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Tubular Structures IX

Edited by

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Height optimization of a triangular CHS truss using an improved cost function

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ABSTRACT: Optimum design of a simply supported, statically loaded triangular CHS truss of parallel chords is dealt with. The members are grouped into 8 groups having the same cross-sectional area. The optimum strut dimensions and the truss height is sought, which minimizes the structural mass or cost. The design constraints relate to the flexural and local buckling of compression members as well as to the strength and eccentricity of joints. The improved cost function includes the costs of material, cutting and grinding of strut ends, assembly, welding and painting. Since the fabrication cost is high compared to the total cost, it influences the optimum height. Therefore, the optimum heights for minimum mass as well as for minimum cost are different.

1 INTRODUCTION

Triangular tubular trusses are used in various structures such as bridges, spatial roofs, towers, cranes, etc. The aim of this study is to work out the minimum cost design process for such a structure.

For welded plated structures we use a relatively simple cost function, in which the fabrication cost includes the cost of assembly, welding and additional costs (deslagging, chipping, electrode changing) (Farkas & Jármai 1997, Jármai & Farkas 1999). A previous study (Tizani et al. 1996) has shown that, for tubular trusses, other cost components are also important. Thus, we formulate a more realistic cost function containing the cost of material, cutting and grinding of strut ends, assembly, welding and painting.

A numerical example of a triangular truss is dealt with in a CIDECT Design Guide (Wardenier et al. 1991) without optimization. Durfee (1987) has optimized the height of a triangular truss bridge welded from rectangular hollow sections minimizing the structural weight.

If we study the structural mass or cost in the function of height of a truss with parallel chords, we can conclude that an optimum height should exist, which minimizes the mass or cost. This fact can be explained as follows. Increasing the height, the chord forces decrease, but the length of the branch increases. On the other hand, by decreasing the height, the chord forces increase, but the branch length decreases. Thus, our aim is to determine the optimum height of a statically loaded, simply supported triangular truss of parallel chords welded from circular hollow sections (CHS) minimizing the structural weight and cost. The optimum design phases are as follows:

(a) problem formulation,
(b) calculation of member forces in the function of truss height,
(c) determination of member groups having the same cross-sectional area and definition of design variables,
(d) formulation of design constraints relating to the members and joints,
(e) formulation of the cost function,
(f) constrained function minimization using an efficient mathematical method,
(g) additional discretization considering the available CHS profiles,
(h) evaluation of results, conclusions.

2 PROBLEM FORMULATION

The investigated triangular CHS truss (Fig.1) has parallel chords, contains 51 struts, is simply supported, statically determined, subject to node forces from a uniformly distributed normal static load. The N-type joints are welded with gap.

