# Minimum Cost Design of a Rectangular Box Column Composed from Cellular Plates with Welded T-Stiffeners

Károly Jármai and József Farkas

University of Miskolc, Hungary
{altjar,altfar}@uni-miskolc.hu

**Abstract.** A cantilever column is loaded by a compression force and a bending moment caused by a horizontal force. It can be derived that, in the case of uniaxial bending, the rectangular cross section is more economic than the square one. In the given numerical case, the plate thicknesses should be too large for fabrication. Therefore stiffened plates should be used. Thus, the aim of the present study is to elaborate the minimum cost design of a column with rectangular cross-section and cellular plate walls. Cellular plates are constructed from two plates and longitudinal stiffeners welded between them. Previous studies have shown that welded T-stiffeners are more economic than the halved rolled I-section stiffeners, thus, welded T-stiffeners are used.

Stress and horizontal deformation constraints are formulated. In the stress constraint the face plate buckling is avoided by using effective widths. Local buckling constraint is used for the web of T-stiffeners. Variables are as follows: heights of welded T-sections, thicknesses of stiffener webs, number of stiffeners in both directions, main dimensions of the rectangular box section, thicknesses of outer and inner face plates in smaller and larger walls. The cost function is formulated according to the fabrication sequence and consists of cost of material, welding and painting. The constrained function minimization is performed by using an effective mathematical optimization method.

**Keywords:** structural optimization, minimum cost design, cellular plates, columns.

## 1 Introduction

Steel columns are widely used for buildings, bridges, as supports of highways etc. A column is loaded by a compression force $N_F$ and a bending moment caused by a horizontal force $H_F = 0.1N_F$ (Fig.1,2). It can be derived that, in the case of uniaxial bending, the rectangular cross section is more economic than the square one. In the given numerical case, the unstiffened plate thicknesses should be too large for fabrication. Therefore stiffened plates should be used.

Results obtained for square box columns have shown that the cellular plate elements are more economic than the plates stiffened on one side (Farkas and Jármai 2008). Thus, the aim of the present study is to elaborate the minimum cost design of a column with rectangular cross-section and cellular plate walls.

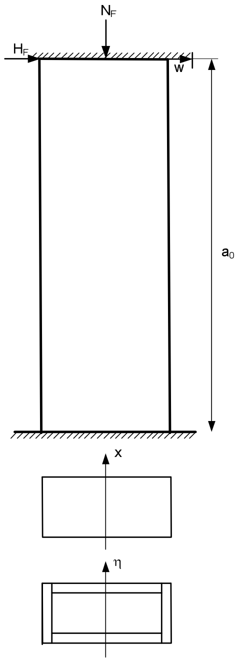



Fig. 1. Box column with walls of unstiffened and cellular plates, the two ends are built-in

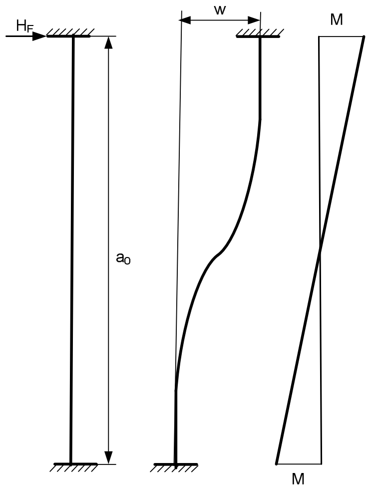



Fig. 2. Deformation and bending moment distribution of the column caused by the horizontal force

## 2 Formulation of the Problem

Numerical data: the factored compression force $N_F = 10^8$ [N], $a_0$ = 15 m, $f_y$ = 355 MPa, $E = 2.1 \times 10^5$ MPa.

Cellular plates are constructed from two plates and longitudinal stiffeners welded between them. Welded T-sections are selected for stiffeners. Figs 3 and 4 show the dimensions of cellular plate walls. Variables are as follows: height of welded T-sections $h_1/2 = h/2 - t_f$, $h_{11}/2 = h_2/2 - t_{f1}$, thickness of stiffener webs $t_w$ and $t_{w1}$, widths of stiffener flanges $b$ and $b_1$, number of stiffeners in both directions $n$ and $n_1$, main dimensions of the rectangular box section $b_0$ and $b_{01}$, thicknesses of outer and inner face plates in smaller and larger walls $t$ and $t_1$.

Ranges of variables are as follows: $t$ = 4 – 40 mm, $h$ = 300 – 1000 mm, $t_f = t$, $t_{f1} = t_1$.

