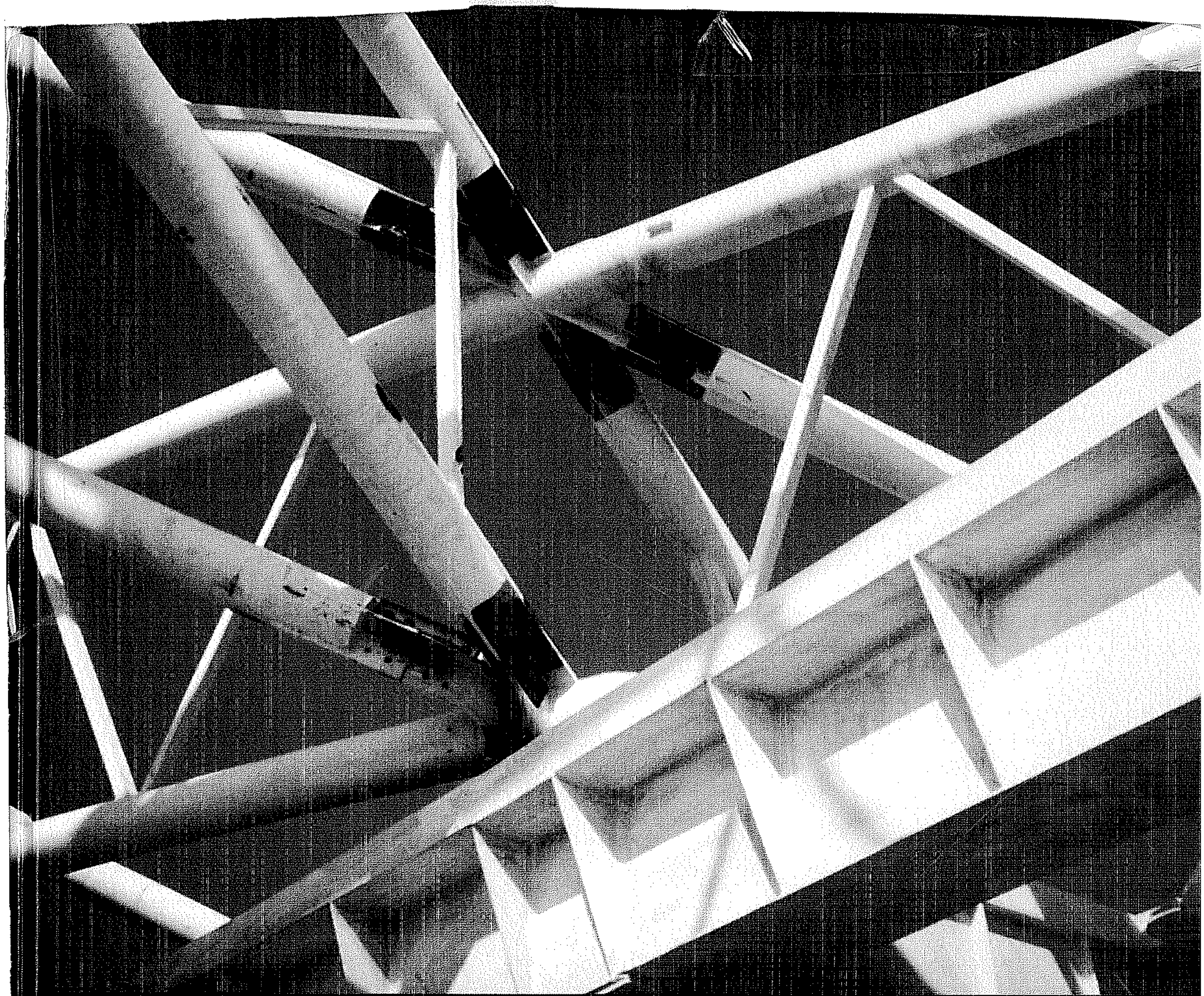

Tubular Structures VII

József Farkas and Károly Jármai, editors

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Cover photo: Tubular roof structure of a new sports hall in Budapest

