



Tubular Structures VII

József Farkas and
Károly Jármai, editors

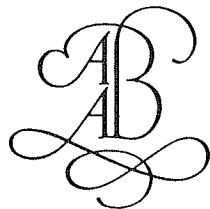
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Cover photo: Tubular roof structure of a new sports hall in Budapest

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Static and dynamic tests on aluminium square hollow section members combined with fiber reinforced plastic and rubber layers

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ABSTRACT: In order to investigate the effect of fiber reinforced plastic (FRP) and rubber layers on the static and dynamic behaviour of aluminium square hollow section (SHS) member five different specimens have been tested by measuring the static deflection and vibration damping. The FRP layers increase the bending stiffness but have no significant effect on damping. The damping can be increased by a rubber layer at the neutral axis of two combined hollow sections, but this layer decreases the bending stiffness. The static and dynamic bending theory of sandwich beams with thick faces is suitable for the calculation of deflections and loss factors of beams constructed from two SHS profiles, a rubber layer glued between them and FRP layers.

1 INTRODUCTION

Fiber reinforced plastics (FRP) are relatively new materials and give many possibilities of application because of their advantageous characteristics. These advantages are the lightweight and high load-carrying capacity. Disadvantages are the manufacturing difficulties and the high costs.

Damping of tubular members is an important design aspect and a lot of investigations have been carried out to increase the damping of tubes. Olcott (1992) has tested tubes with layers of highly dissipative materials constrained by segmented FRP layers. Varying the ply orientation angle in segments creates multiple regions of high shear resulting in high damping. Tubes with loss factors up to 8.5% have been manufactured and tested.

Sattinger and Sanjana (1993) describe tests on internally damped tubes with continuous and segmented constraining layers. Loss factors of 0.08 were measured for global beam-bending and axial vibration modes and 0.05 for torsional mode.

In our research aluminium square hollow section (SHS) rods are combined with FRP and rubber layers. FRP layers increase the static stiffness and rubber layers increase the damping capacity. The aim of the research is to work out a design method for such composite tubular members. On the basis of analytical results it will be possible to work out also the optimum design to develop the most efficient structural versions

Our previous study (Farkas and Jármai 1982) has shown that the poor damping capacity of aluminium SHS beams can be improved by rubber layers because of their high damping factor. As a result of this investigation the static and vibration calculation as well as the minimum cost design has been worked out for sandwich beams constructed from two SHS bars and a rubber layer glued between them.

In the present investigation this study is continued with specimens containing also FRP layers. Static bending tests and vibration damping measurements serve to describe the most important characteristics of the investigated models. For the calculations the static and dynamic bending theory of sandwich beams with thick faces is applied.

2 STATIC BEHAVIOUR

In the static tests the maximum deflection is measured at the midspan of simply supported beams loaded by a concentrated force at midspan. Five specimens have been manufactured and tested (Fig.1).

2.1 Specimen A

An aluminium alloy SHS beam of dimensions 30*2 mm and span length $L=1800$ mm is used. All other specimens contain aluminium SHS bars with these dimensions. The cross-sectional area is $A = 216$

