Volume and cost minimization of a tubular truss with displacement-constraint

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ABSTRACT

The minimum volume and cost of a simply supported planar truss with N-type bracing is optimized. The lower chord of the truss is horizontal, but the symmetric upper chord parts are non-parallel and their inclination angle as well as the cross-sectional area of CHS (circular hollow section) rods are optimized. For the calculation of required cross-sectional area of compression struts closed formulae are used as a good approximation of Eurocode 3 buckling curve. A special method is developed for the minimum volume design considering the deflection constraint. In the case of a strong displacement constraint the cross-sectional areas required for the allowed deflection are larger than those required for stress and buckling constraints. The cost function includes the cost of material, cutting and grinding of CHS strut ends, assembly, welding and painting. Special mathematical methods are used to find the optima in the case of a numerical problem.

Keywords: tubular truss, structural optimization, overall buckling, displacement constraint, minimum cost design

1. INTRODUCTION

Tubular trusses are applied in many load-carrying structures such as roofs, bridges, offshore platforms, towers etc. A literature survey of the optimum design of trusses is given in Section 2.

The aim of the present study is to show the minimum volume and cost optima of a truss and solve the optimum design problem subject to a strong displacement constraint. In the case of stress constraints the tension rods are designed for yield stress by using a safety factor for loading and the compression rods are designed for overall buckling. In the case of a strong displacement constraint the required cross-sectional areas are larger than those required for stress constraints.

In the optimum design process of a truss the optimal value of the cross-sectional areas of struts and the geometric characteristics of the truss are sought which minimize an objective function and fulfil the design and fabrication constraints. The objective function can be the volume (weight) or cost, the design constraints are the limitation of stress and displacement, the fabrication constraints ease the manufacturing (welding) process.

In the case of an active displacement constraint a special method is developed to calculate the required cross-sectional areas and the truss geometry. This method is derived in details in Section 3.

It is shown that the non-parallel chords are more economic than the beam with parallel chords. Thus, in our case the angle of the upper chord (unknowns h_9 and h_{13} in Figure 1) is optimized.

Another problem is the grouping of rods having the same cross-sectional area. The design of all the rods having different cross-sectional areas can cause difficulties in fabrication, but the design of all the rods with the same cross-sectional area would be uneconomic. Thus, the economy depends on grouping of rods. In our case four groups are used.

For the minimization of the structural volume or cost, minimization of the cross-sectional areas of rods is needed. The cross-sectional area of compression rods cannot be calculated from the Eurocode 3 buckling formulae. Therefore approximate formulae of Japan Road Association are used. Stress and buckling constraints are calculated using factored forces, whilst the deflection is calculated with forces without a safety factor.

To obtain comparable optima the required cross-sectional areas are not rounded to available profiles and the most economic $\delta = D/t = 50$ slenderness (diameter/thickness) of CHS is used. The limitation of the angle between CHS struts (minimum 30^0) is taken into account as a fabrication constraint. Another fabrication constraint is that the diameters of the chords should be larger than those of verticals and diagonals of the bracing.

The effect of self mass is neglected in this comparative study.

2. SURVEY OF SELECTED LITERATURE

In order to illustrate the literature of the optimum design of trusses, the characteristics of some articles are summarized in Table 1.

Table 1. Literature survey of selected journal articles about the optimization of trusses

Abbreviations: AISC American Institute of Steel Construction, CHS Circular Hollow Section, AASHTO American Assoc. of State and Highway Transportation Officials, EC3 Eurocode 3 (EN 1993-1-1: 1992), W – American wide flange beam, PSO particle swarm optimizer, ACO ant colony strategy, HS harmony search, MINLP mixed-integer nonlinear programming, alum - aluminium

Author(s)	Examples	Math. Method	Mate-rial	Buckling calculation	Cross-section	Constraint
Gil (2001)	non-parallel chords	conjugates gradient	steel	EC3		stress and geometrical

