Tubular Structures VII

József Farkas and Károly Jármai, editors
Cover photo: Tubular roof structure of a new sports hall in Budapest

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Optimization of tubular columns prestressed by tension ties

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ABSTRACT: In the design of prestressed columns the effect of initial imperfections has been predominantly neglected in the past. The design of tubular columns prestressed by wire ropes is treated now on the basis of the European column buckling formulae which consider the initial imperfections. Columns of one and three intermediate supports are designed. The necessary prestressing is calculated considering the effect of temperature change and creep of cables. The optimal angle between the core tube and cables is sought in the case of one support to minimize the material cost of the core tube, cables and supporting bars.

KEYWORDS: Prestressed columns, prestressed steel structures, creep of cables, column buckling

1 INTRODUCTION

Prestressed columns are used as booms for erection purposes or as parts of higher towers. Their advantage is the high compressive strength and lightweight. A detailed study has been published dealing with the design of such columns by Chu and Berge (1963). On the basis of this analysis Mauch and Felton (1967) have worked out an optimum design procedure for these columns. This study has shown that the weight of the core tube can be significantly decreased by using prestressing with one or three intermediate supports.

The Euler overall buckling formula has been used which does not consider the initial crookedness of the column. Farkas and Jármai have pointed out (Farkas & Jármai 1995, Jármai & Farkas 1996) that the design with Euler formula leads to unsafe and imperfection-sensitive solutions. Thus, in present paper the Ayrton-Perry formulation and the Eurocode 3 (EC3)(1992) calculation method is used which considers the initial imperfections.

Moreover Mauch and Felton (1967) have used a local buckling calculation for aluminium tubes which leads to unrealistic circular hollow section (CHS) dimensions. E.g. in their numerical example a tube of $D^t = 176.5\times0.447\, \text{mm}$ with a local slenderness of 395 has been calculated. On the contrary, the British Standard BS 8118 (1991) prescribes, for an aluminium alloy strut of yield stress 240 MPa, a local slenderness limit of 56 (Jármai & Farkas 1996).

The calculation method used in the book of Belenya (1975) also neglects the initial imperfection and in the numerical example only one support is considered for a 30 m long column loaded by a compressive force of 2000 kN. Round steel bars have been applied as prestressing elements of diameter 30 mm with a limiting tensile stress of 170 MPa instead of high-strength steel wire ropes.

A calculation method based on the Euler solution is treated in Peterson's book (1980) without the effect of initial imperfections. Schock (1976) in his dissertation has considered the initial crookedness and used wire ropes as ties. Several structural versions have been calculated using different complicated systems of ties. For the comparison the material weight has been used since the cost of assembly was not known.

In the case of prestressing with high-strength cables it is necessary to take into account the
decrease of prestress during the service time due to creep and temperature changes. These effects have not been treated in the publications mentioned above.

In the present paper a relatively simple design method is proposed for steel CHS columns prestressed by high-strength cables with one or three intermediate supports. In the calculation of the necessary prestressing the effect of creep and temperature change is also considered. In the optimization the optimal angle between the core tube and cables is sought which minimizes the total cost. Since the cost of assembly of such prestressed structures is not known, in the cost function only the material cost of core tube, cables and supporting bars is considered. The shortening of the core tube and that of supporting bars is neglected. The design method is illustrated by a numerical example.

2 DESIGN OF COLUMNS WITH ONE SUPPORT (Fig.1)

Assume that the core tube is fixed by ties at its midpoint so that it buckles in the second mode (Fig.1). Thus, the core tube can be designed for a given factored external axial compressive force $F$ with an effective buckling length of $L/2$ and for a prescribed limiting local slenderness according to EC3

$$\delta_L = (D/\ell)_L = 90 * 235 / f_y$$

where $f_y$ is the yield stress. The design inequality is expressed by

$$\frac{F_T}{nD^2} / \delta_L \leq \gamma_{mu}$$

where $\gamma_{mu} = 1.1$ is the partial safety factor,

$$F_T = F + 3(F_p + S_e) \cos \alpha$$

$$1 / \lambda = \phi + \sqrt{\phi^2 - \lambda^2}$$

$$\phi = 0.5(1 + 0.34(\lambda - 0.2) + \lambda^2)$$

$$\lambda = \frac{K_L \sqrt{8} / (2D \lambda_e)}; \lambda_e = \frac{\pi \sqrt{E}}{f_y}$$