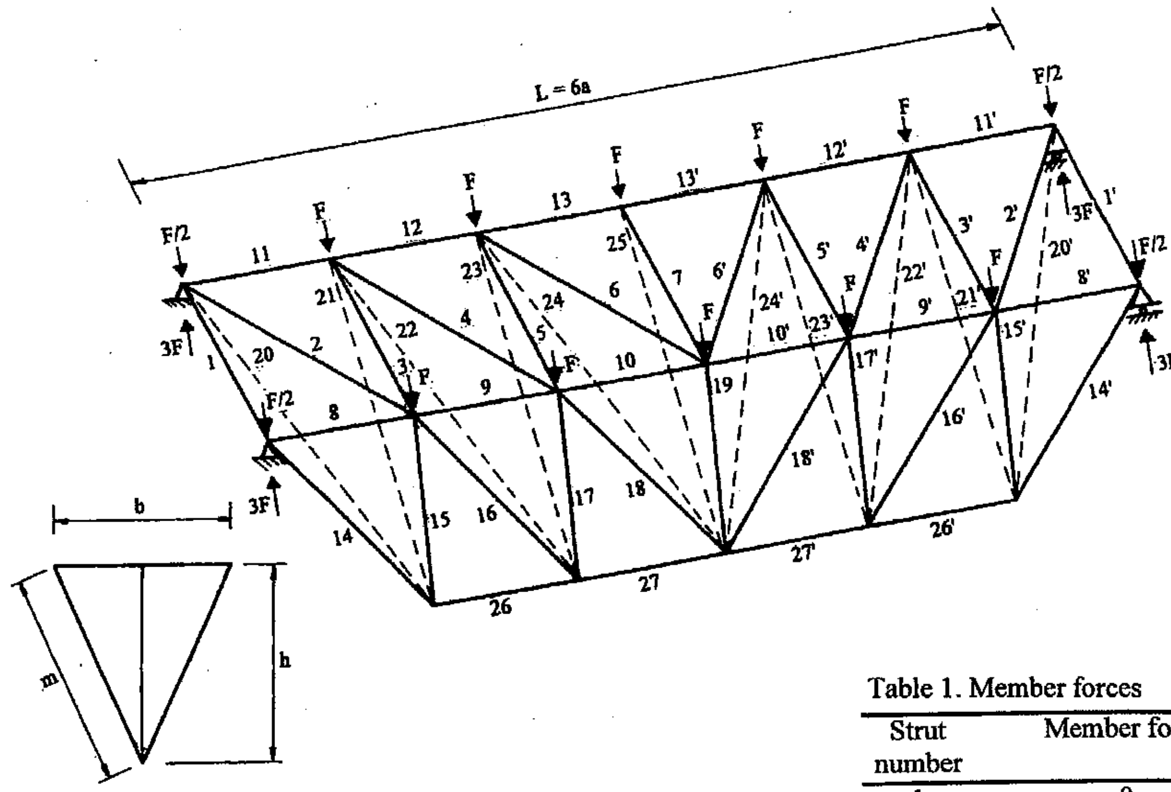

Figure 1. Triangular CHS truss with parallel chords

Data: $a = 7625$, $b = 10{,}675$, $b/a = 1.4$, $L = 45{,}750$ mm, factored load $F = 200$ kN, yield stress of the steel $f_y = 355$ MPa, elastic modulus $E = 2.1\text{x}10^5$ MPa, hot finished CHS profiles according to prEN 10210-2 (1996).

The problem is to determine the optimum cross-section dimensions for member groups and the optimum truss height minimizing the structural mass as well as cost and fulfilling the design constraints.

3 CALCULATION OF MEMBER FORCES

Member forces are expressed in the function of truss height ratio of $\omega = h/a$ and are summarized in Table 1. Minus sign denotes compression.

4 DETERMINATION OF MEMBER GROUPS

It would be uneconomic to use the same cross-sectional area for all the members. On the other hand, it would be unsuitable for fabrication to use different cross-sections for all the members. Thus, based on the previous calculations, we define member groups having the same cross-sectional area. These groups are given in Table 2.

It can be seen that the number of variables is 12. Note that the dimensions of horizontal columns and

Table 1. Member forces

Strut number	Member force
1	0
2	0
3	$0.7F/\omega$
4	0
5	$0.7F/\omega$
6	0
7	$0.7F/\omega$
8	$-2.5F/\omega$
9	$-4F/\omega$
10	$-4.5F/\omega$
11	$-2.5F/\omega$
12	$-4F/\omega$
13	$-4.5F/\omega$
14	$2.5F(1.49+\omega^2)^{0.5}/\omega$
15	$-2.5F(0.49+\omega^2)^{0.5}/\omega$
16	$1.5F(1.49+\omega^2)^{0.5}/\omega$
17	$-1.5F(0.49+\omega^2)^{0.5}/\omega$
18	$0.5F(1.49+\omega^2)^{0.5}/\omega$
19	$-F(0.49+\omega^2)^{0.5}/\omega$
20	$2.5F(1.49+\omega^2)^{0.5}/\omega$
21	$-2.5F(0.49+\omega^2)^{0.5}/\omega$
22	$1.5F(1.49+\omega^2)^{0.5}/\omega$
23	$-1.5F(0.49+\omega^2)^{0.5}/\omega$
24	$0.5F(1.49+\omega^2)^{0.5}/\omega$
25	$-F(0.49+\omega^2)^{0.5}/\omega$
26	$5F/\omega$
27	$8F/\omega$