Tong (2001)	10-,25-bar	combinatorial	alum.			stress and fundamental frequency
Makris (2002)	3-,10-,25-,60- and 132-bar	strain-energy-density	alum.	no buckling		displacement
Hasancebi (2002)	224-bar 3D pyramid, simply supported	simulated annealing	steel	AISC	CHS, W-section	layout optimization
Kripakaran (2007)	10-,18-,21-bar	new algorithm	steel, alum.	AASHTO, Euler	CHS	minimum cost
Lamberti (2008)	18-bar cantilever, 25-bar 3D, 45-,72- and 200-bar	simulated annealing	steel, alum.	Euler		stress, nodal displacement
Silih (2008)	non-parallel chords	MINLP	steel	EC3	CHS	minimum mass or cost
Kaveh (2009)	10-,25-,120-200-, and 244-bar transmission tower	PSO, ACO,HS	steel, alum.	AISC		stress, nodal displacements
Jármai (2004)	Simply supported, parallel chords, 5, 8 spacing	Leap-frog, dynamic-Q	Steel	EC3	CHS	optimum height, effect of loads, min. volume

Remarks:

(1) In trusses the compression members should be designed against overall buckling. The use of Euler-formula gives unsafe design, since it does not take into account the effect of initial imperfections and residual stresses. Therefore buckling formulae of Eurocode 3 or another up-to-date improved buckling formulae should be used.

(2) The type of the investigated cross section should be given, since it has been shown (Farkas & Jármai 1997) that the cross-sectional form affects the optima significantly.

3. MINIMUM VOLUME DESIGN OF THE TUBULAR TRUSS WITH NON-PARALLEL CHORDS

Relatively simple formulae can be derived for trusses to minimize the structural volume and fulfil a displacement constraint.
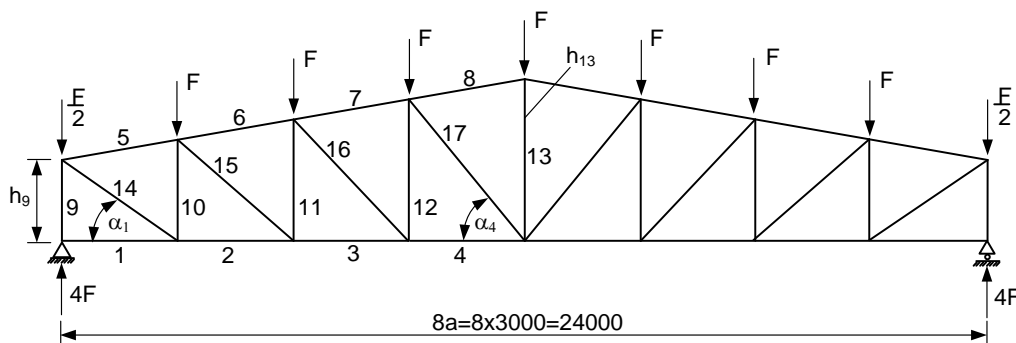

Figure 1. The simply supported truss with non-parallel chords

The truss rods are divided into n-groups having the same cross-sectional areas (A_i), so

$$A_i = \mu_i A, \qquad i = 1\ldots n, \tag{1}$$

where μ_i are multipliers and the displacement constraint is given by

$$w = \frac{1}{EA} \sum_i \frac{S_i s_i L_i}{\mu_i} \leq w_0, \tag{2}$$

where E is the elastic modulus, S_i is the rod force, s_i is the rod force from the unit force acting at the midspan, L_i is the rod length, w_o is the admissible deflection.

From Eqn (2) one obtains

$$A \geq \frac{1}{Ew_0} \sum_i \frac{S_i s_i L_i}{\mu_i}.$$ (3)

The structural volume is calculated as

$$V = \sum_i A_i L_i = \frac{1}{Ew_0} \sum_i \mu_i L_i \sum_i \frac{S_i s_i L_i}{\mu_i} = \frac{v_1 v_2}{Ew_0},$$ (4)

where v_i parameters are

$$v_1 = \sum_i \mu_i L_i, \quad v_2 = \sum_i \frac{S_i s_i L_i}{\mu_i}.$$ (5)

In the minimum volume design the truss geometry is sought, which minimizes

$$V_1 = v_1 v_2.$$ (6)

In the case of the simply supported truss shown in Figure 1 the spacing is constant, the non-parallel upper chord is determined by variable heights h_9 and h_{13}. The truss is subject to a set of vertical static forces F acting on the upper nodes. The displacement of the central lower node is prescribed. It is supposed that all the truss nodes are restrained against transverse deformation.