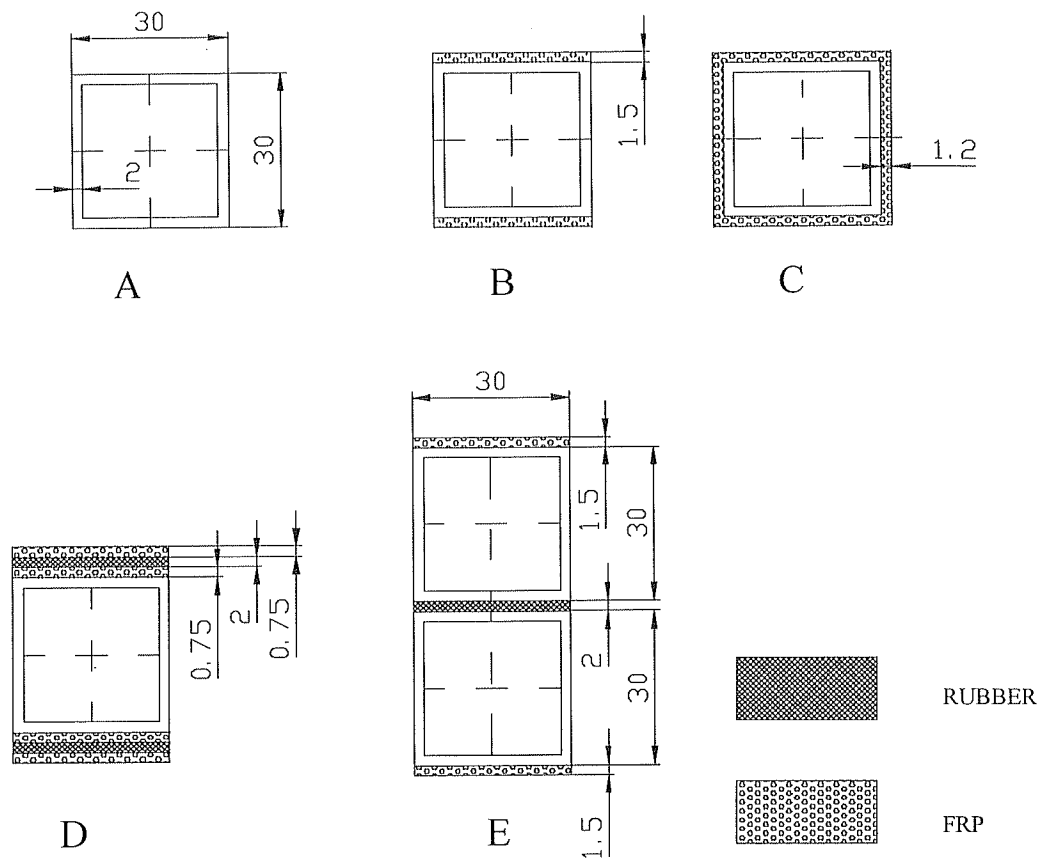


Fig.1. Tested specimens

mm^2 , the moment of inertia is $I_x = 26400 \text{ mm}^4$, the modulus of elasticity is $E = 7 \times 10^4 \text{ MPa}$, the maximum deflection for a force $F = 330 \text{ N}$ is

$w_{\max} = \frac{FL^3}{48EI_x} = 21.7 \text{ mm}$, which agrees with the measured value of 20 mm.

2.2 Specimen B

This specimen has two FRP layers glued on the SHS profile. The characteristics of the FRP layers are as follows. The modulus of elasticity along the fiber direction is (Hoa 1991)

$$E_L = E_f \frac{V_f}{V} + E_m \frac{V_m}{V} \quad (1)$$

where the subscripts f and m relate to the fibers and matrix, respectively. The FRP layers used in our

experiments have glass fibers and the volume fraction of fibers in longitudinal direction is 75% and in transverse one 25%. In a 0.15 mm thick layer of 1 mm^2 area the volume of fibers is $V_f = 0.75 \times 0.15 = 0.105 \text{ mm}^3$ and that of matrix $V_m = 0.24 \times 0.15 = 0.045 \text{ mm}^3$. Thus

$$\frac{V_f}{V} = \frac{0.105 \times 0.75}{0.105 \times 0.75 + 0.045} = 0.6364, \quad \frac{V_m}{V} = 0.3636$$

and using Eq.(1)

$$E_L = 71000 \times 0.6364 + 2200 \times 0.3636 = 45984 \text{ MPa}.$$

One FRP reinforcement contains 7 layers with the fiber orientation angles as follows: for 75% 0° , for 25% 90° . The reduced bending stiffness of the specimen B is calculated as

$$(EI)_{\text{red}} = E_A I_A + E_L I_C \quad (2)$$

where the subscripts *A* and *C* relate to aluminium and composite, respectively.

The manufactured thickness of the composite layer (FRP reinforcement) is $t_c = 1.5$ mm,

$$I_c = 2bt_c \left(\frac{b + t_c}{2} \right)^2 = 22326 \text{ mm}^4,$$

$$(EI)_{red} = 2.874 \times 10^9 \text{ Nmm}^2.$$

The maximum deflection due to a concentric force 210 N acting at midspan of a simply supported beam of span length 1850 mm is

$$w_{max} = \frac{FL^3}{48(EI)_{red}} = 9.64 \text{ mm} \quad (3)$$

the measured deflection was 9.88 mm, the agreement is good.

