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# OPTIMUM DESIGN OF SQUARE HOLLOW SECTION BEAM-COLUMNS

## 1. Introduction

The development of fabrication technology enables the industry to produce thin-walled hollow sections which can be advantegeously applied in steel construction. Circular (CHS), square (SHS) and rectangular (RHS) hollow sections are used as structural elements of beams, columns, trusses and frames. Many theoretical and experimental studies have worked out the analysis and design of such structures [e.g. for CHS (Chen and Sohal, 1988)] and new standards, as the Eurocode 3 (1990) give design rules for calculation.

The aim of this paper is to complete the analytical results with optimization and to give designers calculation methods to select the most economic structural solution. We consider here only the cross-section area (weight) to be minimized, but in other studies we have worked out minimum cost design considering not only the material but also the fabrication cost (e.g. Farkas, 1984, 1990a).

The Eurocode 3 considers 4 classes of cross-sections. We calculate here only the class 3 - elastic stress distribution with max. yield stress without local buckling. Note that in some cases the cross-sections of class 4 may be more economical than those of class 3, but the calculation with effective widths is much more complicated. The optimum design of CHS compressed struts in post-buckling range has been treated in (Farkas, 1990b) and the optimum design of CHS beam-columns in (Farkas, 1992).

## 2. Optimum design of a SHS beam loaded in pure bending

The constraint on bending moment capacity is

$$W_x f_y \geq M_o \qquad (1)$$

where $W_x = 4b^2t/3$, $M_o$ is the factored bending moment and

$f_y$ is the yield stress. The constraint on local buckling of a compressed flange plate

$$b/t \leq 1/\delta \quad (2)$$

According to Eurocode 3 $\quad 1/\delta = 42\varepsilon \; ; \quad \varepsilon = \sqrt{235/f_y} \quad (3)$

Considering (2) as active: $t = \delta b$

and $\quad W_x = 4\delta b^3/3 \quad (4)$

From (1) one obtains

$$b_{opt} = \sqrt[3]{0.75 M_o/(\delta f_y)} \quad (5)$$

and

$$A_{min} = 4bt = \sqrt[3]{36\delta M_o^2/f_y^2} \quad (6)$$

## 3. Optimum design of a compressed SHS column

The overall buckling constraint is

$$N \leq N_b = \chi A f_y/\gamma_{m1} \quad (7)$$

where

$$1/\chi = \phi + \sqrt{\phi^2 - \bar{\lambda}^2}; \quad \phi = 0.5\left[1 + \alpha(\bar{\lambda} - 0.2) + \bar{\lambda}^2\right], \; \chi \leq 1$$

for cold-formed hollow sections $\alpha = 0.34$;

$$\bar{\lambda} = \lambda/\lambda_1 = KL/(i\lambda_1); \quad i = \sqrt{I/A} = b/\sqrt{6}; \quad \lambda_1 = \pi\sqrt{E/f_y} = 93.9\varepsilon$$

for pinned ends $K = 1$, so $\quad \bar{\lambda} = L\sqrt{6}/(b\lambda_1) \quad (8)$

The local buckling constraint is the same as (2), so (7) can be written in the form

$$N \leq 4\delta b^2 \chi f_y/\gamma_{M1} \quad (9)$$

$\gamma_{M1} = 1.1$. (9) is an equation for the unknown $b$, but the solution of this equation cannot be written in closed form.

## 4. Optimum design of a SHS beam-column

According to Eurocode 3 the overall buckling constraint is given in the form of an interaction equation as follows

$$\frac{N}{\chi A f_y} + k \frac{M}{W_x f_y} \leq 1 \quad (10)$$

where the moment amplification factor is

$$k = 1 - \frac{N}{\chi \gamma_{M1} A f_y}; \quad \text{but} \quad 1 \leq k \leq 1.5 \quad (11)$$

and

$$\mu = \bar{\lambda}(2\beta_M - 4), \; \mu \leq 0.90, \; \beta_M = 1.8 - 0.7\psi$$

We consider here two cases (Fig.1):

a/ When the strut is bent in single curvature: $\psi = 1$, $\beta_M = 1.1$, $\mu = -1.8\bar{\lambda}$;

b/ when bent in reverse curvature: $\psi = -1$, $\beta_M = 2.5$, $\mu = \bar{\lambda}$.

For case a/

$$k = 1 + \frac{1.8\bar{\lambda}\,N}{\chi\gamma_{M1}Af_y} \tag{12}$$

for case b/

$$k = 1 - \mu\frac{N}{\chi\gamma_{M1}Af_y}\ ;\ \text{if } \bar{\lambda} \le 0.9,\ \mu = \bar{\lambda};\quad \text{if } \bar{\lambda} > 0.9,\ \mu = 0.9 \tag{13}$$

Note that, for CHS beam-columns, another interaction equations are given by Sohal, Duan and Chen (1989) and by Duan and Chen (1990).

