

PROCEEDINGS OF THE INTERNATIONAL WELDING CONFERENCE

**WELDING SCIENCE  
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**JAPAN - SLOVAK WELDING  
SYMPOSIUM**

**5-7 MARCH 1996  
HOTEL METALURG, HIGH TATRAS MOUNTAINS  
TATRANSKÁ LOMNICA-MATLIARE, SLOVAKIA**

## OPTIMUM FATIGUE DESIGN OF WELDED STEEL AND ALUMINIUM BOX BEAMS

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### Introduction

Optimum design is a structural synthesis which collects all important engineering aspects to work out safe and economic structural versions. The economy is achieved by minimising the cost or weight function while the safety is guaranteed by fulfilling the design constraints on static stress, fatigue, stability, vibration, deflection, technological requirements, etc.

The aim of the present paper is to show how to optimise the main dimensions of a box beam in the case when the beam is subjected not only by static (permanent) but also by pulsating (variable) load which can cause fatigue failure. The constraints are defined according to Eurocode 3 (EC3) (1992) for steel and BS 8118 (1991) for aluminium. The new IIW Recommendations (1995) are also considered.

In the cases when fatigue and buckling constraints should be simultaneously considered the following problem arises. When the fatigue constraint is active, the maximal static compression stress is much smaller than the yield stress, so it is proposed to calculate the buckling characteristics (limiting local plate slendernesses) considering the maximal static stress instead of yield stress.

Box beams are widely used in welded structures. The authors have studied the optimisation problems of steel box beams in their publications (Farkas 1984, Farkas and Jármai 1995a, Farkas and Jármai 1995b) but only for static loading. Now these studies are extended to fatigue loads and also for aluminium beams to show the differences between steel and aluminium beams by means of a numerical example.

### Optimisation of welded steel box beams

A simply supported beam is loaded by a constant, uniformly distributed static load  $p_G$  and by a variable force  $Q$  pulsating between  $+Q$  and  $-Q$  (Fig.1). In order to stiffen the box beam against torsional deformation of the cross-sectional shape, transversal diaphragms should be welded inside by fillet welds. The dimensions of the box beam  $h$ ,  $t_w/2$ ,  $b$  and  $t_f$  should be optimised to minimise the cross-sectional area

$$A = ht_w + 2bt_f \quad (1)$$

and fulfil the design constraints as follows.

The fatigue constraint is defined by

$$\Delta\sigma = \psi_d \frac{2QL}{4W_x} \leq \frac{\Delta\sigma_N}{\gamma_{Mf}} \quad (2)$$

$$W_x = 2I_x / (h + t_f); I_x = h^3 t_w / 12 + 2bt_f (h + t_f)^2 / 4 \quad (3)$$

where  $\psi_d$  is a dynamic factor,  $W_x$  is the elastic section modulus,  $I_x$  is the moment of inertia,  $\Delta\sigma_N$  is the fatigue stress range corresponding to the given number of cycles  $N$ . According to EC3, for diaphragms of box girders welded to the flange or web, for diaphragm thickness  $t < 12$  mm the detail category (the stress range at  $N=2 \cdot 10^6$ ) is  $\Delta\sigma_C = 80$  MPa. For

another number of cycles, for  $N < 5 \cdot 10^6$  the stress range can be calculated using the following formula

$$\log \Delta\sigma_N = \frac{1}{3} \log \frac{2 \cdot 10^6}{N} + \log \Delta\sigma_C \quad (4)$$

$\gamma_{Mf}$  is the safety factor for fatigue. According to EC3, for non "fail-safe" components with poor accessibility it is  $\gamma_{Mf} = 1.35$ .

The static stress constraint is expressed by

$$\sigma_{\max} = (\gamma_G p_G L^2 / 8 + \gamma_Q \psi_d QL / 4) / W_x \leq f_y / \gamma_{M1} \quad (5)$$

where, according to EC3,  $\gamma_G = 1.35$ ,  $\gamma_Q = 1.50$  are safety factors for permanent and variable actions, respectively,  $f_y$  is the yield stress,  $\gamma_{M1} = 1.1$  is the partial safety factor.