Table 2. Member groups, variables, strut lengths

Members	Strut number	Variables	Strut length
upper chords	8,9,10,8',9',10', 11,12,13,11', 12',13'	d_{10}, t_{10}	a
lower chord	26,27,26',27'	d_{27}, t_{27}	a
horizontal columns	1,3,5,7,1',3',5'	d_1, t_1 = constant	b
horizontal braces	2,4,6,2',4',6'	d_2, t_2 = constant	$(a^2+b^2)^{0.5}$
braces	14,20,14',20'	d_{14}, t_{14}	$a(1.49+\omega^2)^{0.5}$
braces	15,21,15',21'	d_{15}, t_{15}	$a(0.49+\omega^2)^{0.5}$
braces	16,22,18,24,16' 22',18',24'	d_{16}, t_{16}	$a(1.49+\omega^2)^{0.5}$
braces	17,23,19,25,17' 23'	d_{17}, t_{17}	$a(0.49+\omega^2)^{0.5}$

braces do not depend on the truss height and will be calculated using the constraint on maximum slenderness.

5 DESIGN CONSTRAINTS

For the horizontal branch we use the rules for maximum slenderness of BS 5950 (1987). For horizontal columns, in which forces arise from vertical load, $KL/r \le 180$, from which r_{min} = 0.75x10625/180 = 44.5 mm; for horizontal diagonals, in which forces do not arise from vertical load, $KL/r \le 250$, from which r_{min} = 0.75x1 3119/250 = 39.4 mm, Thus, for horizontal columns and diagonals we use a CHS profile of 139.7x5 with r = 47.7 mm.

Since we should determine the optimum height, it is necessary to formulate the design constraints and the cost in the function of truss height ratio ω.

Stress constraint for tension members (chord 27, diagonals 14, 16)

$$N_i / A_i \le f_y / \gamma_{M0}; \qquad A_i = \pi(d_i - t_i)t_i; \qquad \gamma_{M0} = \gamma_{M1} = 1.1 \tag{1}$$

Overall buckling constraint for compression members (chord 10, columns 15, 17)

$$N_i / A_i \le \chi_i f_y / \gamma_{M1}; \quad \chi_i^{-1} = \phi_i + \left(\phi_i^2 - \bar{\lambda}_i^2\right)^{0.5};$$

$$\phi_i = 0.5\left[1 + 0.34\left(\bar{\lambda}_i - 0.2\right) + \bar{\lambda}_i^2\right];$$
$$\bar{\lambda}_i = \lambda_i / \lambda_E; \; \lambda_E = \pi\left(E / f_y\right)^{0.5};$$
$$\lambda_i = K_{C,D} L_i / r_i; \qquad r_i = \left(I_i / A_i\right)^{0.5};$$
$$I_i = \pi\left(d_i - t_i\right)^3 t_i / 8 \tag{2}$$

For chords K_C = 0.9, for columns and diagonals K_D = 0.75.

For all the constraints on joint strength we use a multiplier of 0.9 expressing the multiplanarity (Wardenier et al.1991).

Constraints on chord plastification

N-joint of struts 14-15-26

$$N_{14} \le 0.9 \frac{f_y t_{27}^2}{\sin\varphi}\left(1.8 + 10.2\frac{d_{14}}{d_{27}}\right) f_{14}(\gamma_{27}, g') \tag{3}$$

where

$$f_{14}(\gamma_{27}, g') = \gamma_{27}^{0.2}\left[1 + \frac{0.024\gamma_{27}^{1.2}}{\exp(0.5g' - 1.33) + 1}\right];$$

$$\gamma_{27} = \frac{d_{27}}{2t_{27}}; \; g' = \frac{g}{t_{27}}; \; g = t_{14} + t_{15};$$

$$\sin\varphi = \left(\frac{\omega^2 + 0.49}{\omega^2 + 1.49}\right)^{0.5}$$

$$N_{15} \le 0.9 f_y t_{27}^2\left(1.8 + 10.2\frac{d_{15}}{d_{27}}\right) f_{14}(\gamma_{27}, g')$$

Similar constraint is valid also for N-joint of struts 16-17-27. For N-joint of struts 15-16-9 the formulae are also similar, but a multiplier of f_9 should be used expressing that N_9 is a compressive force.