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Optimization of tubular columns prestressed by tension ties

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ABSTRACT: In the design of prestressed columns the effect of initial imperfections has been predominantly neglected in the past. The design of tubular columns prestressed by wire ropes is treated now on the basis of the European column buckling formulae which consider the initial imperfections. Columns of one and three intermediate supports are designed. The necessary prestressing is calculated considering the effect of temperature change and creep of cables. The optimal angle between the core tube and cables is sought in the case of one support to minimize the material cost of the core tube, cables and supporting bars.

KEYWORDS: Prestressed columns, prestressed steel structures, creep of cables, column buckling

1 INTRODUCTION

Prestressed columns are used as booms for erection purposes or as parts of higher towers. Their advantage is the high compressive strength and lightweight. A detailed study has been published dealing with the design of such columns by Chu and Berge (1963). On the basis of this analysis Mauch and Felton (1967) have worked out an optimum design procedure for these columns. This study has shown that the weight of the core tube can be significantly decreased by using prestressing with one or three intermediate supports.

The Euler overall buckling formula has been used which does not consider the initial crookedness of the column. Farkas and Jármai have pointed out (Farkas & Jármai 1995, Jármai & Farkas 1996) that the design with Euler formula leads to unsafe and imperfection-sensitive solutions. Thus, in present paper the Ayrton-Perry formulation and the Eurocode 3 (EC3)(1992) calculation method is used which considers the initial imperfections.

Moreover Mauch and Felton (1967) have used a local buckling calculation for aluminium tubes which leads to unrealistic circular hollow section (CHS) dimensions. E.g. in their numerical example a tube of D*t = 176.5*0.447 mm with a local slenderness of 395 has been calculated. On the contrary, the British Standard BS 8118 (1991) prescribes, for an aluminium alloy strut of yield stress 240 MPa, a local slenderness limit of 56 (Jármai & Farkas 1996).

The calculation method used in the book of Belenya (1975) also neglects the initial imperfection and in the numerical example only one support is considered for a 30 m long column loaded by a compressive force of 2000 kN. Round steel bars have been applied as prestressing elements of diameter 30 mm with a limiting tensile stress of 170 MPa instead of high-strength steel wire ropes.

A calculation method based on the Euler solution is treated in Petersen's book (1980) without the effect of initial imperfections. Schock (1976) in his dissertation has considered the initial crookedness and used wire ropes as ties. Several structural versions have been calculated using different complicated systems of ties. For the comparison the material weight has been used since the cost of assembly was not known.

In the case of prestressing with high-strength cables it is necessary to take into account the decrease of prestress during the service time due to creep and temperature changes. These effects have not been treated in the publications mentioned above.

In the present paper a relatively simple design method is proposed for steel CHS columns prestressed by high-strength cables with one or three intermediate supports. In the calculation of the necessary prestressing the effect of creep and temperature change is also considered. In the optimization the optimal angle between the core tube and cables is sought which minimizes the total cost. Since the cost of assembly of such prestressed structures is not known, in the cost function only the material cost of core tube, cables and supporting bars is considered. The shortening of the core tube and that of supporting bars is neglected. The design method is illustrated by a numerical example.

2 DESIGN OF COLUMNS WITH ONE SUPPORT (Fig.1)

Assume that the core tube is fixed by ties at its midpoint so that it buckles in the second mode (Fig.1). Thus, the core tube can be designed for a given factored external axial compressive force F with an effective buckling length of $L/2$ and for a prescribed limiting local slenderness according to EC3