![Diagram](https://via.placeholder.com/150)

Fig.1. a) Prestressed column with one support. b) Initial crookedness and the elastic deformation of the column without prestressing. c) Second mode of column buckling. d) Forces in ties.e) Measure of prestressing.
\( E \) is the elastic modulus, \( K_e \) is the effective length factor, here for pinned ends \( K_e = 1 \). Note that \( F_p \) and \( S_p \) are cable forces calculated below (Eqs 7 and 13) but not known in advance, so we should guess these values and then correct them using an iteration process.

Eq.(2) can be solved for \( D \) numerically using a computer program. Knowing \( D \) and \( t \) the Euler overall buckling force for pinned ends can be calculated

\[
F_p = \pi^2 E I_e / (L/2)^2; I_e = 4D^3t / 8 \tag{3}
\]

where \( I_e \) is the moment of inertia, for which the table given by the standard DIN 2448 or 2458 can be used (Dutta & Wurker 1988). The EC3 formula is based on the Ayton-Perry formulation (Maquio & Rondal 1978). In this calculation the initial imperfection \( \alpha_0 \) and the elastic deformation \( \gamma_0 \) at the midpoint (Fig.1), which could occur without prestressing, is

\[
\alpha_0 = 0.34 \left( \frac{L}{r_{Ax}} - 0.2 \right) \frac{W_e}{A} \cdot \frac{D}{A} = \frac{D}{4} \tag{4}
\]

\[
\gamma_0 = F_p \alpha_0 / (F_p - F_c) \tag{5}
\]

where \( W_e \) is the elastic section modulus.

The horizontal force \( H_p \) which eliminates the bending moment \( F_2(\alpha_0 + \gamma_0) \) (Fig.1) is obtained from the equation \( F_2(\alpha_0 + \gamma_0) = H_p L / 4 \)

\[
H_p = \frac{4F_p \alpha_0}{L(F_p - F_c)} \tag{6}
\]

The force \( H_p \) causes forces in cables (Fig.1d) of

\[
S_p = H_p / (2 \sin \alpha) \tag{7}
\]

where \( \alpha \) is the angle between the core tube and cables.

The prestressing of wire ropes is realized by turnbuckles in each cable. The measure of prestressing to avoid \( \gamma_0 \) at the midpoint should compensate also the elastic elongation of cables due to the prestressing force \( F_p \), the effect of creep and temperature change

\[
\Delta \gamma \geq F_p L_c / (E_c A_c) + \Delta \gamma + \gamma_0 \sin \alpha \tag{8}
\]

where \( L_c, E_c \) and \( A_c \) are the length, elastic modulus and cross-sectional area of a cable, respectively. The temperature change can be taken as \( \Delta T = \pm 15^\circ C \), thus

\[
\Delta \gamma = \gamma_p \Delta T / c_t, \Delta \gamma = 12 \times 10^{-6} \tag{9}
\]

The creep effect can be calculated using the empirical formula

\[
\Delta \gamma_c = I_c \gamma_c / (I_c - I_c) = I_c c_t \exp(c_t \log I_c) \tag{10}
\]

where \( c_t \) and \( c_t \) are constants for given stress levels in percent of the cable strength 1370 MPa (Table 1). If \( I_c(t) \) is given in log of minutes, then \( \gamma_c \) is obtained in percent. E.g. for \( I_c = 10^6 \) minutes (approx. 2 years) and 50% of 1370 MPa it is

\[
\gamma_c = 0.033555 \exp(0.218934 \times 6) = 0.1248 \% \tag{11}
\]