## 3 Geometric Characteristics for Displacement Constraint

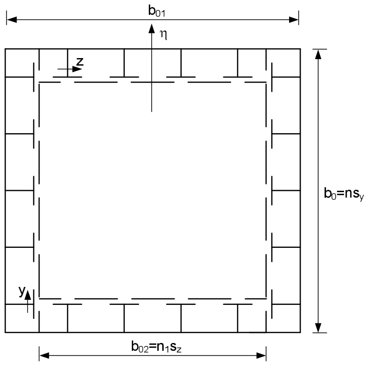
Cross-sectional area for both cellular plate walls

$$A = \frac{h_1 t_w}{2} + bt_f + 2s_y t, s_y = \frac{b_0}{n}, h_1 = h - 2t_f \tag{1}$$

$$A_1 = \frac{h_{11} t_{w1}}{2} + b_1 t_{f1} + 2s_z t_1, s_z = \frac{b_{01} - h - 3t}{n_1}, h_{11} = h_2 - 2t_{f1} \tag{2}$$

Distance of the gravity centre

$$z_G = \frac{1}{A}\left[\frac{h_1 t_w}{2}\left(\frac{h_1 + t}{2}\right) + bt_f \frac{h_1 + t + t_f}{2} + s_y t\left(\frac{h_1}{2} + t + t_f\right)\right] \tag{3}$$



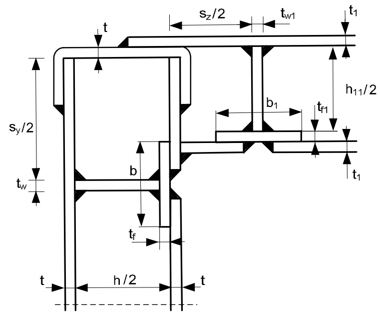




**Fig. 3.** Cross-section of the rectangular box column with cellular plate walls (see also Fig.1)

**Fig. 4.** Details of the corner for the box section with cellular plate walls

Moment of inertia

$$I_y = s_y t z_G^2 + s_y t\left(\frac{h_1}{2} + t + t_f - z_G\right)^2 + \frac{h_1^3 t_w}{96} + I_{y1} \tag{4}$$

$$I_{y1} = \frac{h_1 t_w}{2}\left(\frac{h_1 + t}{2} - z_G\right)^2 + bt_f\left(\frac{h_1 + t + t_f}{2} - z_G\right)^2 \tag{5}$$

Moment of inertia of the whole rectangular box section for axis $\eta$

$$I_\eta = 2nI_y + 2nA\left(\frac{b_{01}}{2} - z_G\right)^2 + 2\frac{b_{01}^3 t_1}{12} + 2\frac{t_1}{12}\left(b_{01} - \frac{h_1}{2} - t - t_f\right)^3 + 2I_{\eta 1} \tag{6}$$

$$I_{\eta 1} = 2\left(\frac{h_{11} t_{w1}}{2} + b_1 t_{f1}\right) s_z^2 \frac{n_1 (n_1 + 2)(n_1 + 1)}{24} \tag{7}$$

The displacement constraint is given as

$$w = \frac{H_F a_0^{\ 3}}{12E\gamma_M I_x} \le \frac{a_0}{\phi} = 15 \text{ mm} \quad I_\eta \ge I_0 = \frac{H_F L^2 \phi}{12E\gamma_M} \tag{8}$$

## 4 Geometric Characteristics for Stress Constraint

The local buckling of face plates is avoided by considering effective plate widths according to Eurocode 3 (2007):

$$A_e = \frac{h_1 t_w}{2} + bt_f + 2s_{ye}t, s_y = \frac{b_0}{n}, h_1 = h - 2t_f, s_{ye} = \rho_y s_y \tag{9}$$

$$\rho_y = \frac{\lambda_{py} - 0.22}{\lambda_{py}^2} \quad \text{if} \quad \lambda_{py} = \frac{s_y}{56.8\varepsilon t} \ge 0.673,\ \varepsilon = \sqrt{\frac{235}{f_y}} \tag{10}$$

$$\rho_y = 1 \quad \text{if} \quad \lambda_{py} < 0.673 \tag{10a}$$

The other formulae are similar to those in Section 3, but $s_e$ should be used instead of $s$.

## 5 Constraint on Local Buckling of Stiffener Webs

The webs are subject to uniform compression. According to Eurocode 3

$$\frac{h_1}{2t_w} \le 42\varepsilon_1, \varepsilon_1 = \sqrt{\frac{235}{\sigma}} \quad \text{and} \quad \frac{h_{11}}{2t_{w1}} \le 42\varepsilon_1 \tag{11}$$

## 6 Fabrication Constraints

In order to allow the welding of stiffener web to the base plate the distance between the stiffener flanges is prescribed as 300 mm:

$$n \le \frac{b_0}{300 + b}, n_1 \le \frac{b_{02}}{300 + b_1} \tag{11a}$$

## 7 Cost Function

The cost function is formulated according to the fabrication sequence.