$$\delta_L = (D/t)_L = 90 * 235 / f_y \tag{1}$$

where f_y is the yield stress. The design inequality is expressed by

$$\frac{F_T}{\pi D^2 / \delta_L} \leq \frac{\chi f_y}{\gamma_{M1}} \tag{2}$$

where $\gamma_{M1} = 1.1$ is the partial safety factor,

$$F_T = F + 3(F_p + S_F)\cos\alpha$$

$$1/\chi = \phi + \sqrt{\phi^2 - \bar{\lambda}^2}$$

$$\phi = 0.5[1 + 0.34(\bar{\lambda} - 0.2) + \bar{\lambda}^2]$$

$$\bar{\lambda} = K_e L\sqrt{8} / (2D\lambda_E); \lambda_E = \pi\sqrt{E/f_y}$$

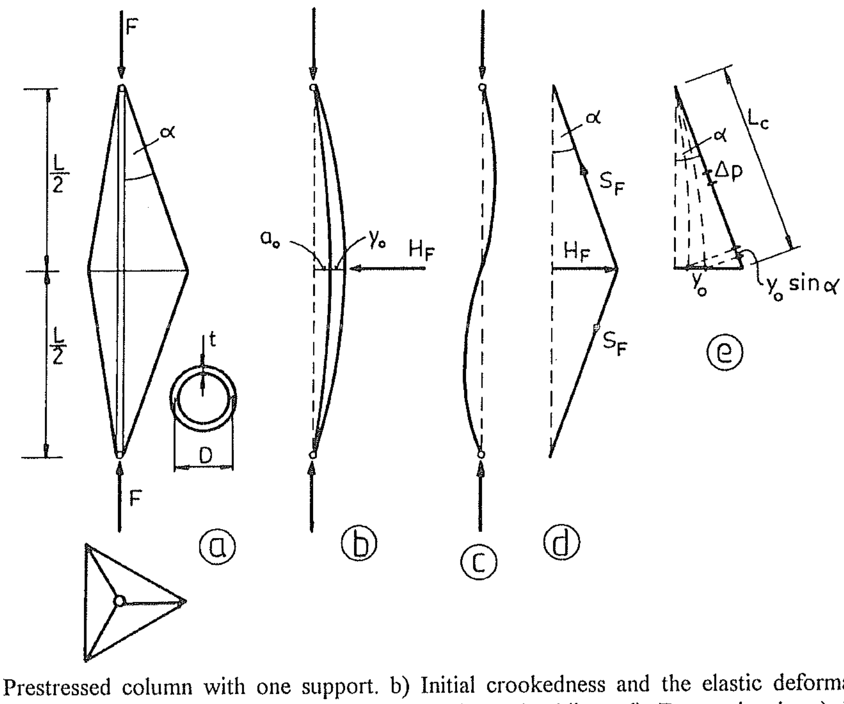

Fig.1. a) Prestressed column with one support. b) Initial crookedness and the elastic deformation of the column without prestressing. c) Second mode of column buckling. d) Forces in ties.e) Measure of prestressing

E is the elastic modulus, K_e is the effective length factor, here for pinned ends $K_e = 1$. Note that F_p and S_F are cable forces calculated below (Eqs 7 and 13) but not known in advance, so we should guess these values and then correct them using an iteration process.

Eq.(2) can be solved for D numerically using a computer program. Knowing D and t the Euler overall buckling force for pinned ends can be calculated

$$F_E = \pi^2 E I_x / (L/2)^2 ; I_x = \pi D^3 t / 8 \qquad (3)$$

where I_x is the moment of inertia, for which the table given by the standard DIN 2448 or 2458 can be used (Dutta & Würker 1988). The EC3 formula is based on the Ayrton-Perry formulation (Maquoi & Rondal 1978). In this calculation the initial imperfection a_0 and the elastic deformation y_0 at the midpoint (Fig.1), which could occur without prestressing, is

$$a_0 = 0.34\left(\frac{L}{r\lambda_E} - 0.2\right)\frac{W_x}{A}; \frac{W_x}{A} = \frac{D}{4} \qquad (4)$$

$$y_0 = F_T a_0 / (F_E - F_T) \qquad (5)$$

where W_x is the elastic section modulus.