The variables to be optimized are the heights h_9 and h_{13} as well as the cross sectional areas of rods (A and μ_i).

The calculations show that, in the case of a strong displacement constraint the necessary rod cross-sectional areas are so large that the stress constraints on tension and overall buckling are fulfilled. In spite of this fact these constraints should be checked.

To facilitate the welding of nodes for tubular trusses a geometric fabrication constraint should be considered that the minimal angle between rods should be equal or greater than 30^0, in our case (Figure 1)

$$tan\,\alpha_1 = \frac{h_9}{a} \geq tan\,30^0, \qquad (7)$$

where a is the distance between columns, from which

$$h_9 \geq a\,tan\,30^0 = 1732\,mm, \qquad (8)$$

and

$$tan\,\alpha_4 \leq 60^0 = \frac{\pi}{3}. \qquad (9)$$

In our case these constraints are always active.

The rod forces and lengths (S_i, s_i, L_i) are expressed in function of h_9 and the inclination angle of the upper chords α.

$$tan\,\alpha = \frac{h_{13} - h_9}{4a}, cos\,\alpha = \frac{1}{\sqrt{(tan\,\alpha)^2 + 1}}, sin\,\alpha = \sqrt{1 - (cos\,\alpha)^2}. \qquad (10)$$

The formulae for S_i, s_i and L_i are given in Tables 2, 3, 4 and 5.

Table 2. Characteristics of rods in the lower chord

i	S_i	s_i	L_i
1	0	0	A
2	$3.5Fa/h_{10}$	$0.5a/h_{10}$	A
3	$6Fa/h_{11}$	a/h_{11}	A
4	$7.5Fa/h_{12}$	$1.5a/h_{12}$	A

Table 3. Characteristics of rods in the upper chord

I	S_i	s_i	L_i
5	$\dfrac{3.5Fa}{h_{10}\cos\alpha}$	$\dfrac{0.5a}{h_{10}\cos\alpha}$	$\dfrac{a}{\cos\alpha}$
6	$\dfrac{6Fa}{h_{11}\cos\alpha}$	$\dfrac{a}{h_{11}\cos\alpha}$	$\dfrac{a}{\cos\alpha}$
7	$\dfrac{7.5Fa}{h_{12}\cos\alpha}$	$\dfrac{1.5a}{h_{12}\cos\alpha}$	$\dfrac{a}{\cos\alpha}$
8	$\dfrac{8Fa}{h_{13}\cos\alpha}$	$\dfrac{2a}{h_{13}\cos\alpha}$	$\dfrac{a}{\cos\alpha}$

Table 4. Characteristics of verticals

i	S_i	s_i	L_i
9	$4F$	0.5	h_9
10	$-3.5F+S_5\sin\alpha$	$-0.5+s_5\sin\alpha$	$h_{10}=h_9+a\tan\alpha$
11	$-2.5F+S_6\sin\alpha$	$-0.5+s_6\sin\alpha$	$h_{11}=h_9+2a\tan\alpha$
12	$-1.5F+S_7\sin\alpha$	$-0.5+s_7\sin\alpha$	$h_{12}=h_9+3a\tan\alpha$
13	$-F+2S_8\sin\alpha$	$2s_8\sin\alpha$	h_{13}

Table 5. Characteristics of diagonals

i	S_i	s_i	L_i
14	$S_5 L_{14}\cos\alpha/a$	$s_5 L_{14}\cos\alpha/a$	$\sqrt{h_9^2 + a^2}$
15	$(2.5F - S_6\sin\alpha)L_{15}/h_{10}$	$(0.5 - s_6\sin\alpha)L_{15}/h_{10}$	$\sqrt{h_{10}^2 + a^2}$
16	$(1.5F - S_7\sin\alpha)L_{16}/h_{11}$	$(0.5 - s_7\sin\alpha)L_{16}/h_{11}$	$\sqrt{h_{11}^2 + a^2}$
17	$(0.5F - S_8\sin\alpha)L_{17}/h_{12}$	$(0.5 - s_8\sin\alpha)L_{17}/h_{12}$	$\sqrt{h_{12}^2 + a^2}$

The rods are divided into four groups having the same cross-section): lower chord (1,2,3,4), upper chord (5,6,7,8), verticals (9,10,11,12,13) and diagonals (14,15,16,17).