(1) Welding of outer face plates with butt welds (SAW – submerged arc welding). A plate element has sizes of 6000x1500 mm or less.

Plate of sizes $a_0$x$b_0$: volume $V_0 = a_0 b_0 t$, weld length $L_{W0} = 2b_0 + (q\text{-}1)a_0$,

$$K_{W0} = k_W \left( \Theta \sqrt{3q\rho V_0} + 1.3 C_W t^{n_0} L_{W0} \right) \quad k_W = 1.0 \text{ \$/min} \tag{12}$$

$q$ is the number of plate elements in the direction of $b_0$ so that $b_0 / q \le 1500$ mm.

The factor of complexity of the assembly is taken as $\Theta = 2$.

$$\text{For } t < 11 \quad C_W = 0.1346x10^{-3}, n_0 = 2 \tag{13}$$

$$\text{for } \quad t \ge 11 \quad C_W = 0.1033x10^{-3}, n_0 = 1.904 \tag{14}$$

Plate of sizes $a_0 x b_{01} : V_{01} = a_0 b_{01} t_1 ; L_{W01} = 2b_{01} + (q_1 - 1)a_0$ (15)

$$K_{W01} = k_W \left( \Theta\sqrt{3q_1 \rho V_{01}} + 1.3C_W t_1^{n_0} L_{W01} \right) \tag{16}$$

$q$ and $q_1$ are the numbers of plate strips of width smaller than 1500 mm.

(2) Welding of stiffeners' webs to outer face plates and to flange with double fillet welds (GMAW-CO2 gas metal arc welding with $C0_2$).

Plate of sizes $a_0$x$b_0$:

$$V_1 = \left( \frac{h_1}{2} t_w + bt_f \right) a_0 n + V_0, L_{W1} = 4a_0 n \tag{17}$$

$$K_{W1} = k_W \left( \Theta\sqrt{(2n+1)\rho V_1} + 1.3x0.3394x10^{-3} a_W^2 L_{W1} \right) \tag{18}$$

$a_W = 0.4t_w$ but $a_{W\min} = 4$ mm

Plate of sizes $a_0$x$b_{01}$:

$$V_{11} = \left( \frac{h_{11}}{2} t_{w1} + b_1 t_{f1} \right) a_0 n_1 + V_{01}, L_{W11} = 4a_0 n_1 \tag{19}$$

$$K_{W11} = k_W \left( \Theta\sqrt{(2n_1+1)\rho V_{11}} + 1.3x0.3394x10^{-3} a_{W1}^2 L_{W11} \right) \tag{20}$$

$a_{W1} = 0.4t_{w1}$ but $a_{W1\min} = 4$ mm

(3) Welding of inner plate strips of width $s_y$ and $s_z$ from 3-3 parts with butt welds excluding the outside strips:

$$V_2 = a_0 s_y t \tag{21}$$

$$K_{W2} = (n-1)k_W \left( \Theta\sqrt{3\rho V_2} + 1.3C_W t^{n_0} 2s_y \right) \tag{22}$$

$$V_{21} = a_0 s_z t_1 \tag{23}$$

$$K_{W21} = (n_1 - 1)k_W \left( \Theta\sqrt{3\rho V_{21}} + 1.3C_W t_1^{n_0} 2s_z \right) \tag{24}$$

(3a) Welding of the outside strips of width $s_y/2$ and $s_z/2$:

$$V_{2a} = a_0 s_y t / 2, V_{21a} = a_0 s_z t_1 / 2 \tag{25}$$

$$K_{W2a} = 2k_W \left( \Theta\sqrt{3\rho V_{2a}} + 1.3C_W t^{n_0} s_y \right) \tag{26}$$

$$K_{W21a} = 2k_W \left( \Theta\sqrt{3\rho V_{21a}} + 1.3C_W t_1^{\,n_0} s_z \right) \tag{27}$$

(4) Welding of inner face plate strips to the stiffener flanges with double fillet welds:

$$V_3 = V_1 + a_0 b_0 t, L_{W2} = 2a_0 n \tag{28}$$

$$K_{W3} = k_W \left( \Theta\sqrt{(n+2)\rho V_3} + 1.3x0.3394x10^{-3} a_{W2}^2 L_{W2} \right) \tag{29}$$