The horizontal force H_F which eliminates the bending moment $F_T(a_0 + y_0)$ (Fig.1) is obtained from the equation $F_T(a_0 + y_0) = H_F L/4$

$$H_F = \frac{4 F_T a_0 F_E}{L(F_E - F_T)} \qquad (6)$$

The force H_F causes forces in cables (Fig.1d) of

$$S_F = H_F / (2 \sin\alpha) \qquad (7)$$

where α is the angle between the core tube and cables.

The prestressing of wire ropes is realized by turnbuckles in each cable. The measure of prestressing to avoid y_0 at the midpoint should compensate also the elastic elongation of cables due to the prestressing force F_p, the effect of creep and temperature change

$$\Delta_p \geq F_p L_C / (E_C A_C) + \Delta_C + \Delta_T + y_0 \sin\alpha \qquad (8)$$

where L_C, E_C and A_C are the length, elastic modulus and cross-sectional area of a cable, respectively. The temperature change can be taken as $\Delta T = \pm 15^0 C$, thus

$$\Delta_T = \alpha_T \Delta T L_C ; \alpha_T = 12 * 10^{-6} \qquad (9)$$

The creep effect can be calculated using the empirical formula

$$\Delta_C = L_C \varepsilon_C (t_C) = L_C c_1 \exp(c_2 \log t_C) \qquad (10)$$

where c_1 and c_2 are constants for given stress levels in percent of the cable strength 1370 MPa (Table 1). If in Eq.(10) the time t_C is given in log of minutes, then ε_C is obtained in percent. E.g. for $t_C = 10^6$ minutes (approx. 2 years) and 50% of 1370 MPa it is

$$\varepsilon_C = 0.033555 \exp(0.218934 * 6) = 0.1248\% \qquad (11)$$

Table 1. Constants for the calculation of creep in cables (Kmet' 1989)

% of 1370 MPa	c_1	c_2
25	0.0018746	0.424056
30	0.0026588	0.373725
35	0.0054372	0.295726
40	0.0110466	0.248691
45	0.0201812	0.227701
50	0.0335550	0.218934
55	0.0470003	0.196371
60	0.0580456	0.180206
65	0.0662387	0.171571
70	0.0705592	0.168776
75	0.0717165	0.169366
80	0.0.737764	0.176920

Assuming that

$$\Delta_p = \eta(\Delta_C + \Delta_T + y_0 \sin\alpha); (\eta > 1) \qquad (12)$$

(η can be optionally chosen between 1 and 2), from Eq.(8) we obtain

$$F_p = \frac{E_C A_C}{L_C}(\eta - 1)(\Delta_C + \Delta_T + y_0 \sin\alpha) \qquad (13)$$

The required cross-sectional area of a cable can be calculated from the condition

$$(F_p + S_F)/A_C \le f_C/\gamma \tag{14}$$

where f_C is the cable ultimate tensile strength, and γ is a safety factor. Using Eqs (13) and (14) one obtains

$$A_C = \frac{S_F}{\dfrac{f_C}{\gamma} - \dfrac{E_C}{L_C}(\eta - 1)(\Delta_C + \Delta_T + y_0 \sin\alpha)} \tag{15}$$

The supporting bars are designed as compressed CHS struts of an effective length factor $K_e = 2$ and subject to an axial compression force

$$F_S = 2(F_p + S_F)\sin\alpha \tag{16}$$

Note that approximate closed-form equations for the design of compressed CHS struts can be found in (Farkas & Jármai 1994).

The material cost of the prestressed column is

$$K = k_{CHS}\rho_{CHS}(LA_{core} + 3L_S A_S) + k_C \rho_C A_C L_C \tag{17}$$

where k_{CHS} and k_C are material cost factors ($/kg) for CHS column, supporting bars and cables, respectively, ρ are material densities (kg/m^3), $L_S = (L\tan\alpha)/2$ is the length and A_S is the cross-sectional area of tubular supporting bars.

