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# OPTIMIZATION WITH AN IMPROVED PSO ALGORITHM

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## Abstract

Two new methods for improving the PSO algorithm are proposed. One of the methods doesn't need more function evaluations than the standard algorithm, and it's efficiency doesn't depends on the initial state of the process. The efficiency of the method depends on how and when we change the particle's speed knowing the gradient information in the previous sample points. These parameters can be different for every objective function. The other method improves the technique which uses an operator called Crazy Bird. We have applied the methods on several two dimensional test problems, and on a structural optimization problem.

**Keywords:** Optimization, PSO algorithm, Gradient estimation

## 1. INTRODUCTION

Optimization problems can be found in various fields of science, among which there are a lot of problems that can't be solved by analytical methods because of their complexity. Over the years the researchers have developed a lot of algorithms to solve these problems among which evolutionary algorithms became the most popular because of their simplicity and efficiency. These methods can find an approximate solution of these problems. In the literature a lot of evolutionary algorithms can be found for example the ant colony algorithm [1] which simulates the behaviour of ants, genetic algorithms [6] which solves the problem by simulating the process of evolution, Particle Swarm Optimization (PSO) algorithm, and hybrid techniques which are created as a mixture of algorithms to compound their beneficial features.

In the literature we can meet wide scale of PSO variants, and PSO improvement techniques [3]. The efficiency of a lot of techniques depends on the stochastic nature of the process, and these methods don't use the local features of the objective function. One of our new methods uses the gradient information in the previous sample points to change the speed of the particle. The tests have shown that this method improves the convergence speed of the algorithm in a lot of test cases, and we can use it in any optimization problem where the gradient information exists. The other method (Section 4.) improves the technique called Crazy Bird which can be described as a stochastic process.

## 2. PSO ALGORITHM

PSO algorithm [4, 7] was developed by Kennedy and Eberhart in 1995. Their goal was to visualize the social behaviour of bird flocks, and later they realized that the algorithm can be efficiently used to solve optimization problems. Over the years the researchers have developed a lot of PSO variants which can solve wide scale of optimization problems. These algorithms have become popular in practice [3, 8, 9] because of their simplicity, efficiency and easy implementation.

In the first step the algorithm generates particles in a predefined interval of the objective function. Every particle has a position ***x***, and a velocity ***v*** vector. The length of these vectors equals to the dimension of the objective function. The algorithm generates the position vectors by uniform distribution in the predefined interval. The particles move in the interval and search for the optimal solution. Every particle stores the best solution and its position during the particle's movement. These are called local best value and local best position. The algorithm selects the best of the local bests which are called global best value and position. Every particle in every iteration step evaluates the objective function, and they change their positions using the following equations:

$$v_i^{k+1} = v_i^k + c_1 r_1 (pbest_i - x_i^k) + c_2 r_2 (gbest_i - x_i^k) \qquad (1)$$

$$x_i^{k+1} = x_i^k + v_i^{k+1} t, \qquad (2)$$

where $v_i$ is the *i*-th element of the velocity vector, $x_i$ is the *i*-th element of the position vector, $c_1$ and $c_2$ are positive constants, $r_1$, $r_2$ are uniformly distributed number in [0,1], $pbest_i$ is the *i*-th element of the local best position vector, $gbest_i$ is the *i*-th element of the global best position vector of a given particle, *k* is the iteration number, and *t* stands for the unit time. The following flowchart shows the steps of the algorithm:

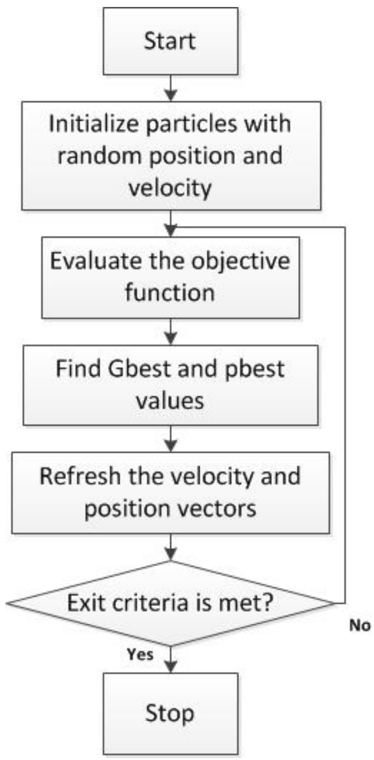


**Figure 1.** Flowchart of the PSO algorithm

## 3. IMPROVING PSO ALGORITHM WITH GRADIENT ESTIMATION

PSO algorithm has a lot of variants. The efficiency of lots of techniques depends on the random nature of the process. We can work with multiple swarms which communicate, or we can use the Crazy Bird approach which sends some of the particles to random direction instead of using equation (1), but the effectiveness of these methods are different in every objective function.

We propose a new method that doesn't need more function evaluations than the standard algorithm, and its efficiency doesn't depend on the initial state of the process. This method uses the gradient information in the previous sample points. Using the gradient information we can change the speed of the particles. If there are a lot of positive gradients in a particle's history, we raise its speed, because we conclude that the particle is moving towards the optimum point. This statement is true for a lot of objective functions, but not in all cases. Using the backward difference gradient estimation

$$f'(x_0) \approx \frac{f(x_0) - f(x_0 - h)}{h} \qquad (3)$$

which can be derived from the Taylor expansion of a one dimensional function we can easily compute the sign of the gradient knowing only the sample values at the previous sample points.

We have tested the method using twelve optimization test problems. We have run the algorithm one hundred times for one objective function and we have computed the average of the global best values in every iteration step. The algorithm has found the optimum faster for nine test functions, for two functions the speed was approximately the same as the standard algorithm, and for one test function it was worse.

## 4. IMPROVING PSO ALGORITHM WITH ELITIST CRAZY BIRD

We can improve the technique by introducing an operator called Crazy Bird [10] which can be described as a stochastic process. This approach sends some of the particles to random direction, or puts them in a random position instead of using equation (1), but the effectiveness of this method is different for every objective function, furthermore, if we raise the dimension of the objective function, the probability of finding a better result is highly decreasing.

If we put some of the particles in the local neighborhood of the actual global best position, we can find better result than the actual global best. In equation (2) the $t$ unit time is a predefined constant. If we choose a big number for this constant, the particles will miss the optimum, and if we choose a small number, we have to run the algorithm for a very long time to find the optimum. Using the elitist crazy bird method we can get information about the objective function at the local neighborhood of the global best position. The method slightly depends on the predefined $t$ constant.

## 5. TEST RESULTS

We have tested the gradient based technique using twelve optimization test problems [5, 2]. We have run the algorithm one hundred times for one objective function and we have calculated the average of the global best values in every iteration step. The light grey curve represents the gradient based technique the dark grey represents the standard algorithm.