In order to facilitate the fabrication, the lower and upper chords have the same cross-section ($\mu_1 = \mu_2 = 1$) and the optimal values of μ_3 (multiplier for verticals) and μ_4 (multiplier for diagonals) are sought, which should be smaller than μ_1.

The components of $V_1 = v_1 v_2$ to be minimized are as follows.

$$v_1 = 8(a + L_7) + 2\mu_3 \sum_{i=9}^{12} h_i + \mu_3 h_{13} + 2\mu_4 \sum_{i=14}^{17} L_i \,, \tag{11}$$

$$v_2 = 2a \sum_{i=2}^{4} S_i s_i + 2L_7 \sum_{i=5}^{8} S_i s_i + \frac{2}{\mu_3} \sum_{i=9}^{12} S_i s_i h_i + \frac{S_{13} s_{13} h_{13}}{\mu_3} + \frac{2}{\mu_4} \sum_{i=14}^{17} S_i s_i L_i \,, \tag{12}$$

With the optimum values of h_9, h_{13}, μ_3 and μ_4

$$A_1 = A_2 = \frac{v_{2opt}}{Ew_{adm}}, \quad A_3 = \mu_{3opt}A_1, \quad A_4 = \mu_{4opt}A_1 \,. \tag{13}$$

The minimum structural volume is

$$V_{min} = v_1 A_1. \tag{14}$$

For a circular hollow section (CHS of diameter D and thickness t)

$$A = \pi D t = \pi D^2 / \delta, \delta = D / t \,, \tag{15}$$

from which

$$D = \sqrt{\frac{A\delta}{\pi}}, \quad t = \frac{D}{\delta} \,. \tag{16}$$

In the design we should use the maximum value of δ, but it is limited to 50 (Wardenier et al. 1991). In the case of available CHS profiles according to (EN 10210-2. 2006) δ is varied between 10-50. In order to obtain realistic optima in all cases the optimum $\delta = 50$ is used.

4. CHECK OF THE COMPRESSION RODS FOR OVERALL BUCKLING

For checking the overall buckling, the approximate formulae of the Japan Road Association (JRA) (Hasegawa et al. 1985) can be used instead of EC3 curve (b). In this case closed formulae can be given for cross-sectional sizes.

$$N / A \le \chi f_y, \tag{17}$$

$$\chi = 1 \quad \text{for} \quad 0 \le \bar{\lambda} \le 0.2 \,, \tag{18a}$$

$$\chi = 1.109 - 0.545\bar{\lambda} \quad \text{for} \quad 0.2 \le \bar{\lambda} \le 1, \tag{18b}$$

$$\chi = \frac{1}{0.773 + \bar{\lambda}^2} \quad \text{for} \quad \bar{\lambda} \ge 1, \tag{18c}$$

where f_y is the yield stress, N is the compression force, χ is the overall buckling parameter.

Introducing the symbol

$$\vartheta = 100 D / L \ , \tag{19}$$

and using $\overline{\lambda} = c / \vartheta$ the closed formulae are as follows.

For $0.2\vartheta \le c \le \vartheta$

$$\vartheta = 0.24572c \left[1 + \sqrt{1 + \frac{14.93475v}{c^2}} \right] , \tag{20a}$$

and for $\vartheta \le c$

$$\vartheta = \left\{ 0.3865v \left[1 + \sqrt{1 + \frac{6.69424c^2}{v}} \right] \right\}^{1/2} , \tag{20b}$$

for CHS

$$c = \frac{100K\sqrt{8}}{\lambda_E}, \quad v = \frac{10^4 S}{L^2} \cdot \frac{\delta}{\pi f_y} , \tag{21}$$

where the limiting value of $\delta = D/t = 50$ is used

$$D = \frac{\vartheta L}{100} . \tag{22}$$

In the case of very long struts with small compressive force, the limitation of the strut slenderness can be governing. From the limitation of

$$\lambda = K_R L / r \le \lambda_{max}, \tag{23}$$

the required radius of gyration is

$$r \ge K_R L / \lambda_{max} . \tag{24}$$

According to (BS 5400 1983) $\lambda_{max} = 180$.