Table 1. Constants for the calculation of creep in cables (Kmet' 1989)

<table>
<thead>
<tr>
<th>% of 1370 MPa</th>
<th>( c_t )</th>
<th>( c_t )</th>
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</thead>
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<tr>
<td>25</td>
<td>0.0018746</td>
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<td>0.218934</td>
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<tr>
<td>55</td>
<td>0.0470003</td>
<td>0.196371</td>
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<td>0.180206</td>
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<td>0.0705592</td>
<td>0.168776</td>
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<tr>
<td>75</td>
<td>0.0711765</td>
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</tr>
<tr>
<td>80</td>
<td>0.0737764</td>
<td>0.176920</td>
</tr>
</tbody>
</table>

Assuming that

\[
\Delta \gamma = \eta(\Delta \gamma_c + \Delta \gamma + \gamma_0 \sin \alpha), (\eta > 1) \tag{12}
\]

( \( \eta \) can be optionally chosen between 1 and 2), from Eq.(8) we obtain

\[
F_p = E_c A_c (\eta - 1)(\Delta \gamma_c + \Delta \gamma + \gamma_0 \sin \alpha) \tag{13}
\]

The required cross-sectional area of a cable can be calculated from the condition
\[(F_r + S_p)/A_C \leq f_c/\gamma\]  \hspace{1cm} (14)

where \(f_c\) is the cable ultimate tensile strength, and \(\gamma\) is a safety factor. Using Eqs (13) and (14) one obtains

\[A_C = \frac{S_p}{\frac{f_c}{\gamma} - \frac{E_C(\eta - 1)(\Delta_C + \Delta_r + \gamma_k \sin \alpha)}{I_C}}\]  \hspace{1cm} (15)

The supporting bars are designed as compressed CHS struts of an effective length factor \(K_e = 2\) and subject to an axial compression force

\[F_z = 2(F_r + S_p) \sin \alpha\]  \hspace{1cm} (16)

Note that approximate closed-form equations for the design of compressed CHS struts can be found in (Parkas & Járai 1994).

The material cost of the prestressed column is

\[K = k_{CHS} \rho_{CHS} (L_{nax} + 3L_o A_o) + k_c \rho_c A_c I_c\]  \hspace{1cm} (17)

where \(k_{CHS}\) and \(k_c\) are material cost factors ($/kg$) for CHS column, supporting bars and cables, respectively, \(\rho\) are material densities (kg/m$^3$), \(L_o = (L \sin \alpha)/2\) is the length and \(A_o\) is the cross-sectional area of tubular supporting bars.

3. DESIGN OF A COLUMN WITH THREE SUPPORTS (Fig.2)

The design method, detailed for one support in the previous section, can be generalized for more symmetrically arranged and equally spaced supports. The method is treated here for three supports.

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**Fig.2.** a) Prestressed column with three supports. b) Forces in ties. c) Measures of prestressing.
First the core tube is designed for a compressive force

\[ F_z = F + 3(F_{pl} + S_i) \cos \alpha_i \]  \hspace{1cm} (18)

and buckling length \( L/A \). The second term in Eq.(18) should be guessed in advance and then corrected. Then calculate the Euler buckling force

\[ F_E = \pi^2 E I_s / (L/A)^4 \]  \hspace{1cm} (19)

and \( a_0 \) and \( y_0 \) with Eq.(4) and (5). For the forces \( H_t \) and \( H_o \) which eliminate the bending moments \( M_t \) and \( M_o \) (Fig.2) a system of two equations can be written as follows

\[ M_t = \left( H_t + \frac{H_o}{2} \right) \frac{L}{4} = F_t (a_0 + y_0) \sin \frac{\pi}{4} \]  \hspace{1cm} (20a)

\[ M_o = \left( H_t + \frac{H_o}{2} \right) \frac{L}{2} - \frac{H_t L}{4} = F_t (a_0 + y_0) \]  \hspace{1cm} (20b)