Y-joint of struts 14-8

$$N_{14} \le 0.9 \frac{f_y t_{10}^2}{\sin\varphi}\left(2.8 + 14.2\frac{d_{14}^2}{d_{10}^2}\right)\gamma_{10}^{0.2} f\left(n_8'\right) \tag{4}$$

where

$$\gamma_{10} = \frac{d_{10}}{2t_{10}}; \qquad f\left(n_8'\right) = 1 - 0.3n_8'(1 + n_8');$$

$$n_8' = \left|\frac{N_8}{A_8 f_y}\right|$$

The calculations have shown that the constraint on punching shear is passive.

Constraints on joint eccentricity

Eccentricity in longitudinal direction

$e / d_0 \le 0.25$ or

$$\left(\frac{d_i}{2\sin\varphi}+t_i+t_k+\frac{d_k}{2}\right)\tan\varphi \le 0.75d_j \quad (5)$$

$$\tan\varphi = \left(\omega^2+0.49\right)^{0.5}$$

For joint 14-15-27 it is $i = 14$, $j = 27$, $k = 15$, for joint 15-16-9 it is $i = 15$, $j = 10$, $k = 16$.
Eccentricity in transverse direction (Fig.2)

From

$$\tan\varphi_1 = \frac{\frac{d_i}{2}+\frac{g_i}{2\cos\varphi_1}}{\frac{d_0}{2}+e_0\cos\varphi_1}$$ one obtains

$$e_0 = \frac{d_i\cos\varphi_1+g_i}{2\cos^2\varphi_1\tan\varphi_1}-\frac{d_0}{2\cos\varphi_1}\le 0.25d_0 \quad (6)$$

where $g_i = 2t_i$; $\tan\varphi_1 = 0.7/\omega$;

$$\cos\varphi_1 = \frac{\omega}{\left(\omega^2+0.49\right)^{0.5}}$$

Local buckling constraint

$$d_i/t_i \le 50;\ i = 10, 14, 15, 16, 27 \quad (7)$$

Size limitation to enable the fabrication

$$d_i < d_{10} \text{ and } d_i < d_{27};\ i = 14, 15, 16, 17 \quad (8)$$

6 COST FUNCTION

The costs of material, cutting and grinding of strut ends, assembly, welding and painting are considered as follows.

$$K = K_M + K_C + K_A + K_W + K_P \quad (9)$$

It is assumed that in the material cost of

$$K_M = \rho\sum_i k_{Mi}A_iL_i \quad (10)$$

the material cost factors k_{Mi} depend only on the strut diameter. On the basis of the British Price List (1995) for hot finished CHS of yield stress $f_y = 355$ MPa, taking into account that 1 £ = 1.6$, the following material cost factors are calculated (Table 3).

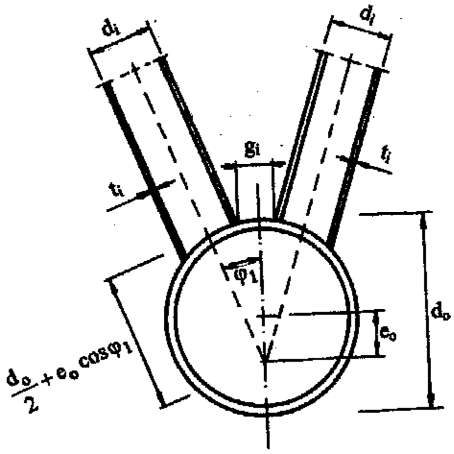

Figure 2. Joint eccentricity in transverse direction

Table 3. Material cost factors for available CHS diameters

d (mm)	k_M ($/kg)
88.9, 101.6, 114.3	1.0553
139.7, 168.3, 177.8, 193.7	1.1294
219.1, 244.5, 273.0, 323.9	1.2922
355.6, 406.4	1.3642
457.0, 508.0	1.4081

Since the horizontal branch dimensions do not depend on the truss height, it is possible to calculate its cost in advance. We denote the values of horizontal branch by index HB.