The local buckling constraints are as follows:

$$\text{for flange buckling } (b - 40) / t_f \leq 42 \sqrt{235 / \sigma_{\max}} \quad (\sigma_{\max} \text{ in Mpa}) \quad (6)$$

$$\text{and for web buckling } 2h / t_w \leq 124 \sqrt{235 / \sigma_{\max}} \quad (7)$$

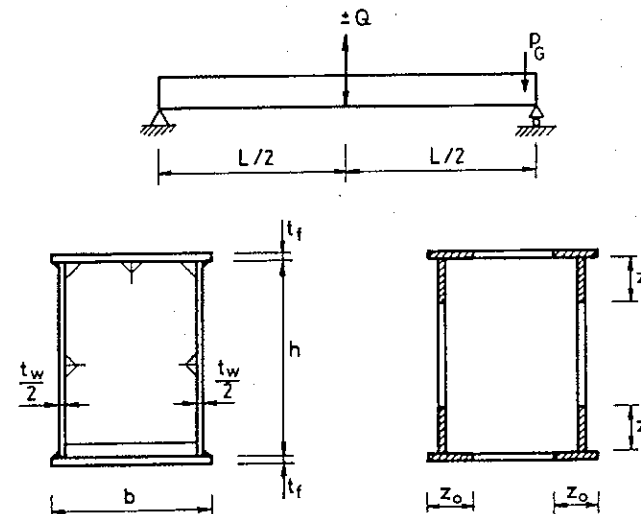


Fig.1 Main dimensions of a box beam and reduced cross-section of the aluminium beam.

It should be noted that we use in Eqs (6) and (7) the maximal static stress instead of yield stress, since the static stress may be much smaller than the yield stress when the fatigue constraint is active. Note that the shear buckling of webs should also be checked, but, in our numerical example, this constraint will be always passive.

The deflection constraint is, according to EC3 for floor beams

$$\frac{5p_G L^4}{384E_s I_x} + \frac{QL^3}{48E_s I_x} \leq \frac{L}{300} \quad (8)$$

where  $E_s$  is the elastic modulus of steel.

### Optimisation of welded aluminium box beams

The fatigue constraint is the same as for steel beams, Eqs.(2) and (3). The difference is, that, according to the IIW Recommendations (1995), for plates with transverse stiffeners  $\Delta\sigma_C = 28$  MPa. Eq.(4) can also be used. Note that, according to the BS 8118 (1991), in the fatigue constraint the section properties should not be reduced for HAZ (heat affected zone).

The static stress constraint. In Eq.(5), according to BS 8118  $\gamma_G = 1.20, \gamma_Q = 1.33$ , and the section modulus should be reduced considering the HAZ softening effect. According to BS 8118

$$W_{x,red} = \frac{2I_{x,red}}{h+t_f}, I_{x,red} = I_x - 4z_0 \frac{t_f}{2} \left( \frac{h+t_f}{2} \right)^2 - 4z_0 \frac{t_w}{4} \left( \frac{h}{2} - \frac{z_0}{2} \right)^2 \quad (9)$$

where the width of the HAZ is  $z_0 = 3t_B^2 / t_A$  (10)

$$\begin{aligned} \text{if } t_w/2 \leq t_f \quad t_B = t_w/2; \quad \text{if } t_w/2 > t_f \quad t_B = t_f \\ \text{if } 0.5(t_w/2 + t_f) \leq 1.5t_B \quad t_A = 0.5(t_w/2 + t_f); \quad \text{if } 0.5(t_w/2 + t_f) > 1.5t_B \\ t_A = 1.5t_B \end{aligned}$$

Furthermore, in Eq.(5), instead of  $f_y / \gamma_{M1}$  the value of  $p_0 / \gamma_m$  should be used, where  $p_0$  is the limiting stress for bending and overall yielding,  $\gamma_m = 1.2$  is the material factor.