Solving Eqs (20) we obtain

\[ H_o = \frac{8F_t}{L} (a_0 + y_o) \left( 1 - \sin \frac{\pi}{4} \right) \]  \hspace{1cm} (21a)

\[ H_t = \frac{8F_t}{L} (a_0 + y_0) \left( \sin \frac{\pi}{4} - \frac{1}{2} \right) \]  \hspace{1cm} (21b)

The tension forces in cables due to \( H_t \) and \( H_o \) are as follows (Fig.2)

\[ S_t = \left( H_t + \frac{H_o}{2} \right) / \sin \alpha_i \]  \hspace{1cm} (22a)

\[ S_o = H_o / (2 \sin \alpha_o) \]  \hspace{1cm} (22b)

For a given angle \( \alpha_i \), the angle \( \alpha_o \) can be calculated (Fig.2)

\[ \tan \alpha_o = H_o / (2Y), Y = \left( H_t + \frac{H_o}{2} \right) / \tan \alpha_i \]  \hspace{1cm} (23)

The measure of prestressing in cables, necessary to avoid the elastic deformations \( y_i \) and \( y_o \) can be calculated similarly to Eq.(8)

\[ \Delta_{pl} \geq \frac{F_{pl} L}{A_c E_c} + \Delta_{ct} + \Delta_{r1} + y_i \sin \alpha_i \]  \hspace{1cm} (24a)

\[ \Delta_{po} \geq \frac{F_{po} L_o}{A_c E_c} + \Delta_{ct} + \Delta_{r2} + (y_o - y_i) \sin \alpha_o \]  \hspace{1cm} (24b)

Assuming that

\[ \Delta_{pl} = \eta_1 (\Delta_{ct} + \Delta_{r1} + y_i \sin \alpha_i) \]  \hspace{1cm} (25a)

\[ \Delta_{po} = \eta_0 (\Delta_{ct} + \Delta_{r2} + (y_o - y_i) \sin \alpha_o) \]  \hspace{1cm} (25b)

\( \eta_1 > 1; \eta_0 > 1 \)

the prestressing forces can be calculated from Eqs (24) and (25) similarly to Eq.(13)

\[ F_{pl} = \frac{A_c E_c}{L_1} (\eta_1 - 1) (\Delta_{ct} + \Delta_{r1} + y_i \sin \alpha_i) \]  \hspace{1cm} (26a)

\[ F_{po} = \frac{A_c E_c}{L_0} (\eta_0 - 1) (\Delta_{ct} + \Delta_{r2} + (y_o - y_i) \sin \alpha_o) \]  \hspace{1cm} (26b)

Using the condition Eq.(14) we get

\[ A_{ct} = 2 \left( \frac{S_t}{\gamma - \frac{E_c}{L_1} (\eta_1 - 1) (\Delta_{ct} + \Delta_{r1} + y_i \sin \alpha_i) } \right) \]  \hspace{1cm} (27a)

\[ A_{co} = 2 \left( \frac{S_o}{\gamma - \frac{E_c}{L_0} (\eta_0 - 1) (\Delta_{ct} + \Delta_{r2} + (y_o - y_i) \sin \alpha_o) } \right) \]  \hspace{1cm} (27b)

The compressive forces acting on supporting bars are as follows: at midpoint

\[ F_{po} = 2(S_o + F_{po}) \sin \alpha_o \]  \hspace{1cm} (28a)

others

\[ F_{pi} = (S_i + F_{pi}) \sin \alpha_i - (S_o + F_{po}) \sin \alpha_o \]  \hspace{1cm} (28b)

4 NUMERICAL EXAMPLE

Data: \( F = 440 \) kN, \( L = 10 \) m, \( f_t = 355 \) MPa, \( E_c = 1.5 \times 10^5 \) MPa, \( f_c = 1500 \) MPa, \( \gamma = 15, \eta = 2 \).
4.1 The case of one support

We guess that the total compressive force will be \( F_T = 590 \) kN. We calculate first with an angle of \( \alpha = 15^\circ \), but this angle will be varied to find an optimal value corresponding to the minimal total material cost of the structure.