3 DESIGN OF A COLUMN WITH THREE SUPPORTS (Fig.2)

The design method, detailed for one support in the previous section, can be generalized for more symmetrically arranged and equally spaced supports. The method is treated here for three supports.

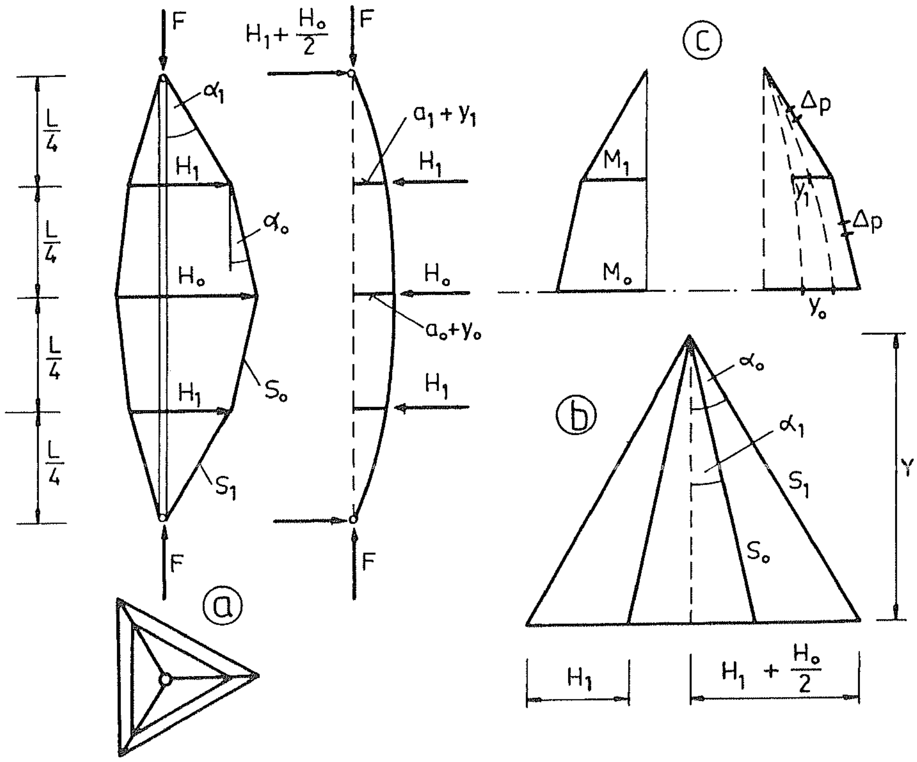

Fig.2. a) Prestressed column with three supports. b) Forces in ties. c) Measures of prestressing

First the core tube is designed for a compressive force

$$F_T = F + 3(F_{p1} + S_1)\cos\alpha_1 \qquad (18)$$

and buckling length $L/4$. The second term in Eq.(18) should be guessed in advance and then corrected. Then calculate the Euler buckling force

$$F_E = \pi^2 EI_x / (L/4)^2 \qquad (19)$$

and a_0 and y_0 with Eq.(4) and (5). For the forces H_1 and H_0 which eliminate the bending moments M_1 and M_0 (Fig.2) a system of two equations can be written as follows

$$M_1 = \left(H_1 + \frac{H_0}{2}\right)\frac{L}{4} = F_T(a_0 + y_0)\sin\frac{\pi}{4} \qquad (20a)$$

$$M_0 = \left(H_1 + \frac{H_0}{2}\right)\frac{L}{2} - \frac{H_1 L}{4} = F_T(a_0 + y_0) \qquad (20b)$$

Solving Eqs (20) we obtain

$$H_0 = \frac{8F_T}{L}(a_0 + y_0)\left(1 - \sin\frac{\pi}{4}\right) \qquad (21a)$$

$$H_1 = \frac{8F_T}{L}(a_0 + y_0)\left(\sin\frac{\pi}{4} - \frac{1}{2}\right) \qquad (21b)$$