K_R is the strut end restraint factor, for chords $K_R = 0.9$, for verticals and diagonals $K_R = 0.75$ (Rondal et al. 1992).

For the check of overall buckling the following constraint should be fulfilled for all compression rods

$$A_i \geq \frac{\pi D_i}{\delta},$$
(25)

where A_i is the optimum cross-sectional area for displacement constraint and D_i is the required diameter from overall buckling calculation.

5. THE COST FUNCTION

The cost function contains the cost of material, cutting and grinding of CHS strut ends, assembly, welding and painting.

The cost of material is given by

$$K_M = k_M \rho V_2,$$
(26)

where an average specific cost of $k_M = 1.0$ \$/kg is considered, $\rho = 7.85 \times 10^{-6}$ kg/mm^3 for steel. V_2 is the actual structural volume (see Eqn (35)).

The cost of cutting and grinding of CHS strut ends is calculated with a formula proposed by Glijnis (Farkas & Jármai 2003).

$$K_{CG}(\$) = k_F \Theta_{CG} \frac{2.5 \pi D}{(350 - 2t)0.3 \sin \alpha},$$
(27)

where $k_F = 1.0$ \$/min is the specific fabrication cost, $\Theta_{CG} = 3$ is a factor for work complexity, 350 mm/min is the cutting speed, 0.3 is the efficiency factor, diameter D and thickness t are in mm, α is the inclination angle of diagonal braces.

In our case for verticals

$$K_{CG} = \Theta_{CG} 2.5 \pi 9 D_3 \frac{1 + \dfrac{1}{\cos \alpha}}{(350 - 2t_3)0.3}.$$
(28)

For diagonals at the lower strut ends

$$K_{CG1} = \Theta_{CG} 2.5\pi 2D_4 \frac{\sum_{i=1}^{4} \frac{1}{\cos \alpha_i}}{(350 - 2t_4)0.3}, \tag{29}$$

where

$$\tan \alpha_1 = h_9/a, \quad \tan \alpha_2 = h_{10}/a, \quad \tan \alpha_3 = h_{11}/a, \quad \tan \alpha_{.4} = h_{12}/a. \tag{30}$$

For diagonals at the upper strut ends

$$K_{CG2} = \Theta_{CG} 2.5\pi 2D_4 \frac{\sum_{i=1}^{4} \frac{1}{\cos \beta_i}}{(350 - 2t_4)0.3}, \tag{31}$$

where

$$\beta_i = 90^0 - \alpha - \alpha_i, \quad i = 1,2,3,4. \tag{32}$$

The general formula for the welding cost is as follows (Farkas & Jármai 1997, 2003, 2008).

$$K_w = k_w \left(C_1 \Theta \sqrt{\kappa \rho V} + 1.3 \sum_i C_{wi} a_{wi}^n C_{pi} L_{wi} \right), \tag{33}$$

where k_w [\$/min] is the welding cost factor, C_1 is the factor for the assembly usually taken as $C_1 = 1$ min/kg$^{0.5}$, Θ is the factor expressing the complexity of assembly, the first member calculates the time of the assembly, κ is the number of structural parts to be assembled, ρV is the mass of the assembled structure. The second member estimates the time of welding, C_w and n are the constants given for the specified welding technology and weld type.

Furthermore C_{pi} is the factor for the welding position (download 1, vertical 2, overhead 3), L_w is the weld length, the multiplier 1.3 takes into account the additional welding times (deslagging, chipping, changing the electrode, etc.).

