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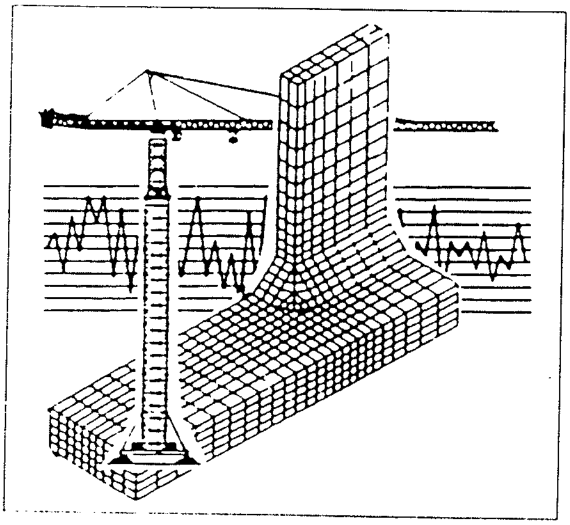

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# Fatigue Constraints in the Optimum Design of Welded Structures

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**Abstract**
The optimum design of welded structures is treated in the case when, in addition to permanent loads, fluctuating forces also act. Constraints on fatigue as well as on overall and local buckling are simultaneously considered. When the fatigue constraint is active, the design stress can be smaller than the yield stress, thus, the limiting plate slendernesses for local buckling may be calculated on the basis of static design stress. In the first example a welded I-section cantilever, in the second one a compressed tubular strut is optimized taking into account the fatigue constraint for connecting fillet welds. The required weld sizes are also calculated according to new IIW recommendations.

## 1. INTRODUCTION

In the case when fluctuating forces act in addition to permanent static load on thin-walled structures containing welds, the fatigue and buckling constraints should be simultaneously considered. The effect of instabilities on low-cycle fatigue is well investigated since it is important for the design of seismic resistant structures (see e.g. [1]). The breathing of thin plate elements due to fluctuating loads is also studied (e.g. [2]), but it is concentrated only to thin webs. There are no design rules for cases when permanent and high-cycle fluctuating loads can cause instability in structures containing welded joints in which fatigue cracks can occur.

In the optimum design fatigue and buckling constraints can simultaneously defined and fulfilled. Note that the first author has treated the optimum design of welded tubular trusses with buckling and fatigue constraints [3], but the buckling and fatigue constraints have been separately considered. The simultaneous consideration of buckling and fatigue constraints means that in the local buckling formulae the maximal static stress can be used instead of the yield stress. When the fatigue constraint is active this maximal stress can be much lower than the yield stress which results in different optimal solutions than those obtained by static design.

In order to illustrate this special optimization procedure two numerical examples are worked out. The fatigue constraint is defined using the fatigue stress range concept according to the IIW Recommendations [4]. The Eurocode 3 (EC3)(1992) is also used for static design rules, for safety factors and for the interaction fatigue formula when normal and shear stress components occur in a weld.

## 2. ILLUSTRATIVE EXAMPLES

### 2.1. Example: optimum design of a welded I-section cantilever connected to a column by fillet welds (Fig.1)

.For the calculation of the optimal dimensions of the welded I-section cantilever the constraint on fatigue is used. The objective function is the cross-sectional area

$$A = ht_w + 2bt_f \qquad (1)$$

*The fatigue constraint* for the parent material at the toes of fillet welds connecting the flanges, in the case of a force $F$ fluctuating between $+F$ and $-F$, is defined by

$$\Delta\sigma = 2FL / W_x \leq \Delta\sigma_N / \gamma_{Mf} \qquad (2)$$

The moment of inertia is expressed by

$$I_x \cong h^3 t_w / 12 + 2bt_f (h/2)^2$$

Eq.(2) can be written in the form [5]

$$W_x \cong I_x / (h/2) = h^2 t_w / 6 + bt_f h \geq W_0 = 2FL / (\Delta\sigma_N / \gamma_{Mf}) \qquad (3)$$

where $\Delta\sigma_N$ is the fatigue stress range, $\gamma_{Mf} = 1.25$ is the safety factor, $W_0$ is the required section modulus. From Eq.(1) one obtains