The volume of the structure is given by

$$V = V_{HB} + 12A_{10}L_{10} + 4A_{27}L_{27} + 4A_{14}L_{14} + 8A_{16}L_{16} + 4A_{15}L_{15} + 6A_{17}L_{17} \quad (11)$$

where $V_{HB} = 325.29\text{x}10^6$ mm^3, $K_{M,HB} = 2884$ $.

For hand cutting and machine grinding of two strut ends we use the following formula (Farkas & Jármai 1997)

$$K_C = k_F\Theta_C\sum_i\frac{2\pi d_i}{\sin\varphi}\left(4.54+0.4229t_i^2\right) \quad (12)$$

where the fabrication cost factor is taken on the basis of Tizani et al. (1996) as £25/h = 40$/h = 0.6667 $/min, and the difficulty factor is considered as $\Theta_C = 3$. d_i is in m, t_i is in mm. For the horizontal branch it is $K_{C,HB}$ = 381$. Without the horizontal branch it is

$$\sum_i\frac{2d_i}{\sin\varphi_i}f(t_i) = \frac{8d_{14}}{\sin\varphi}f(t_{14}) + 8d_{15}f(t_{15}) +$$

$$+\frac{16d_{16}}{\sin\varphi}f(t_{16})+12d_{17}f(t_{17})$$

$$f(t_i)=4.54+0.4229t_i^2$$

Note that Glijnis (1999) proposed a formula for one strut end in the case of oxyfuel cutting on CNC machine as follows:

$$K_C(\$)=\frac{2.5\pi d_i}{(350-2t_i)0.3\sin\varphi_i} \tag{13}$$

where 350 mm/min is the cutting speed, 0.3 is the efficiency factor, d_i and t_i are in mm.

It should be noted that the cutting cost values of Tizani et al.(1996) relate to a diameter of 60 mm, so they cannot be used for our example.

The cost of assembly is calculated with the formula of (Farkas & Jármai 1997)

$$K_A=k_F C_A \Theta_A(\kappa\rho V)^{0.5} \tag{14}$$

C_A =1.0min/kg$^{0.5}$

where $\kappa=38$ is the number of structural elements to be assembled. Note that the 3 chords are assumed to be welded to whole length before the assembly. The difficulty factor is taken as $\Theta_A=3.5$ expressing the complexity of the assembly. Note that K_A for horizontal branch cannot be calculated separately because of the non-linearity of formula (14).

For cost of welding the following formula is used

$$K_W=k_F\Theta_W\sum_i C_{Wi}a_{Wi}^n L_{Wi} \tag{15}$$

where $C_W a_W^n$ is given for different welding technologies and weld types on the basis of COSTCOMP software (Bodt 1990, COSTCOMP 1990, Farkas & Jármai 1997, Jármai & Farkas 1999). Here we use for SMAW (shielded metal arc welding) of fillet welds

$C_W a_W^n=0.7889x10^{-3}t_i^2$; considering also the additional welding costs such as electrode changing, deslagging, chipping, the difficulty factor is taken as $\Theta_W=4$;

$L_{Wi}=2\pi d_i/\sin\varphi_i$; L_W in mm.

For the horizontal branch it is $K_{W,HB}$ = 664\$.

Without the horizontal branch it is

$$\sum_i a_{Wi}^n L_{Wi}/(2\pi)=\frac{4t_{14}^2 d_{14}}{\sin\varphi}+4t_{15}^2 d_{15}+$$

$$+\frac{8t_{16}^2 d_{16}}{\sin\varphi}+6t_{17}^2 d_{17}$$

The cost of painting is defined by

$$K_P=k_P\Theta_P\sum_i \pi d_i L_i \tag{16}$$

where, according to Tizani et al. (1996) k_P = 14.4 \$/m^2, $\Theta_P=2$. For the horizontal branch it is $K_{P,HB}$ = 1939\$.