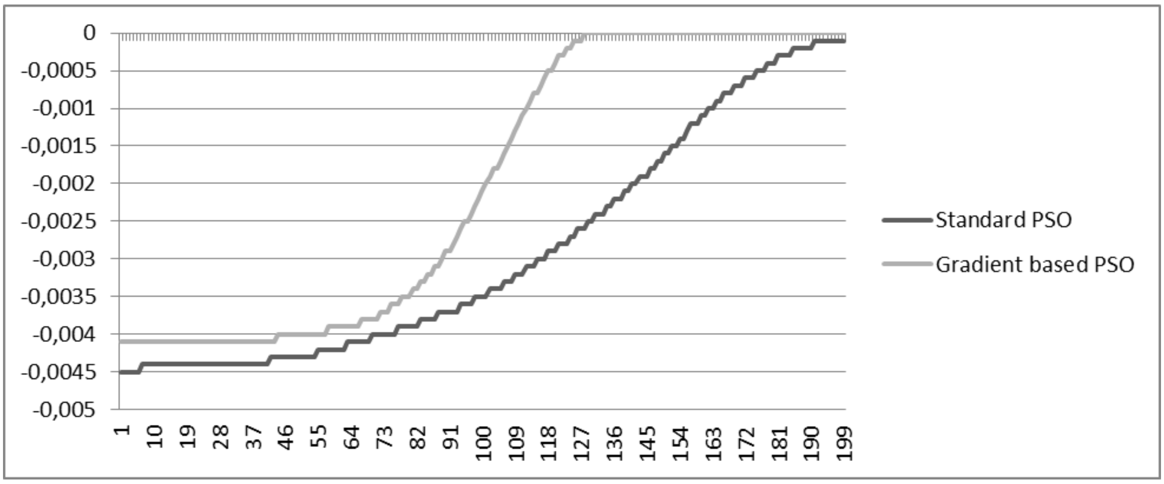


**Figure 2.** Global best values in the function of iterations. De-Jong test function with 1000 particles $f(x,y) = x^2 + y^2$.

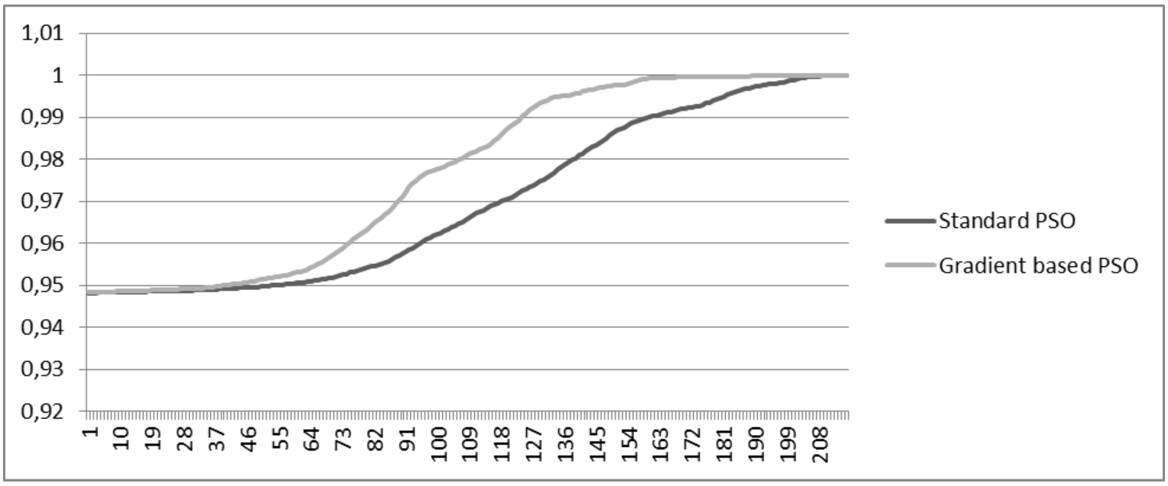


**Figure 3.** Global best values in the function of iterations. Drop Wave test function with 1000 particles. $f(x,y)=-(1+\cos(12(x^2+y^2)^{0.5}))/(0.5(x^2+y^2)+2)$

We can see on the plots that the gradient based method has higher global best value at almost every iteration level in these test functions. We have created tests using the elitist crazy bird method. If we change the parameters (crazy bird probability, local neighboring volume of global best, exit criteria) the results can be different. Testing with the De-Jong function (theoretical maximum value is 0), after averaging 100 test run's global best values the standard algorithm's result was -0.000001727, and with elitist crazy bird the result was -0.00000004.

## 6. MINIMUM COST DESIGN OF A CELLULAR PLATE

Cellular plates are constructed from two base plates and an orthogonal grid of stiffeners welded between them. Halved rolled I-section stiffeners are used for fabrication aspects. The torsional stiffness of cells makes the plate very stiff. In the case of uniaxial compression the buckling constraint is formulated on the basis of the classic critical stress derived from the Huber's equation for orthotropic plates. The cost function contains the cost of material, assembly and welding and is formulated according to the fabrication sequence. The unknown variables are the base plate thicknesses, height of stiffeners and numbers of stiffeners in both directions.