$$bt_f = A/2 - ht_w/2 \qquad (4)$$

Substituting Eq.(4) into (3) we get

$$A \geq 2W_0 / h + 2ht_w / 3 \qquad (5)$$

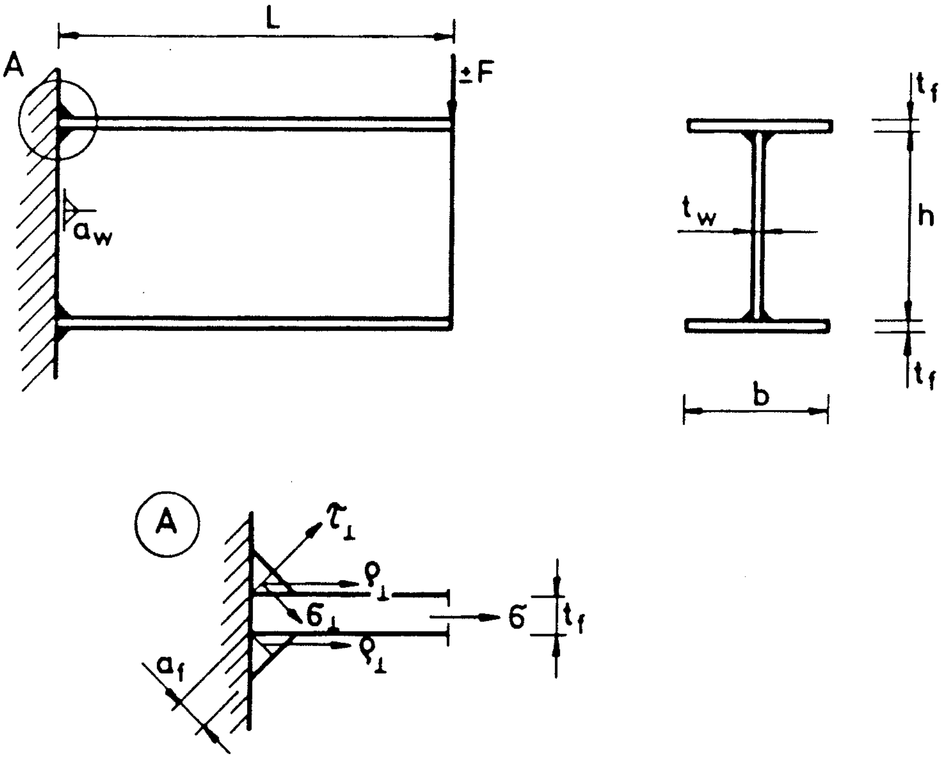


Fig.1. Welded I-section cantilever and the stress components in the connecting fillet welds

*The local buckling constraint* can be expressed by means of the limiting plate slenderness which can be derived from the basic formula for the critical local plate buckling stress. This stress should be larger than the $\sigma_{max}$ static design stress

$$\sigma_{cr} = \frac{k\pi^2 E}{12\left(1-\nu^2\right)}\left(\frac{t}{b}\right)^2 \geq \sigma_{max} \tag{6}$$

where $k$ is the plate buckling factor, $\nu$ is the Poisson`s ratio, $b$ and $t$ are the width and thickness of the plate, respectively. The limiting plate slenderness is

$$\left(\frac{b}{t}\right)_L = \sqrt{\frac{k\pi^2 E}{12\left(1-\nu^2\right)\sigma_{max}}} \tag{7}$$

For a bent web plate of an I-beam the theoretically calculated value of $k$ = 23.9, with the elastic modulus of $E$=2.1*10$^5$ MPa, $\nu = 0.3, \sigma_{max} = f_y = 235$ MPa one obtains

$$h/t_w \leq 138.9 \tag{8}$$

This value should be decreased according to EC3 to 124 considering the effect of initial imperfections and residual welding stresses. For another $f_y$ values and different design stresses Eq.(8) can be generalized as

$$\frac{h}{t_w} \leq \frac{1}{\beta} = 124\sqrt{\frac{235}{f_y}}\sqrt{\frac{f_y}{\sigma_{max}}} \tag{9}$$

or using the notation of EC3

$$1/\beta = 124\varepsilon, \varepsilon = \sqrt{235/\sigma_{max}} \tag{10}$$

EC3 gives a smaller value for the case when the designer wants to neglect the shear buckling check of the web

$$1/\beta = 69\varepsilon \tag{11}$$

Since in the present example the maximal compressive stress is caused by $-F$, so

$$\sigma_{max} = \Delta\sigma_N / \left(2\gamma_{Mf}\right) \tag{12}$$

and *the local buckling constraint for the web* can be written as

$$t_w \geq \beta h; 1/\beta = 69\varepsilon; \varepsilon = \sqrt{\frac{235}{\Delta\sigma_N/(2\gamma_{Mf})}} \tag{13}$$