2.3 Specimen C

This specimen is similar to B, but the FRP layers are wound around the SHS profile. Thus, the moment of inertia of composite layers of manufactured thickness 1.2 mm is:

$$I_c = 2 \times 1.2 \times 30^3 / 12 + 2 \times 1.2 \times 32.4 \times 15.6 = 24324 \text{ mm}^4, \quad (EI)_{red} = 2.967 \times 10^9 \text{ Nmm}^2, \text{ the span length is 1800 mm, the maximum deflection due to a force 670 N is } w_{max} = 27.4 \text{ mm, which agrees with the measured 26.9 mm.}$$

2.4 Specimen D

Three layers are glued to the upper and lower flange of the SHS profile. Between two FRP layers a 2 mm thick rubber layer is glued to improve the vibration damping of the beam.

The two FRP layers are segmented as proposed by Olcott (1992) to increase the shear damping of the rubber layer. The fiber orientation for 75% of fibers is 27° (first layer; the values corresponding to this layer are denoted with last subscript 1) and for 25% perpendicular to 75° . (2nd layer, last subscript 2).

This 27° is positive for the layers above the rubber layer and minus for layers below it. The segmentation is performed with segment length of 40 mm and 10 mm overlaps.

The stiffness characteristics can be calculated according to the theory of FRP laminates (e.g. Hoa 1991).

$$V_{m1} = \frac{V_m}{2} = 0.02254 \text{ mm}^3$$

$$V_{f1} = 0.75V_f = 0.07875 \text{ mm}^3$$

$$V_1 = V_{m1} + V_{f1} = 0.10125 \text{ mm}^3$$

$$E_{L1} = E_f \frac{V_{f1}}{V_1} + E_m \frac{V_{m1}}{V_1} = 55711 \text{ MPa}$$

$$\nu_{L1} = \nu_f \frac{V_{f1}}{V_1} + \nu_m \frac{V_{m1}}{V_1} = 0.233 \quad (4)$$

$$\frac{1}{E_{T1}} = \frac{1}{\frac{V_{f1}}{V_1} + 0.5 \frac{V_{m1}}{V_1}} \left[\frac{V_{f1}}{V_1 E_f} + 0.5 \frac{V_{m1}}{V_1 E_m} \right]$$

$$E_{T1} = 14593 \text{ MPa}$$

$$\nu_{T1} = \nu_{L1} \frac{E_{T1}}{E_{L1}} = 0.06$$

where the subscripts *L* and *T* indicate the longitudinal and transverse Young's moduli, respectively, and ν - s denote the major and minor Poisson ratio.

The shear moduli of fibers and matrix, respectively, are

$$G_f = \frac{E_f}{2(1 + \nu_f)} = 29600 \text{ MPa}$$

$$G_m = \frac{E_m}{2(1 + \nu_m)} = 820 \text{ MPa}$$

$$\frac{1}{E_s} = \frac{1}{\frac{V_{f1}}{V_1} + 0.5 \frac{V_{m1}}{V_1}} \left[\frac{V_{f1}}{V_1 G_f} + 0.5 \frac{V_{m1}}{V_1 G_m} \right] \quad (5)$$

$$E_s = 2560 \text{ MPa}$$

$$m = \frac{1}{1 - \nu_{L1} \nu_{T1}} = 1.014$$

where E_s is the in plane-shear modulus of the composite.