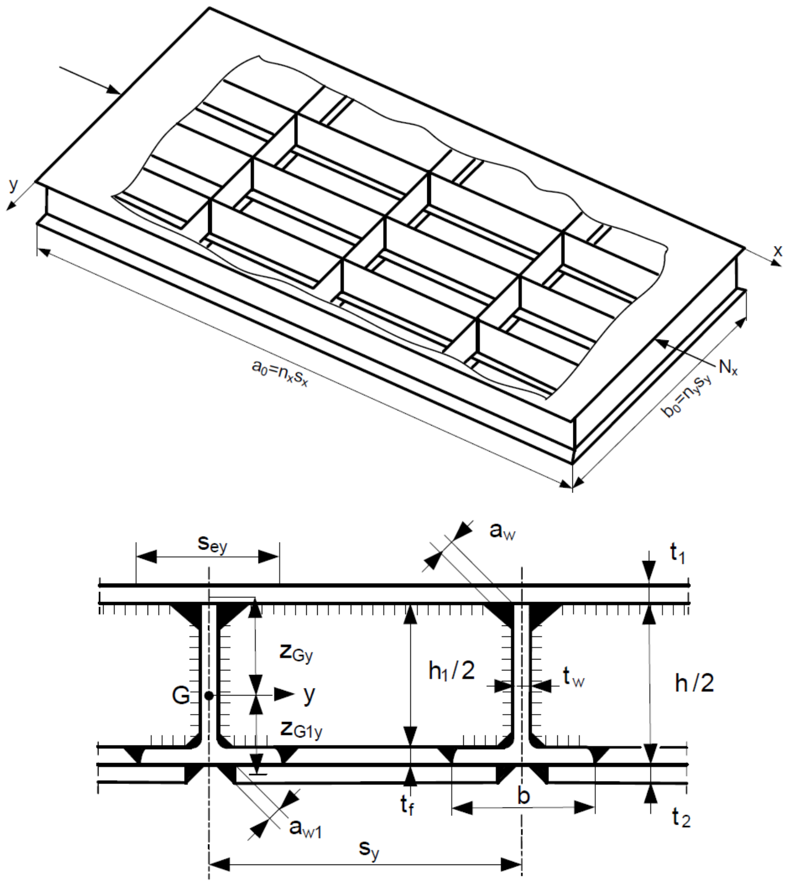


**Figure 4.** Orthogonally stiffened cellular plate and its cross-section

| Method | $x_1=t_1$ [mm] | $x_2=t_2$ [mm] | $x_3=h$ [mm] | $x_4=n_x$ | $x_5=n_y$ | cost [$] | Particle number |
|---|---|---|---|---|---|---|---|
| PSO | 8 | 5 | 403.2 | 2 | 14 | 42308.18 | 1000 |
| PSO | 9 | 5 | 403.2 | 2 | 14 | 44364.36 | 10000 |
| **GPSO** | **5** | **5** | **454.6** | **2** | **14** | **41679.97** | **1000** |
| **GPSO** | **7** | **5** | **403.2** | **2** | **15** | **41442.72** | **10000** |

**Table 1.** Optimum values for a cellular plate. The discrete values are found after finding the continuous ones.

## 7. CONCLUSION

Two new methods for improving the PSO algorithm are proposed. One of the methods doesn't need more function evaluations than the standard algorithm, and its efficiency doesn't depend on the initial state of the process. We have tested the method using twelve optimization test problems. The algorithm has found the optimum faster for nine test functions, for two functions the speed was approximately the same as the standard algorithm, and for one test function it was worse. We have applied the gradient based method in a structural optimization problem, and the results were better than the standard algorithm. The other method improves the technique called Crazy Bird. By means of the gradient based method, the particle can find the optimum faster, and with the Elitist Crazy Bird we can get closer to the theoretical optimum. Further research is needed in order to define the parameters of these methods.

## 8. ACKNOWLEDGEMENT

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