The calculations show that this constraint is always active thus it can be treated as equality. Then the objective function (5) is

$$A = 2W_0/h + 2\beta h^2 \tag{14}$$

and the condition $\mathrm{d}A/\mathrm{d}h = 0$ gives the optimal web height

$$h_{opt} = \sqrt[3]{3W_0/\left(2\beta\right)} \tag{15}$$

The local buckling constraint for the compression flange is given by

$$t_f \geq \delta b \tag{16}$$

where, with the theoretical buckling factor of $k = 0.4$

$$\left[b/\left(2t_f\right)\right]_L = 18\varepsilon \tag{17}$$

EC3 takes into account the above mentioned effects and gives the value of 14 instead of 18, thus

$$1/\delta = 28\varepsilon \tag{18}$$

and from Eq.(4) one obtains

$$b_{opt} = h_{opt}\sqrt{\frac{\beta}{2\delta}} \quad (19)$$

Finally, using Eq.(14) we get

$$A_{min} = \sqrt[3]{18\beta W_0^2} \quad (20)$$

which shows that the cross-sectional area (weight) can be decreased using higher values of $1/\beta$.

*Numerical data*: $F = 150$ kN fluctuating between $+F$ and $-F$, $L = 2$ m, the number of cycles $N = 3*10^5$. The fatigue detail category for toe cracking is $\Delta\sigma_C = 71$ MPa, for the given number of cycles, using the interpolation formula as follows

$$\log\Delta\sigma_N = \frac{1}{3}\log\frac{2*10^6}{N} + \log\Delta\sigma_C \quad (21)$$

we get $\Delta\sigma_N = 133.6$ MPa, thus, with Eq.(3) $W_0 = 5.61*10^6$ mm$^3$. Using Eq.(13) one obtains $1/\beta = 144.7$ and Eq.(15) gives $h_{opt} = 1068$ rounded 1070 mm. Furthermore $t_w = 8$ mm, $1/\delta = 28\varepsilon = 58.7, b_{opt} = 485, t_f = 9$ mm.

The stress range in the upper flange is (with $\beta = 8/1070$)

$$\Delta\sigma = \frac{M_{max}}{W_x} = \frac{300*2*10^6}{2\beta h^3/3} = 98.3 \text{ MPa}.$$

To calculate the required fillet weld throat size $a_f$ we reduce the normal stress in parent metal into the fillet weld based on the following equality (see Fig.1)

$$\Delta\sigma t_f = 2\rho_\perp a_f$$

from which $\rho_\perp = \Delta\sigma t_f/(2a_f)$

and the two stress components are $\Delta\sigma_\perp = \Delta\tau_\perp = \frac{\rho_\perp}{\sqrt{2}} = \frac{312.8}{a_f}$

*For root cracking of partial penetration fillet welds* and for $N = 3*10^5$ cycles we use $\Delta\sigma_N = 52.6$ and $\Delta\tau_N = 116.9$ MPa. According to the EC3, the fillet weld should be checked using the following interaction formula

$$\left(\frac{\gamma_{Ff}\Delta\sigma}{\Delta\sigma_N/\gamma_{Mf}}\right)^3 + \left(\frac{\gamma_{Ff}\Delta\tau}{\Delta\tau_N/\gamma_{Mf}}\right)^5 \le 1 \quad (22)$$

With $\Delta\sigma_\perp = \Delta\tau_\perp = 312.8/a_f$, $\gamma_{Ff} = 1.0$ and $\gamma_{Mf} = 1.25$ one obtains

$$411/a_f^3 + 418/a_f^5 \le 1$$

from which $a_f = 8$ mm (rounded value).

*In the fillet welds connecting the web plate*, in addition to the perpendicular stress components

$$\sigma_\perp = \tau_\perp = \frac{\Delta\sigma t_w}{2\sqrt{2}a_w} = \frac{278}{a_w}$$

a parallel stress component should be calculated as well

$$\tau_{//} = \frac{2F}{2a_w h} = \frac{140.2}{a_w}$$

In this case we use, according to the EC3, the following stress components in the interaction formula

$$\Delta\sigma = \sqrt{\sigma_\perp^2 + \tau_\perp^2} = \rho_\perp = \sqrt{2}\sigma_\perp = 393 / a_w$$

and $\Delta\tau = \tau_{//} = 140.2 / a_w$

From $$\left(\frac{393}{52.6a_w / 1.25}\right)^3 + \left(\frac{140.2}{116.9a_w / 1.25}\right)^5 \leq 1$$

we obtain $a_w = 9.3$, rounded 10 mm.