The local buckling constraint is the same as (2). Considering it as active $t = \delta b$, $1/\delta = 42\varepsilon$,

$$A = 4\delta b^2 = b^2/(10.5\varepsilon) \tag{14}$$

and

$$W_x = 4\delta b^3/3 = b^3/(31.5\varepsilon) \tag{15}$$

Instead of the unknown width b we use here the dimensionless variable

$$\beta = 100b/L \tag{16}$$

so

$$\bar{\lambda} = \frac{L\sqrt{6}}{93.9\varepsilon b} = \frac{2.61}{\varepsilon\beta} \tag{17}$$

and (10) can be written as

$$\frac{10.5\varepsilon \times 10^4 N/L^2}{\chi f_y \beta^2} + k\ \frac{31.5\varepsilon \times 10^6 M/L^3}{f_y\beta^3} \le 1 \tag{18}$$

This yields for $\beta$ a higher order equation which can be solved only numerically.

Note that, according to Usami (1989), the cross-sections of class 4 can be more economic than those of class 3 when $\bar{\lambda} > 1$.

<u>Numerical example</u>

$N = 950$ kN, $L = 7$ m, pinned ends $K = 1$.

$10^4 N/L^2 = 193.88$ N/mm$^2$. $A = 466.67\beta^2/\varepsilon$.

To show the effect of the bending moment M, we calculate the $\beta_{opt}$ values for $10^3 M/(NL) = 0;\ 5$ and $10$, this means

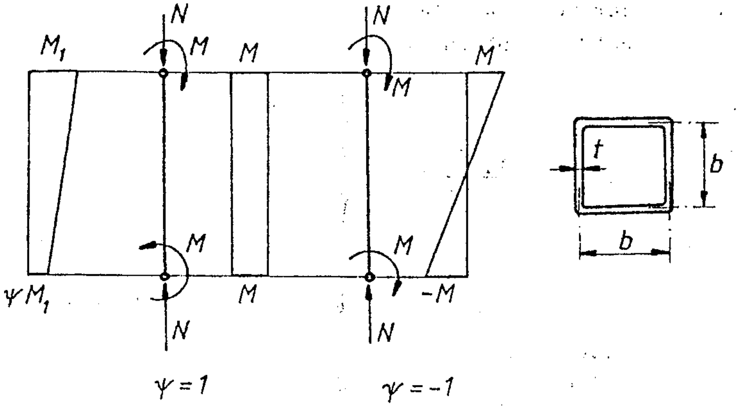


Fig.1 Bending moment distributions on a SHS beam-column

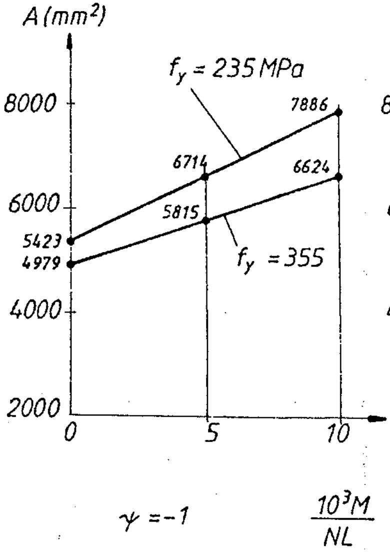


Fig.2 Effect of yield stress

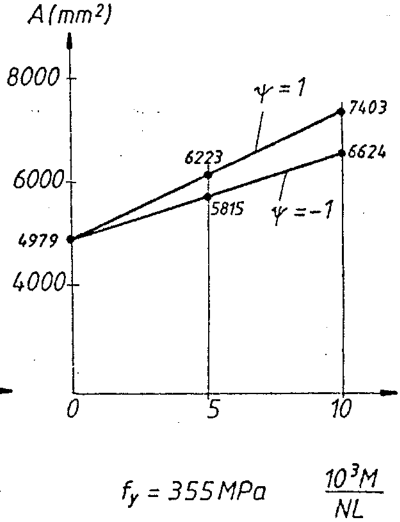


Fig.3 Effect of moment distribution

for $10^6 M/L^3$ = 0; 96.94 and 193.88 N/mm$^2$. To study the effect of yield stress, we calculate with $f_y$=235 and 355MPa. In another diagram we show the effect of the bending moment distribution for $\psi = 1$ and $\psi = -1$.

Fig.2 shows the effect of yield stress. By using a higher strength steel of yield stress 355 MPa instead of 235 MPa, designers can achieve 13-19% savings in weight.

Fig.3 shows the effect of moment distribution. For the case of $\psi = 1$ 7-12% greater cross sectional area is needed than for $\psi = -1$.

## 5. Conclusions

Cross-section minimization is treated for SHS beams, columns and beam-columns. The calculations are performed according to Eurocode 3, considering cross-sections of class 3. For the calculation of the unknown width and thickness the overall and local buckling constraints are considered as active. The computations have been carried out using the Rosenbrock hillclimb mathematical programming method. The effect of yield stress and the moment distribution is shown in Figs 2 and 3.

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