The local buckling constraints, according to BS 8118, are as follows:

$$\text{for flange } (b-40)/t_f \leq 18\sqrt{250/\sigma_{max}} \quad (11)$$

$$\text{and for webs } 2h/t_w \leq 18/0.35\sqrt{250/\sigma_{max}} \quad (12)$$

The deflection constraints, according to BS 8118 for beams in buildings, are as follows:

$$\frac{5p_G L^4}{384E_a I_x} \leq \frac{L}{200} \quad (13)$$

$$\frac{5p_G L^4}{384E_a I_x} + \frac{QL^3}{48E_a I_x} \leq \frac{L}{100} \quad (14)$$

where  $E_a$  is the elastic modulus for aluminium alloys.

### Numerical example

The following data are given:  $Q = 6$  kN,  $L = 12$  m,  $\psi_d = 2$ ; the values of  $p_G$  are varied to show that for low  $p_G/Q$  ratios the fatigue constraint, for large ratios the static stress or deflection constraint is active.  $N = 3 \cdot 10^6$ , so, using Eq.(4), we get for steel beam  $\Delta\sigma_N = 69.8$  MPa, for aluminium beam  $\Delta\sigma_N = 24.5$  MPa. Take  $f_y = 235$  MPa for steel Fe 360 and  $p_0 = 240$  MPa for 6082-T6 heat treatable aluminium alloy (ISO: AISi1MgMn) plates with thicknesses of  $t = 3 - 25$  mm. For steel  $E_s = 2.1 \cdot 10^5$  and for aluminium alloys  $E_a = 7 \cdot 10^4$  MPa.

The Rosenbrock's Hillclimb mathematical programming method has been used for computerised optimisation. Rounded values are computed by a complementary special program. The results are summarised in Tables 1-2. It can be seen that, depending on the  $p_G/Q$  ratio, the fatigue or the static stress as well as the deflection constraint is active.

Tab.1 Optimal dimensions (mm) and minimal cross-sectional areas of welded steel box beams for  $Q = 6$  kN and various values of  $p_G$

$p_G$ (N/mm)	3	6	9	10	12
$h \cdot t_w/2$	595*6	535*7	510*8	540*9	550*9
$b \cdot t_f$	230*8	245*8	230*9	200*9	215*10
$A$ (mm <sup>2</sup> )	7250	7665	8220	8460	9250
active constraint	fatigue	fatigue	fatigue	fatigue and static stress	static stress

Tab.2 Optimal dimensions (mm) and minimal cross-sectional areas of welded aluminium box beams for  $Q = 6$  kN and various values of  $p_G$

$p_G$ (N/mm)	3	12	21	25	30
$h \cdot t_w/2$	705*10	625*14	665*19	665*19	695*20
$b \cdot t_f$	245*18	290*17	280*14	260*19	290*19
$A$ (mm <sup>2</sup> )	15870	18610	20275	22515	24920
active constraint	fatigue	fatigue	fatigue and deflection	deflection	deflection

### Conclusions

When fatigue and buckling constraints are simultaneously considered, the buckling constraints should be calculated with the actual maximal static stress which can be much smaller than the yield stress. Depending on the ratio of permanent and pulsating load intensity, the fatigue or the static stress as well as the deflection constraint can be active. The differences between steel and aluminium beams are as follows: the fatigue stress ranges as well as the limiting plate slendernesses are for aluminium much smaller than those for steel beams. In the static stress constraint for aluminium beam the softening effect of HAZ should be considered. For aluminium beams the deflection constraint can be active instead of the static stress constraint.

### References

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### Acknowledgements

This research work has been supported by the grants OTKA T-4479 and OTKA T-4407 of the Hungarian Fund for Scientific Research.