In our case $k_w = 1.0$ \$/min, $\Theta = 3$,

the cost of assembly and welding using SMAW (shielded metal arc welding) fillet welds is given by for verticals

$$K_W = k_w \left[\Theta \sqrt{21 \rho V_2} + 1.3x0.7889x10^{-3} x9\pi D_3 \left(1 + \frac{1}{\cos \alpha}\right) t_3^2 \right], \tag{34}$$

$$V_2 = 8aA_1 + 8L_7A_2 + 2A_3\sum_{i=9}^{12}h_i + A_3h_{13} + 2A_4\sum_{i=14}^{17}L_i . \tag{35}$$

For diagonals at the lower strut ends

$$K_{W1} = 1.3x0.7889x10^{-3}x2\pi D_4 t_4^2 \sum_{i=1}^{4}\frac{1}{\cos\alpha_i} . \tag{36}$$

For diagonals at the upper strut ends

$$K_{W1} = 1.3x0.7889x10^{-3}x2\pi D_4 t_4^2 \sum_{i=1}^{4}\frac{1}{\cos\beta_i} . \tag{37}$$

The cost of painting is calculated as

$$K_P = k_P S_P, k_P = 28.8x10^{-6} \ \$/mm^2. \tag{38}$$

The superficies to be painted is

$$S_P = 8a\pi D_1 + 8L_7\pi D_2 + 2\pi D_3\sum_{i=9}^{12}h_i + \pi D_3 h_{13} + 2\pi D_4\sum_{i=14}^{17}L_i . \tag{39}$$

The total cost is given by

$$K = K_M + K_{CG} + K_{CG1} + K_{CG2} + K_W + K_{w1} + K_{w2} + K_P . \tag{40}$$

6. NUMERICAL DATA

Loads for displacement calculation (without safety factor) F = 120000 N, for stress and buckling constraints $F_0 = 1.5F = 180000$ N (safety factor of 1.5). Yield stress of steel $f_y = 355$ MPa, elastic modulus $E = 2.1x10^5$ MPa, span length $L = 24$ m, allowable displacement at the middle of the span $w_0 = 32$ mm = $L/750$.

7. THE OPTIMIZATION PROCESS

Calculate the optimum values of h_9, h_{13}, μ_3 and μ_4 to obtain V_{min} or K_{min} and fulfil the constraints on displacement, on minimum angle α_1 [Eqn. (8)], on maximum angle α_4 [Eqn. (9)] as well as on stress and overall buckling.

The ranges of unknowns are as follows: $1732 < h_9 < 5000$ mm, $4000 < h_{13} < 8000$ mm and $h_9 < h_{13}$, $0.5 < \mu_3 < 1$, $0.5 < \mu_4 < 1$.

In the case of minimum volume design Eqns. (13) and (14) give the results and Eqn. (25) should be fulfilled. In the case of minimum cost Eqn. (40) should be minimized, for which Eqns. (11), (12), (13), (16) and (35) should be used.

8. RESULTS OF THE OPTIMIZATION

The fabrication constraints [Eqn. (7) and (8)] determine the optimal pair of unknowns h_9 and h_{13} as follows: for a given h_9 a value of h_{13} smaller than h_{13opt} gives larger $v_1 v_2$, larger does not fulfil the fabrication constraint Eqn. (8). Table 6 shows the max h_{13} in function of h_9.

Table 6. Maximum h_{13} values in function of h_9. Values in mm

h_9	1750	1850	1950	2000	2100	2200	2300
h_{13opt}	6340	6310	6280	6260	6220	6190	6160

Furthermore the calculations show that the best value for μ_3 and μ_4 is 0.6, since the value of 0.5 gives cross-sections which do not fulfil the buckling constraints. Thus, the remaining unknown h_9 can be optimized using the MathCAD program. Table 7 gives the volume and cost in function of h_9. The optimum h_9 minimizes thee product $v_1 v_2$ (fulfilling the deflection constraint) and also V and K.

Table 7. Volume and cost in the function of h_9. h in mm. Optima are marked with bold letters

h_9	h_{13}	$v_1 v_2 \times 10^{-15}$	$V \times 10^{-8}$ mm^3	K \$
1750	6390	2.331	3.469	7854
1850	6310	2.324	3.459	7830
1950	**6280**	**2.321**	**3.454**	**7825**
2000	6260	2.322	3.456	7829
2100	6220	2.327	3.463	7843

Table 7 shows that the following optima are determined: in the case of $\mu_3 = \mu_4 = 0.6$, $h_{9opt} = 1950$, $h_{13opt} = 6280$, $v_1 v_{2min} = 2.321 \times 10^{15}$, $V_{min} = 3.454 \times 10^8$ mm^3, $K_{min} = 7825$ \$, $A_1 = A_2 = 3708$, $A_3 = A_4 = 2225$ mm^2. Table 7 shows that the sensitivity of V and K is small.