$$Q_{LL1} = mE_L = 56500 \text{ MPa}$$

$$Q_{TL1} = mE_T \nu_L = 3450 \text{ MPa}$$

$$Q_{SS1} = E_s = 5500 \text{ MPa} \quad (6)$$

$$Q_{LT1} = mE_L \nu_T = 3390 \text{ MPa}$$

$$Q_{TT1} = mE_T = 14800 \text{ MPa}$$

where Q_{MNI} are the on-axis moduli appearing in the on-axis stress-strain relation

$$U_{1Q1} = \frac{1}{8} [3Q_{LL1} + 3Q_{TT1} + 2Q_{LT1} + 4Q_{SS1}]$$

$$U_{2Q1} = \frac{1}{2} [Q_{LL1} - Q_{TT1}]$$

$$U_{3Q1} = \frac{1}{8} [Q_{LL1} + Q_{TT1} - 2Q_{LT1} - 4Q_{SS1}] \quad (7)$$

$$U_{4Q1} = \frac{1}{8} [Q_{LL1} + Q_{TT1} + 6Q_{LT1} - 4Q_{SS1}]$$

$$U_{5Q1} = \frac{1}{8} [Q_{LL1} + Q_{TT1} - 2Q_{LT1} + 4Q_{SS1}]$$

where U_{iQ} are the combined moduli. These facilitate the transformation of material properties from on-axis coordinate system to off-axis.

$$\begin{aligned} Q_{111} &= U_{1Q1} + U_{2Q1} \cos 2\theta + U_{3Q1} \cos 4\theta \\ Q_{221} &= U_{1Q1} - U_{2Q1} \cos 2\theta + U_{3Q1} \cos 4\theta \\ Q_{121} &= U_{4Q1} - U_{3Q1} \cos 4\theta \\ Q_{661} &= U_{5Q1} - U_{3Q1} \cos 4\theta \\ Q_{161} &= 0.5U_{2Q1} \sin 2\theta + U_{3Q1} \sin 4\theta \\ Q_{261} &= 0.5U_{2Q1} \sin 2\theta - U_{3Q1} \sin 4\theta \\ \theta &= 27^\circ \end{aligned} \quad (8)$$

where Q_{ij1} are the off-axis moduli appearing in the off-axis stress-strain relation.

$$\begin{aligned} \Delta &= \det Q_{ij1} \\ S_{111} &= [Q_{221}Q_{661} - Q_{261}^2] / \Delta \\ S_{221} &= [Q_{111}Q_{661} - Q_{161}^2] / \Delta \\ S_{121} &= [Q_{111}Q_{661} - Q_{121}Q_{661}] / \Delta \\ S_{661} &= [Q_{111}Q_{221} - Q_{121}^2] / \Delta \\ S_{161} &= [Q_{121}Q_{261} - Q_{221}Q_{161}] / \Delta \\ S_{261} &= [Q_{121}Q_{161} - Q_{111}Q_{261}] / \Delta \end{aligned} \quad (9)$$

where S_{ij1} are the off-axis compliances, $S_{ij1} = Q_{ij1}^{-1}$.

$$\begin{aligned} E_{11} &= \frac{1}{S_{111}} = 23440 \text{ MPa} \\ E_{21} &= \frac{1}{S_{221}} = 13830 \text{ MPa} \\ E_{61} &= \frac{1}{S_{661}} = 8000 \text{ MPa} \end{aligned} \quad (10)$$

the moduli of elasticity of FRP layers in longitudinal direction are E_{11} and in the 2nd layer $E_{12} = 11920 \text{ MPa}$. The reduced bending stiffness of the whole beam is $(EI)_{red} = 2.45 \times 10^9 \text{ Nmm}^2$. Note that in calculation the rubber layers are neglected because of the low value of their elastic modulus.

The maximum deflection for a force of 480 N is 23.8 mm, which agrees well with the measured value of 22.4 mm.

2.5 Specimen E

This beam is constructed from two SHS profiles, a rubber layer glued between them and two FRP layers glued on the upper and lower flanges. The deflection should be calculated according to the theory of sandwich beams with thick faces, developed e.g. by Stamm and Witte (1974), which considers also the shear deformation of the core. The following formulae are given also in Farkas (1984). Here we consider the reduced stiffness of SHS and FRP layers as faces.

