \qquad \lambda_p \ge 0.526 \tag{24b}$$

where

$$\lambda_p = \left(\frac{4\pi^2 E}{10.92 f_y}\right)^{1/2} \frac{b_1}{t_F} = \frac{b_1 / t_F}{56.8\varepsilon} \tag{25}$$

THE COST FUNCTION

The objective function to be minimized is defined as the sum of material and fabrication costs

$$K = K_m + K_f = k_m \rho V + k_f \sum T_i \tag{26}$$

or in another form

$$\frac{K}{k_m} = \rho V + \frac{k_f}{k_m}\left(T_1 + T_2 + T_3\right) \tag{27}$$

where ρ is the material density, V is the volume of the structure, K_m and K_f as well as k_m and k_f are the material and fabrication costs as well as cost factors, respectively, T_i are the fabrication times as follows:
time for preparation, tacking and assembly

$$T_1 = \Theta_d \sqrt{\kappa \rho V} \tag{28}$$

where Θ_d is a difficulty factor expressing the complexity of the welded structure, κ is the number of structural parts to be assembled;
T_2 is time of welding, and T_3 is time of additional works such as changing of electrode, deslagging and chipping. $T_3 \approx 0.3T_2$, thus,

$$T_2 + T_3 = 1.3 \sum C_{2i} a_{wi}^n L_{wi} \tag{29}$$

where L_{wi} is the length of welds, the values of $C_{2i} a_{wi}^n$ can be obtained from formulae or diagrams constructed using the COSTCOMP software (Bodt 1990, COSTCOMP 1990, Farkas & Jármai 1997, Jármai & Farkas 1999), a_w is the weld dimension.

In our case, the volume of the structure is

$$V = BLt_F + (\varphi - 1) A_S L, \tag{30}$$

the weld length is

$$L_W = 2(\varphi - 1)L \tag{31}$$

and for SAW (Submerged Arc Welding) fillet welds it is

$$C_W a_W^n = 0.3258x10^{-3} a_W^2 \tag{32}$$

The optima are calculated for $k_F/k_M = 0$ and 1.5 kg/min. $k_F/k_M = 0$ corresponds to minimum mass design.

NUMERICAL DATA

$B = 4000$, $L = 6000$ mm, $N = 1.974x107$ [N], $p = 0.2$ MPa, $f_y = 355$ MPa, $E = 2.1x10^5$ MPa,

$\rho = 7.85x10^{-6} kg/mm^3$.

THE OPTIMIZATION PROCEDURE AND RESULTS

For the constrained function minimization the following two mathematical methods are used. The *Rosenbrock's method* (Rosenbrock 1960), which has been modified to be able to handle discrete values. This is a direct search mathematical programming method without derivatives. Instead of continually searching in the co-ordinate space corresponding to the directions of the independent variables, the method achieves an improvement after one cycle of co-ordinate searches by lining the search directions up into an orthogonal system, with the overall step of the previous stage as the first building block for the new set of orthogonal directions.

The dynamic trajectory, more commonly known as the *leap-frog method* (Snyman 1982, 2000), was originally proposed for the unconstrained minimization of a scalar function. The algorithm has recently been modified to handle constraints by means of a penalty function formulation. The method seeks *low local minima* and can thus be used as a basic component in a methodology for global optimization. The method usually converges very quickly to the neighbourhood of the optimum. This is because the fundamental physical principles underlying the method, ensures controlled and stable convergence along a dynamic trajectory towards the optimum.

Since the stiffener thickness is limited to t_{Smax} =10 mm regarding the cold-forming, the optima for t_F are determined for constant t_S and for stepwise changed discrete values of φ. The results are shown in Table 1. The optima are marked by bold letters.

TABLE 1
Optimum values of unknowns and cost data

			K/k_m (kg)	
φ	t_S (mm)	t_F (mm)	k_F/k_m=0	k_F/k_m=1.5
8	10	20	6103	8431
7	10	21	5958	8020
6	10	22	5812	7606
5	**10**	**22**	**5479**	**6986**
4	10	26	5899	7162
3	10	31	6507	7517

It can be seen from the Table 1 that the optimum values marked by bold letters give the minimum mass and minimum cost solutions. It should be mentioned that the mass and cost values corresponding to the optima in the case of t_S = 6 or 8 mm are larger that those for t_S = 10 mm.

CONCLUSIONS

The secondary deflection due to compression and lateral pressure is calculated using the solution of the differential equation of orthotropic plate theory. In the stress constraint the effect of average compression stress, lateral pressure and deflection due to shrinkage of longitudinal eccentric welds are taken into account. The local buckling strength of the faceplate strips is incorporated into the stress constraint as well. The cost function is formulated considering material and fabrication costs. The welding cost includes the times necessary for assembly, tacking, welding, deslagging, electrode changing and chipping.

It can be seen from Table 1 that the cost difference between the best and worst solution in the investigated region of stiffener numbers is 100(8431-6986)/6986 = 21%, thus, it is worth to use an optimization procedure to find the most economic solution.

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