According to Eq. (1) \( \delta_L = 60^\circ \). From Eq. (2) we obtain a CHS of 215.1*4 with an outside diameter of 219.1 mm. The radius of gyration is \( r = 76.1 \) mm, the cross-sectional area is \( A = 2700 \) mm\(^2\).

With Eq. (3) we get \( F_{se} = 1296.6 \) kN. From Eqs. (4) and (5) \( \alpha_0 = 27.79^\circ \) and \( \beta_0 = 23.20^\circ \). The required prestressing is \( \Delta_p = 12.0 \) and \( S_p = 23.2 \) kN. According to Eqs. (9) and (10), with \( L_c = 5176.4 \) mm, it is \( \Delta_r = 0.93\beta_0\sin\alpha = 6.00 \) mm.

The creep effect is calculated for a year i.e. for 525600 min. for the stress level \( \sigma_c = 1000 \) MPa, it is approx. 75% of 1370 MPa, thus from Table 1 \( \sigma_c = 0.0717165 \) and \( \sigma_c = 0.169366 \).

Using the approximate formula valid for steel wire ropes \( A_c = 0.79\pi d_c^2 / 4 \) one obtains \( d_c = 8.5 \) mm. According to the data of the German standard DIN 3052 (for helical ropes of one stranding) the mass of a cable of diameter 9 mm is \( m = 0.407 \) kg/m and the cost of this cable (with zinc cover of wires) is approx. 1.25 $/kg.

With \( A_c = 50.3 \) mm\(^2\), Eq. (13) yields \( F_r = 30A_c(10.35 + 23.2\sin\alpha\cos\alpha) = 24.35 \) kN (29).

With Eq. (2) \( F_T = 578 \) kN, thus the guess of 590 kN was good and an iteration is not needed.

The material cost of cables is (Eq.17)

\[
K_c = 6k_c\rho_c A_c L_c
\]

and the material cost of tubular supporting bars is (Eq.17)

\[
K_s = 3k_{CHS}\rho_{CHS} A_s L_s
\]

To find the optimal angle \( \alpha \), the total cost is calculated for several angles between 10 and 30\(^\circ\) and the calculated values are summarized in Table 2. It can be seen that the cost of cables decreases and the cost of supporting bars increases with the increase of the angle. Furthermore the total cost slightly increases when the angle increases, so that the optimal angle may be selected in the range of 10-20\(^\circ\).

4.2 The case of three supports

We calculate with the angle \( \alpha_1 = 20^\circ \). For an approx. compressive force \( F_T = 520 \) kN and an effective buckling length \( L/4 = 2.5 \) m one obtains for the core tube a CHS of 193.7*3.2 mm with \( A = 1920 \) mm\(^2\), \( r = 67.4 \) mm, \( I_s = 8.69*10^4 \) mm\(^4\). With Eq. (19) we get \( F_{se} = 2.8817*10^5 \) kN. Eq. (4) gives \( \alpha_0 = 28.20^\circ \), \( \alpha_0 + \beta_0 = 34.41^\circ \). Eqs. (21a,b) give \( H_0 = 4.19 \) and \( H_1 = 2.96 \) kN. Eq. (23) yields \( \alpha_0 = 8.6^\circ \). Eqs. (22) give \( S_j = 14.78 \) and \( S_0 = 14.01 \) kN. Furthermore \( L_i = \)