$$w_{max} = \frac{FL^3}{48B} + \frac{FL}{4B_q} \left(1 - \frac{B_f}{B} \right) \left(1 - \frac{\tanh \chi}{\chi} \right) \quad (11)$$

where

$$\begin{aligned} \chi &= \frac{1}{2} \left[\frac{B_f}{B_q L^2} \left(1 - \frac{B_f}{B} \right) \right]^{-1/2} \\ B &= B_f + B_s; \quad B_f = 2(EI)_{red} \\ B_s &= 2 \sum_i E_i A_i e_i^2 \\ B_q &= G_s b (h + t_2)^2 / t_2 \\ G_s &= 1.5 \text{ MPa being the static shear modulus of the rubber, } b = h = 30, \quad t_2 = 2 \text{ mm. } B_q = 23040, \quad B_f = 3.696 \times 10^9, \quad B_s = 1.048 \times 10^{10}, \quad B = 1.4176 \times 10^{10} \text{ Nmm}^2, \quad \chi = 2.6134, \quad w_{max} = 25.3 \text{ mm, which agrees well with the measured } 22 \text{ mm.} \end{aligned}$$

The deflections are summarized in Table 1 unified for a force 100 N and span length of 1800 mm.

Table 1. Deflections due to a force of 100 N

Specimen	Deflection (mm)
A	6.57
B	4.23
C	4.09
D	4.96
E	1.77
F	1.16

The difference of 35% between deflections for specimens A and B shows how the FRP layers increase the static bending stiffness of the SHS beam. For comparison we have calculated the deflection for a specimen F that means two SHS profiles rigidly connected to each other without any layers. It can be seen that the shear deformation of the soft rubber layer increases the deflection from 1.16 to 1.77 mm i.e. decreases the bending stiffness by 52%.

The applied calculation method gives suitable results as compared to the measured deflections. When the SHS profile is combined with FRP layers, the reduced bending stiffness can be used. In the case of a sandwich beam with soft core material (rubber), the theory of sandwich beams with thick faces gives good results.

3 DYNAMIC BEHAVIOUR

In order to determine the eigenfrequencies and vibration damping or loss factors the Brüel-Kjaer vibration measuring devices have been used in our laboratory. The loss factors are obtained by the evaluation of the automatically registered diagrams of acceleration versus frequency using the half-power bandwidth method developed by Oberst (1952) and described also in Nashif et al.(1985). The formula for the loss factor at the i th eigenfrequency f_i is

$$\eta_i = \Delta f / f_i \quad (12)$$

where Δf is the frequency bandwidth.

Fig.2 shows a frequency diagram. Fig.3 shows a photo from the measuring device with a specimen.

Table 2 gives the measured results.

Remarks to the data: specimen A: the material damping of aluminium profile is very low. Specimens B and C: the FRP layers do not increase the damping

Table 2. Measurement results: eigenfrequencies and loss factors

Specimen	i	f_i (Hz)	η_i
A	1	32	0.0125
	2	196	0.0028
	3	536	0.0015
B	1	33	0.0120
	2	200	0.0032
	3	543	0.0028
C	1	31	0.0014
	2	200	0.0031
	3	590	0.0032
D	1	28	0.0265
	2	171	0.0082
	3	460	0.0098
E	1	52	0.0520
	2	255	0.0570
	3	648	0.0530

capacity of SHS beams, since the damping of individual FRP layers is also low. Specimen D: The rubber layers of high damping capacity and the effect of segmentation of FRP layers causes an increase of the loss factor.

Specimen E: The rubber layer at the neutral axis causes a significant increase of the loss factor due to the shear damping, compared to specimen A the loss