The cross-sectional areas required for stress and buckling constraints are as follows: $A_1 = A_2 = 2195$, $A_3 = 2084$, $A_4 = 2094$ mm^2. It can be seen that the cross-sectional areas determined for a strong displacement constraint are larger than those required for stress or buckling constraints.

In addition the calculation results for $\mu_3 = 0.7$ and $\mu_4 = 0.5$ are given.

Table 8. Results in the case of $\mu_3 = 0.7$ and $\mu_4 = 0.5$. Optimum is marked by bold letters

h_9	h_{13}	$v_1 v_2 \times 10^{-15}$
1750	6390	2.335
1850	6310	2.329
1950	**6280**	**2.326**
2000	6260	2.328
2100	6220	2.334

Another optimum values for $h_9 = 1950$ mm: $A_1 = A_2 = 3728$, $A_3 = 2610$, $A_4 = 1864$ mm^2. $V = 3.462 \times 10^8$ mm^3, $K = 7818$ \$. Since $A_4 = 2094$ mm^2 is necessary for buckling constraint, the value of $\mu_4 = 0.5$ is too small and $\mu_4 = 0.6$ should be used.

For comparison the optimum data for the truss of parallel chords: $h_{9opt} = h_{13opt} = 5000$ mm, $V_{min} = 5.852 \times 10^8$ mm^3. $K_{min} = 11350$ \$. It can be seen that the truss of non-parallel chords is much more economic than the truss of parallel chords.

9. CHECK OF STRENGTH OF A TUBULAR JOINT

After the optimization the optimal cross-sections should be replaced by available profiles according to EN 10291-2 and the joints should be checked for strength according to new IIW rules (Static design 2009). To illustrate this procedure a tubular joint of the truss optimized for strength is shown in Figure 2.

The related rod forces are as follows: $S_3 = 787.4$ kN (tension), $S_{12} = 11.1$ kN (tension), $S_{15} = 233.4$ kN (tension), governing for diagonals, also for rod 16, for which $S_{16} = 13820$ N compression.

The available CHS profiles for the optimized truss are as follows: chords: Ø273.0x5 mm, verticals and diagonals: Ø 139.7x6 mm.

According to Figure 2 the joint is designed an overlap K-joint, with the eccentricity of $e = 0.25 \times 273 = 68$ mm, the overlap is $Ov = 100q/p = 100 \times 34.3/161.6 = 21.2\%$.
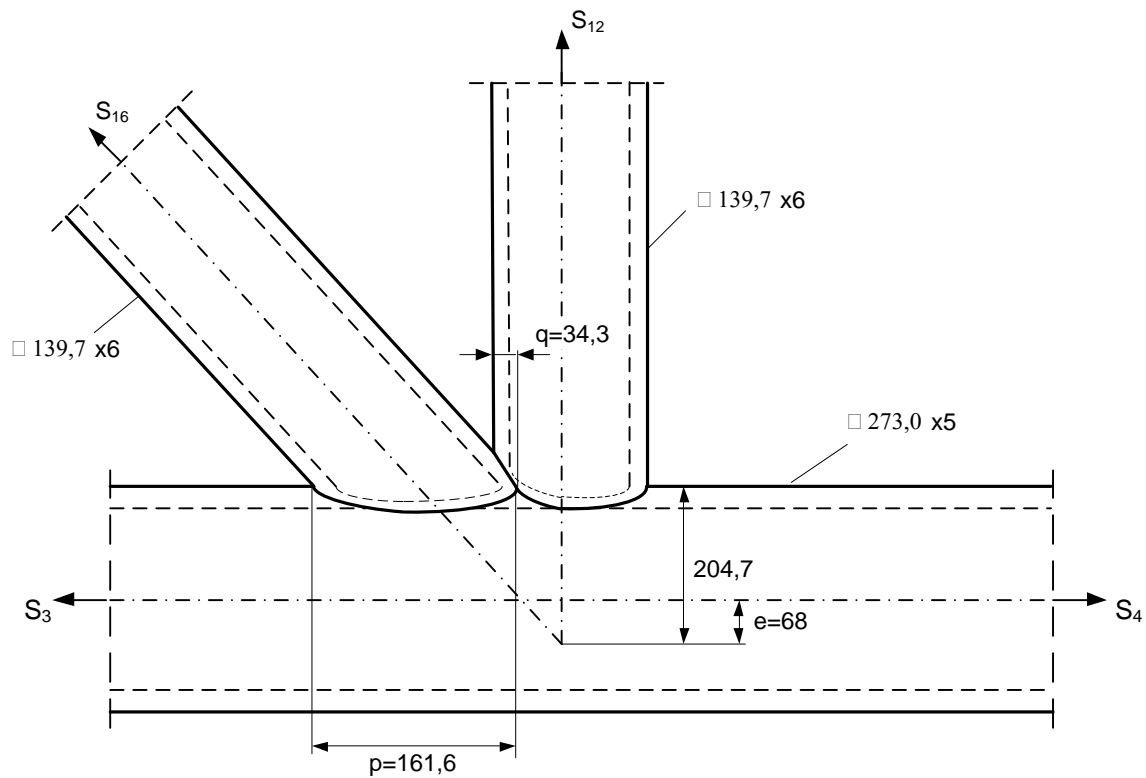
Figure 2. The overlapped tubular joint