Without the horizontal branch it is

$$\sum_i d_i L_i=12d_{10}L_{10}+4d_{27}L_{27}+$$
$$+4d_{14}L_{14}+4d_{15}L_{15}+8d_{16}L_{16}+$$
$$+6d_{17}L_{17} \tag{17}$$

7 MATHEMATICAL CONSTRAINED FUNCTION MINIMIZATION AND RESULTS

It would be possible to search for optimum truss height by function minimization. The cost function is rather complicate, so it is simpler to use the mathematical method only for cross-sectional variables for given series of ω-values, and then to select the optimum height ratio based on the calculated objective function values.

The determination of optimum values of variables, which minimize the structural mass as well as cost, is performed by using the Rosenbrock's hillclimb method (Farkas & Jármai 1997) with an additional search for discrete optimum values corresponding to prEN 10210 (1996).

The results are summarized in Tables 4, 5 and 6.

Table 4. Mass and total cost

$\omega=h/a$	ρV (kg)	K (\$)
0.7	18572	37188
0.8	17806	36520
0.9	17709	**35775**
1.0	**17326**	36264
1.1	20198	40679

The optima are marked by bold letters. It can be seen that the optimum heights corresponding to the minimum mass as well as to minimum cost are different. This is caused by the high fabrication cost as it is given in Table 5.

Table 5. Parts of the total cost for $h/a = 0.9$ in \$

K_M	21879
K_A	1914
K_C	1324
K_W	2466
K_P	8192
K	35775

Table 6. Optimum dimensions in mm of member groups for h/a=0.9

Member group	Optimum dimensions
d_{10}, t_{10}	273x12.5
d_{27}, t_{27}	355.6x12.5
d_1, t_1	139.7x5
d_2, t_2	139.7x5
d_{14}, t_{14}	177.8x5
d_{15}, t_{15}	193.7x8
d_{16}, t_{16}	88.9x6
d_{17}, t_{17}	193.7x5

In this case the ratio fabrication cost/total cost is 100(13896/35775) = 39%. Note that the painting cost is relatively high compared to the other fabrication costs.

The optimum strut dimensions for h/a = 0.9, according to member groups of Table 2 are given in Table 6.

8 CONCLUSIONS

One of the main structural characteristics of trusses with parallel chords is the distance between chords, i.e the truss height. In the optimization of truss geometry this height is sought, which minimizes the structural mass or cost. This height optimization is shown for a simply supported, statically loaded triangular CHS truss.

Design constraints relate to overall and local buckling of truss members as well as to the strength and eccentricity of joints. An advanced cost function is formulated including the cost of material, cutting and grinding of strut ends, assembly, welding and painting.

The following general conclusions can be drawn. Parallel chord trusses always have an optimum height, which minimize the mass or cost. The optimum height for minimum mass differs from that for minimum cost. The difference depends on the ratio of fabrication/total cost. In the fabrication cost of tubular trusses the cost components considered in this study play an important role.

Designers want to use rules concerning the h/L ratio. In our case this ratio is for minimum weight design h/L = 1/6, for minimum cost design 1/6.7. In a numerical example of a uniplanar CHS truss with parallel chords (Farkas & Jármai 1997, Sect.11.2) optimized for minimum weight it is h/L = 1/9. This difference can be explained by the fact that a triangular truss should have higher height, since it has only three chords. In another example (Farkas & Jármai 1997, Sect. 13.2) a N-type SHS truss of parallel chords optimized for minimum weight for a belt-conveyor bridge has an optimum ratio of h/L = 1/8. Unfortunately, a general rule cannot be given, since this ratio depends on type of structure and on loads.

The constraints relating to the member and joint strength as well as the geometric limitations of joint eccentricity can be active, so it is important to consider them in the optimization process.

ACKNOWLEDGEMENTS

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