### 2.2 Example: compressed strut of circular hollow section (CHS) with a welded splice (Fig.2)

A CHS strut is concentrically compressed by a permanent force $-F_G$ (minus denotes compression) and a variable load $F_Q$ pulsating between $+F_Q$ and $-F_Q$. $F_Q$ contains also a dynamic factor. The strut is constructed with a splice fillet welded end-to-end with an intermediate transverse plate. In the optimization of the strut section the unknown dimensions $D$ and $t$ are sought to minimize the strut cross-sectional area.

In the case of two unknowns the grapho-analytical optimization method can be advantageously applied. It is based on the theorem that, in the coordinate-system of the two unknowns, one of contours of the objective function touches the feasible region in the optimum point (Fig.3). This theorem can be derived by means of the method of Lagrange-multipliers [6].

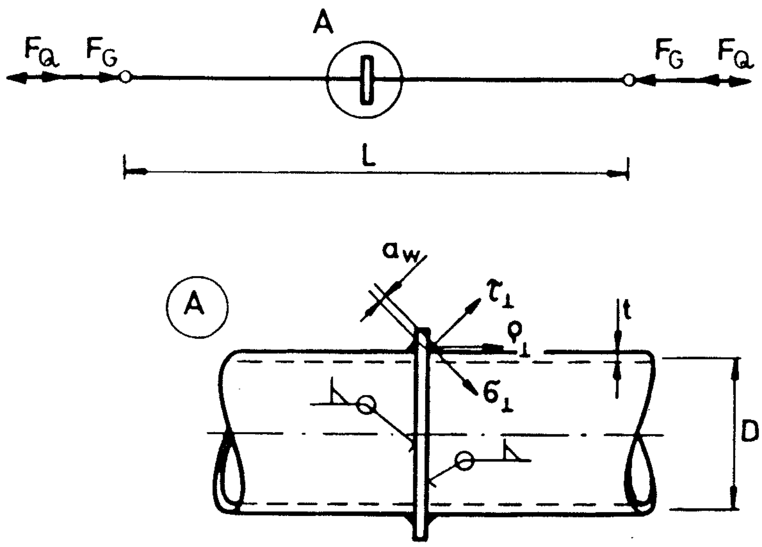


Fig.2. CHS compressed strut with a welded splice

In the optimum design of a compressed strut we use the unknowns $\vartheta = 100D / L$ and $\delta_C = D / t$ instead of $D$ and $t$ since the objective function can be expressed by

$$A = \pi D t = \frac{\pi D^2}{\delta_C} = \frac{\pi L^2 \vartheta^2}{10^4 \delta_C} \tag{23}$$

and the contours of the objective function are defined by

$$\delta_C = const * \vartheta^2 \tag{24}$$

which are straight lines in the coordinate-system $\delta_C - \vartheta^2$, so it is easy to find the touching point.

*The fatigue constraint* is expressed by

$$2F_Q / A \le \Delta\sigma_N / \gamma_{Mf} \quad \text{or} \quad \delta_C \le \frac{L^2 \pi \Delta\sigma_N}{2F_Q 10^4 \gamma_{Mf}} \vartheta^2 \tag{25}$$

According to the Recommendations [4] the fatigue stress range at $N = 2*10^6$ is $\Delta\sigma_C = 50$ MPa for splice of CHS with intermediate plate, toe crack, wall thickness smaller than 8 mm.

*The local buckling constraint* is defined by using the limiting local slenderness (according to EC3 for section class 1)

$$\delta_C \le \delta_{CL} = 50*235/f_{y1}; f_{y1} = f_y / \gamma_{M1}; \gamma_{M1} = 1.1 \tag{26}$$

*The static overall buckling constraint* is given by

$$(\gamma_G F_G + \gamma_Q F_Q)/A \le \chi f_{y1} \tag{27}$$

where $\gamma_G, \gamma_Q$ are the safety factors for permanent and variable load, respectively.