(a) Check of local yielding of overlapping brace

$$N^* = f_y t_i L_{b.eff} \, ,$$

$$L_{b.eff} = \frac{\pi}{4} \left(2d_i + d_{ei} + d_{e.ov} - 4t_i \right),$$

$$d_{ei} = \frac{12}{d_0 / t_0} \frac{t_0}{t_i} d_i \le d_i \, .$$

Indices: for overlapping brace i, for overlapped brace j

$$d_{e.ov} = \frac{12}{d_j / t_j} \frac{t_j}{t_i} d_i \le d_i \, .$$

In our case $d_0 = 273, d_i = d_j = 139.7, t_i = t_j = 6, t_0 = 5$

$$d_{ei} = 25.6, d_{eov} = 72, L_{b.eff} = 277.2 \, .$$

$$N* = 355x6x277.2 = 5900533 > 233400 \text{ N, OK.}$$

(b) Check of local chord member yielding

$$\left(\frac{N_0}{N_{pl.0}}\right)^{1.7} \leq 1, \quad N_{pl.0} \geq N_0, \quad N_{pl.0} = A f_y,$$

$$N_{pl.0} = 3781x355 = 1342255 > 787400 \text{ N, OK.}$$

(c) Check of brace shear

$$N_i \cos\theta_i + N_j \cos\theta_j \leq N_S *$$

$$N_S* = \frac{\pi}{4}\left(0.58 f_{ui} \frac{\frac{100 - Ov}{100} 2d_i + d_{ei}}{\sin\theta_i} t_i + 0.58 f_{ui} \frac{2d_j + d_{ej}}{\sin\theta_j} t_j\right).$$

In our case $Ov = 21.2$, $f_{ui} = 510$, $d_{ei} = d_{ej} = 25.6$, $d_i = d_j = 139.7$, $t_i = t_j = 6$,

$$S_{15} \cos\theta_j = 116700 < N_S* = 833513 \text{ N, OK.}$$

10. CONCLUSIONS

The optimization problem to be solved is the following: find the optimal geometry and cross-sectional areas of rods which minimize the structural volume or cost for a simply supported tubular truss with non-parallel chords for a strong displacement constraint.

For the solution of this problem a developed calculation method is used. Besides the displacement constraint the rods are checked for tension stress and overall buckling. It is shown that, in the case of a strong displacement constraint the cross-sectional areas are larger than those required for constraints on stress and buckling.

The fabrication (welding) constraints on minimal angle between tubular rods (30^0) have been also active. In the calculation of overall buckling the Eurocode 3 formulae are approximated

by formulae of Japan Road Association enabling the explicit expression of the necessary cross-sectional area.

Special formulae are used for the cost calculation. The cost function expresses the cost of material, cutting and grinding of the tubular (CHS) rod ends, assembly, welding and painting. It is shown that, in this case, the structural optima for minimum volume and minimum cost are the same.

Check of strength of a tubular joint shows that the chords and braces of available CHS profiles fulfil the requirements.

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