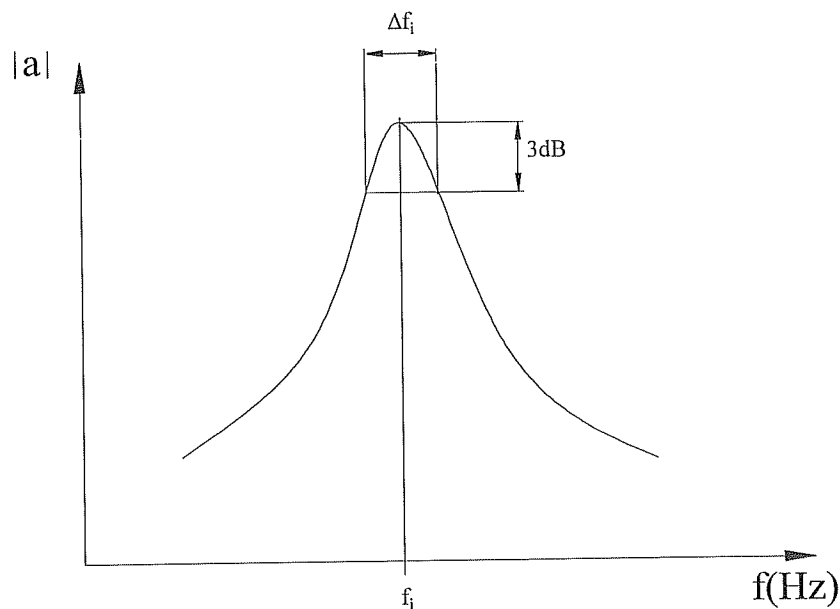


Fig.2.Measured acceleration as a function of frequency

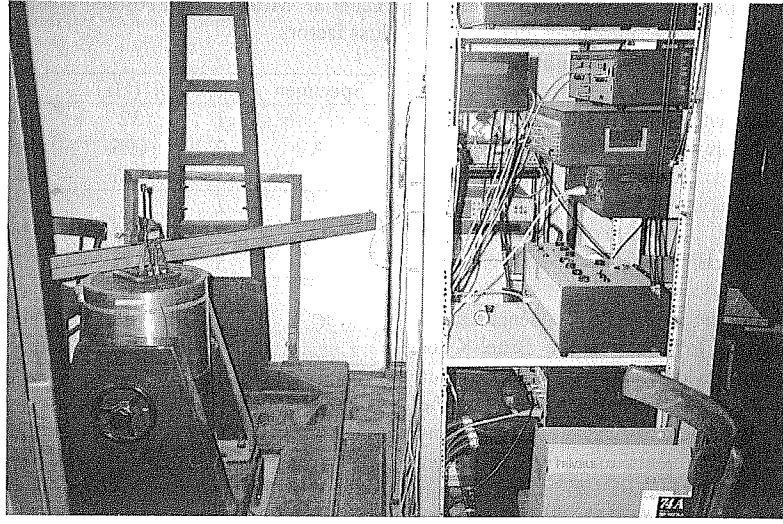


Fig.3.Vibration measurement of specimen E on Brüel-Kjaer devices

factor is quadrupled. This loss factor can be calculated using the formula for sandwich beams with thick faces developed by Ungar (1962) (also in Farkas 1984).

$$\eta = \frac{\eta_2 XY}{1 + (2 + Y)X + (1 + Y)(1 + \eta_2^2)X^2} \quad (13)$$

where

$$X = g_0 \left(\frac{C_d L}{2\pi} \right)^2$$

$$g_0 = \frac{2G_d b}{t_2 \sum_i A_i E_i}$$

$$Y = \frac{(b + t_2)}{2B_f} \sum_i A_i E_i$$

For rubber $G_d = 4.5$ MPa, according to Yin et al. (1967) $C_d = 1.1$, $L = 1800$ mm. $X = 0.8125$, $g_0 = 8.1818 \cdot 10^{-6} \text{ mm}^{-2}$, $Y = 2.2857$, $\eta = 0.0497$. This value is near to the measured ones.

4 CONCLUSIONS

The static bending stiffness of a SHS aluminium beam can be significantly increased by using FRP

layers. This increase was in the case of our investigated specimens about 35%, without any increase in weight.

The FRP layers do not increase the vibration damping, the loss factor is only about 1%.

The static behaviour of a SHS profile with FRP layers can be calculated by the reduced bending stiffness.

The damping can be significantly increased by applying a rubber layer of high damping capacity. This damping capacity can be increased by segmentation of the FRP constrained layers.

When the rubber layer is located at the neutral axis the damping capacity increases significantly due to high shear damping. In our case the loss factor has been quadrupled (comparison between specimens A and E). Due to a soft rubber layer the static bending stiffness decreased by 52%.

The static and dynamic behaviour of specimen E - a sandwich beam with two SHS profiles, a rubber layer between them and stiffening FRP layers - can be calculated with sufficient accuracy by the bending theory of sandwich beams with thick faces.

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