$a_{W2} = 0.7t$ but $a_{W2\min} = 3$ mm

$$V_{31} = V_{11} + a_0 (b_{01} - h - 3t) t_1, L_{W21} = 2a_0 n_1 \tag{30}$$

$$K_{W31} = k_W \left( \Theta\sqrt{(n_1+2)\rho V_{31}} + 1.3x0.3394x10^{-3} a_{W21}^2 L_{W21} \right) \tag{31}$$

$a_{W21} = 0.7t_1$ but $a_{W21\min} = 3$ mm

(5) Welding of 2 U-elements to the ends of the smaller wall with 2-2 fillet welds

$$A_U = \left( \frac{h_1}{2} + t_f + 2t + 80 \right) t, \quad V_4 = 2A_U a_0 + V_3 \tag{32}$$

$$K_{W4} = k_W \left( \Theta\sqrt{3\rho V_4} + 1.3x0.3394x10^{-3} a_{W2}^2 4a_0 \right) \tag{33}$$

(6) Welding of larger walls to the smaller ones with fillet welds

$$V_5 = 2V_4 + 2V_{31}, L_{W3} = 8a_0 \tag{34}$$

$$K_{W5} = k_W \left( \Theta\sqrt{4\rho V_5} + 1.3x03394x10^{-3} a_{W21}^2 L_{W3} \right) \tag{35}$$

The material cost

$$K_M = k_M \rho V_5, k_M = 1.0 \text{ \$/kg} \tag{36}$$

The painting cost is calculated as

$$K_P = k_P \Theta S_P, k_P = 14.4x10^{-6} \text{ \$/mm}^2 \tag{37}$$

Surface to be painted

$$S_P = 2a_0 \left( 2b_0 + 2b_1 - \frac{h_1}{2} - t_f - 2t - \frac{h_{11}}{2} - t_{f1} - 2t_1 \right) \tag{38}$$

The total cost

$$K = K_M + 2(K_{W0} + K_{W01} + K_{W1} + K_{W11} + K_{W2} + K_{W2a} + K_{W21} + K_{W21a}) + \\ + 2(K_{W3} + K_{W31}) + K_{W4} + K_{W5} + K_P \tag{39}$$

## 8 Results of the Optimization

The optimization is made using the Particle Swarm Optimization (PSO) technique. PSO is a population based stochastic optimization technique developed by Eberhart and Kennedy (1995), inspired by social behaviour of bird flocking or fish schooling. If one of the members of the swarm sees a desirable path to go, the rest of the swarm will follow quickly. Every member of the swarm searches for the best in its locality - learns from its own experience.

Each particle keeps track of its coordinates in the problem space which are associated with the best solution (fitness) it has achieved so far. (The fitness value is also stored.) This value is called *pbest*. Another "best" value that is tracked by the particle swarm optimizer is the best value, obtained so far by any particle in the neighbours of the particle. This location is called *lbest*. when a particle takes all the population as its topological neighbours, the best value is a global best and is called *gbest*.

The particle swarm optimization concept consists of, at each time step, changing the velocity of (accelerating) each particle toward its p*best* and *lbest* locations. In the calculation the number of particles was 500. The detailed description of the technique is available in Farkas and Jármai (2008). The results are as follows:

| | |
|---|---|
| Widths of the column | $b_0$ = 3800 mm, $b_{01}$ = 5400 mm |
| Stiffeners height | $h$ = 1000 mm, and $h_1$= 1000 mm |
| Number of stiffeners | $n$ = 8, $n_1$ = 9 |
| Thicknesses of stiffeners web | $t_w = t_{w1}$ = 15 mm |
| Widths of the stiffeners flange | $b = b_1$ = 150 mm |
| Thicknesses of cover plates | $t$ = 5 mm, $t_1$ = 5 mm |
| The total cost of the structure | 120600 $ |
| Stress constraint | 354 MPa < 355 MPa |
| Displacement constraint | 4 mm < 15 mm |
| Stiffeners web buckling constraint | 33<34 |
| Fabrication constraints | $n<8.44$, $n_1<9.7$ |

## 9 Conclusions

Cellular plates are constructed from two base plates and longitudinal stiffeners welded between them. A cantilever column is loaded by a compression force and a bending moment caused by a horizontal force. It can be derived that, in the case of uniaxial bending, the rectangular cross section is more economic than the square one. Stress and horizontal deformation constraints are formulated. In the stress constraint the face plate buckling is avoided by using effective widths. Local buckling constraint is used for the web of T-stiffeners. The cost function contains the cost of material, assembly and welding and is formulated according to the fabrication sequence. Particle Swarm Optimization is used to find the optimum.

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