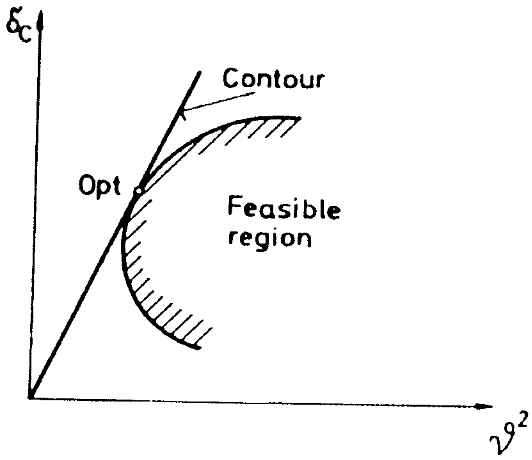


Fig.3. Grapho-analytical optimization method

According to EC3

$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}}; \phi = 0.5\left[1 + 0.34(\bar{\lambda} - 0.2) + \bar{\lambda}^2\right]; \bar{\lambda} = \frac{KL\sqrt{8}}{D\lambda_E} = \frac{100K\sqrt{8}}{\lambda_E \vartheta}; \lambda_E = \pi\sqrt{\frac{E}{f_{y1}}} \tag{28}$$

$K = 1$ for pinned ends [7]. Thus, Eq.(27) can be written as

$$\delta_C \le \frac{\pi \chi f_{y1}}{10^4 (\gamma_G F_G + \gamma_Q F_Q) L^2} \vartheta^2 \tag{29}$$

In a *numerical example* the given data are as follows. $F_G$ =300 kN, $2F_Q$ =145 kN, $\Delta\sigma_C = 50$ MPa, with Eq.(21) $\Delta\sigma_N = 43.7$ MPa, L = 5 m, $f_y$ = 235 MPa, $\gamma_{M1} = 1.1, \gamma_{Mf} = 1.25, \gamma_G = 1.35, \gamma_Q = 1.50$. $N$=3*$10^6$.

The fatigue constraint Eq.(25) is

$$\delta_C \le 1.8958 \vartheta^2 \tag{30}$$

The local buckling constraint Eq.(26) is

$$\delta_C \le 55 \tag{31}$$

In the overall buckling constraint the buckling factor can be calculated with simpler formulae of the Japanese Road Association which give values near to the EC3 column curve "b" [8].

$$\chi = 1 \quad \text{for} \quad 0 \le \bar{\lambda} \le 0.2 \tag{32a}$$

$$\chi = 1.109 - 0.545\bar{\lambda} \quad \text{for} \quad 0.2 \le \bar{\lambda} \le 1 \tag{32b}$$

$$\chi = 1/\left(0.773 + \bar{\lambda}^2\right) \quad \text{for} \quad \bar{\lambda} \ge 1 \tag{32c}$$

In our example from Eq.(29) we get

$$\delta_C \le 3.2654\vartheta^2(1.109 - 1.5649/\vartheta) \quad \text{for} \quad \vartheta^2 \ge 8.2447 \tag{33}$$

$$\delta_C \le 3.2654\vartheta^2/\left(0.773 + 8.2447/\vartheta^2\right) \quad \text{for} \quad \vartheta^2 \le 8.2447 \tag{34}$$

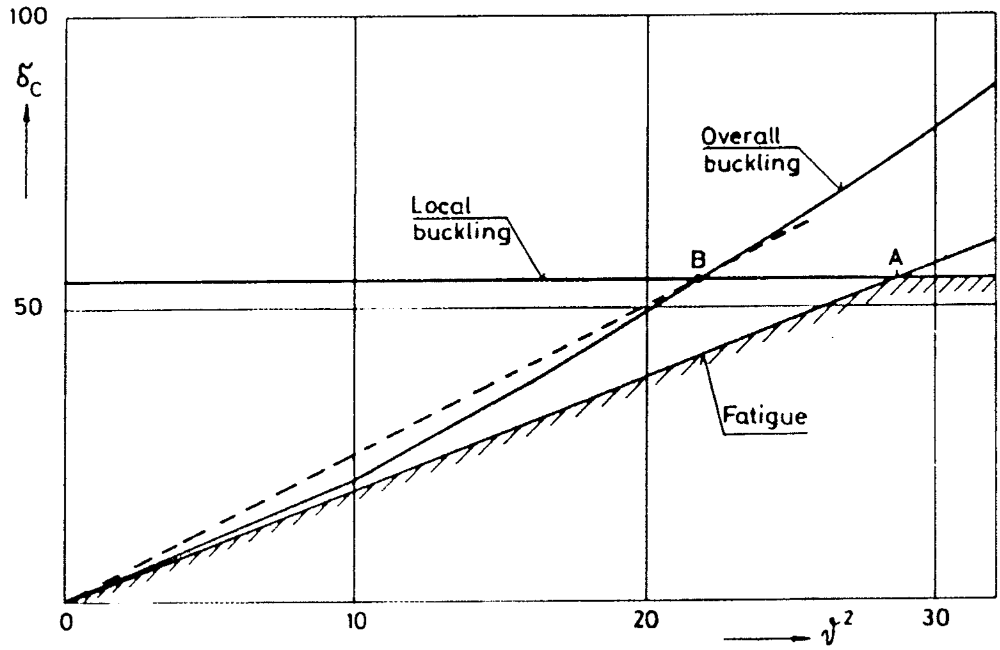


Fig.4. Graphoanalytical optimization of the strut shown in Fig.2.