| Table 2. Cost calculation as a function of the cable slope angle |
|---------------------------------|-----|-----|-----|-----|-----|
| \( \alpha^\circ \)             | 10  | 15  | 20  | 25  | 30  |
| \( A_c (\text{mm}^2) \) Eq. (15)| 60.73 | 45.03 | 37.76 | 33.65 | 30.99 |
| rounded \( d_c (\text{mm}) \)  | 10  | 9   | 8   | 8   | 7   |
| \( A_c \) for rounded \( d_c \) | 62.05 | 50.26 | 39.71 | 39.71 | 30.40 |
| \( F_T \) (kN) Eq. (29)        | 26.65 | 24.35 | 21.21 | 22.92 | 18.60 |
| \( F_T \) (kN) Eq. (2)         | 588  | 578  | 565  | 565  | 549  |
| \( K_c \) (kN) Eq. (30)        | 18.6 | 15.3 | 12.4 | 12.9 | 10.3 |
| \( F_T \) (kN) Eq. (16)        | 17.36 | 24.67 | 30.32 | 38.88 | 41.67 |
| CHS supporting bars            | 51*1.2 | 63.5*1.4 | 82.5*1.6 | 101.6*2 | 108*2 |
| \( K_s \) (kN) Eq. (31)        | 3.9  | 8.6  | 17.4 | 34.3 | 45.3 |
| \( K \) (kN) Eq. (17)          | 234.5 | 235.9 | 241.8 | 259.2 | 267.6 |
2660 and \( L_d = 2528 \text{ mm} \), \( \Delta y_t = 0.48; \Delta y_s = 0.46 \text{ mm} \);
\( y_1 = y_0 \cdot \sin(\pi / 4) = 4.39; y_0 - y_1 = 182 \text{ mm} \). As in Section 4.1
\( e_c = 0.189\%; \Delta C_1 = 5.03; \Delta C_2 = 4.78 \text{ mm} \). From Eqs (27) we get \( A_C = 24.44 \) and \( A_{C_2} = 20.82 \text{ mm}^2 \).

For all ties we select wire ropes of \( d_c = 7 \text{ mm} \), \( A_c = 30.4 \text{ mm}^2 \), \( m = 0.246 \text{ kg/m} \), \( k_c = 1.25 \text{ S/kg} \). Eqs (25) give \( \Delta y_c = 14.0; \Delta y_{as} = 11.0 \text{ mm} \). With Eqs (26) one obtains \( F_{p_1} = 12.02 \text{ and } F_{p_2} = 9.94 \text{ kN} \), with Eqs (28) \( F_{p_3} = 7.16 \text{ and } F_{p_4} = 5.59 \text{ kN} \).

For a supporting bar at the midpoint we use a CHS profile designed for a compressive force \( F_{p_2} = 7.16 \text{ kN} \) and an effective buckling length \( 2 \times 1288 = 2576 \text{ mm} \). The selected CHS profile for all supporting bars is \( 42.4 \times 12 \text{ mm} \) with \( A_s = 155 \text{ mm}^2 \).

The total length of supporting bars is \( 3 \times 1288+6 \times 910=9324 \text{ mm} \).

The total material cost of the prestressed structure, with \( k_{CHS} = 1.0 \text{ S/kg} \), \( L_c = 31.128 \text{ m} \), is

\[ K = K_{c_1} + K_c + K_s = 150.7+9.6+11.4 = 171.75 \text{ S} \]

For the column without prestressing the CHS profile of \( 273 \times 4.5 \text{ mm} \) is selected with the material cost of \( 298.35 \text{ S} \), thus, it can be concluded that, in this numerical example, the material cost savings achieved by using one support with \( \alpha = 10^6 \) is \( 21\% \) and with three supports is \( 42\% \).

5 CONCLUSIONS

The weight of a compressed tubular column can be significantly decreased by prestressing it with ties. The larger the number of supports the smaller the weight with the same compressive strength.

The necessary prestressing can be calculated by relatively simple closed-form equations based on the overall buckling relations considering the initial crookedness.

In the calculation of prestress the temperature change and the creep of cable ties is taken into account. The supporting CHS bars are designed for overall buckling.

The weight of these bars and the cable ties is small compared with the core tube and the change of their weight with the varied angle between the core tube and ties does not significantly affect the total weight or total material cost. The material cost calculations in the case of one support show that the optimal value of this angle is about \( 10-20^\circ \).

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