Fig.4 shows the limiting lines of constraints in the coordinate-system of the two unknowns. These lines define the feasible region and the optimum point at the intersection of the constraint on fatigue and local buckling (point A), since the contour touches the feasible region in this point. This means that the overall buckling constraint is in this case passive. The result is

$$\vartheta^2_{opt} = 29.01, \vartheta_{opt} = 5.3862, D_{opt} = 269.3, t = D/55 = 4.9 \text{ mm},$$

we use the available section 273*5 mm.

The point B gives optimum when the fatigue constraint is not considered. The solution of Eq.(33) with $\delta_C = 55$ is $\vartheta = 4.6660, \vartheta^2 = 21.77, D = 233.3, t = 4.24$ mm.

If the value of $F_Q$ is kept constant, it is easy to find $F_G$ corresponding to point A, this means, when all three constraints are active $(\vartheta = 5.3862)$. From Eq.(33) $F_G = 455.9$ kN.

Finally, *the required fillet weld size* can be calculated from the fatigue constraint for root cracking. The pulsating load causes a stress range in the section

$$\Delta\sigma = 2F_Q / A = 145000/4210 = 34.4 \text{ MPa}$$

and stress components in the fillet weld (Fig.2)

$$\Delta\rho_\perp = \Delta\sigma t / a_w = 172.2/a_w; \Delta\sigma_\perp = \Delta\tau_\perp = \Delta\rho_\perp / \sqrt{2} = 121.8/a_w$$

According to the Recommendations [4] $\Delta\sigma_C = 40, \Delta\tau_C = 80$ MPa. For the given number of cycles $N = 3*10^6$ one obtains $\Delta\sigma_N = 34.9, \Delta\tau_N = 73.8$ MPa. Using the interaction formula Eq.(22) we get

$$83.02/a_w^3 + 37.37/a_w^5 \le 1$$

from which $a_w = 5$ mm.

Note that the static stress constraint for fillet welds is in this case also passive.

## 3. CONCLUSIONS

Two numerical examples illustrate the optimum design procedure when the fatigue and buckling constraints should be treated simultaneously for welded structures. In the case of a welded I-section cantilever the local buckling constraints are calculated with the actual maximal static stress instead of yield stress. The optimal beam dimensions can be calculated on the basis of the fatigue stress range for toe cracking of the connecting welds, while the required weld sizes are determined from the fatigue stress range for root cracking. In the case of a compressed CHS strut the grapho-analytical optimization method is advantageous, since it clearly shows the active and passive constraints. The fatigue or the overall buckling constraint may be active depending on the ratio of permanent and variable load.

**Acknowledgements**

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## References

[1] Farkas,J., "Absorbed energy of CHS and SHS braces cyclically loaded in tension-compression", Tubular Structures V. (Spon, London etc. 1993) pp. 607-614.

[2] Skaloud,M., *Thin-Walled Structures* **20** (1994) 83-95.

[3] Farkas,J., "Minimum cost design of tubular trusses considering buckling and fatigue constraints", Tubular Structures, (Elsevier, London-New York, 1990) pp. 451-459.

[4] Recommendations on fatigue of welded components, IIW-Doc. XIII-1539-95/XV-845-95.

[5] Farkas,J., Optimum design of metal structures. (Akadémiai Kiadó, Budapest, Ellis Horwood, Chichester, 1984)

[6] Farkas,J., "Optimum design of circular hollow section beam-columns" 2nd Internat. Offshore and Polar Engineering Conference, San Francisco 1992 (Internat. Society of Offshore and Polar Engineers, Golden, Colorado, USA. 1992) **IV** pp. 494-499.

[7] Farkas,J. and Jármai,K., "Savings in weight by using CHS or SHS instead of angles in compressed struts and trusses", Tubular Structures VI. (Balkema, Rotterdam, 1994) pp. 417-422.

[8] Hasegawa,A., Abo,H. et al. *Proc. JSCE Structural Engineering/Earthquake Engineering,* **2** (1985) 121-129.