The tension forces in cables due to H_1 and H_0 are as follows (Fig.2)

$$S_1 = \left(H_1 + \frac{H_0}{2}\right) / \sin\alpha_1 \qquad (22a)$$

$$S_0 = H_0 / (2\sin\alpha_0) \qquad (22b)$$

For a given angle α_1 the angle α_0 can be calculated (Fig.2)

$$\tan\alpha_0 = H_0 / (2Y); Y = \left(H_1 + \frac{H_0}{2}\right) / \tan\alpha_1 \qquad (23)$$

The measure of prestressing in cables, necessary to avoid the elastic deformations y_1 and y_0 can be calculated similarly to Eq.(8)

$$\Delta_{p1} \geq \frac{F_{p1} L_1}{A_{C1} E_C} + \Delta_{C1} + \Delta_{T1} + y_1 \sin\alpha_1 \qquad (24a)$$

$$\Delta_{p0} \geq \frac{F_{p0} L_0}{A_{C0} E_C} + \Delta_{C0} + \Delta_{T0} + (y_0 - y_1)\sin\alpha_0 \qquad (24b)$$

Assuming that

$$\Delta_{p1} = \eta_1(\Delta_{C1} + \Delta_{T1} + y_1 \sin\alpha_1) \qquad (25a)$$

$$\Delta_{p0} = \eta_0\left[\Delta_{C0} + \Delta_{T0} + (y_0 - y_1)\sin\alpha_0\right] \qquad (25b)$$

$$\eta_1 > 1; \eta_0 > 1$$

the prestressing forces can be calculated from Eqs (24) and (25) similarly to Eq.(13)

$$F_{p1} = \frac{A_{C1} E_C}{L_1}(\eta_1 - 1)(\Delta_{c1} + \Delta_{T1} + y_1 \sin\alpha_1) \qquad (26a)$$

$$F_{p0} = \frac{A_{C0} E_C}{L_0}(\eta_0 - 1)\left[\Delta_{C0} + \Delta_{T0} + (y_0 - y_1)\sin\alpha_0\right] \qquad (26b)$$

Using the condition Eq.(14) we get

$$A_{C1} = \frac{S_1}{\dfrac{f_C}{\gamma} - \dfrac{E_C}{L_1}(\eta_1 - 1)(\Delta_{C1} + \Delta_{T1} + y_1 \sin\alpha_1)} \qquad (27a)$$

$$A_{C0} = \frac{S_0}{\dfrac{f_C}{\gamma} - \dfrac{E_C}{L_0}(\eta_0 - 1)\left[\Delta_{C0} + \Delta_{T0} + (y_0 - y_1)\sin\alpha_0\right]} \qquad (27b)$$

The compressive forces acting on supporting bars are as follows: at midpoint

$$F_{S0} = 2(S_0 + F_{p0})\sin\alpha_0 \qquad (28a)$$

others

$$F_{S1} = (S_1 + F_{p1})\sin\alpha_1 - (S_0 + F_{p0})\sin\alpha_0 \qquad (28b)$$

4 NUMERICAL EXAMPLE

Data: $F = 440$ kN, $L = 10$ m, $f_y = 355$ MPa, $E_C = 1.5{*}10^5$ MPa, $f_C = 1500$ MPa, $\gamma = 1.5, \eta = 2$.

4.1 *The case of one support*

We guess that the total compressive force will be F_T= 590 kN. We calculate first with an angle of $\alpha = 15^0$, but this angle will be varied to find an optimal value corresponding to the minimal total material cost of the structure.

According to Eq.(1) $\delta_L = 60$. From Eq.(2) we obtain a CHS of 215.1*4 with an outside diameter of 219.1 mm. The radius of gyration is r = 76.1 mm, the cross-sectional area is A = 2700 mm^2 Check of Eq.(2): 218.5 < 221.7 MPa, OK.

Using Eq.(3) we get F_E = 1296.6 kN. Eqs (4) and (5) yield a_0 = 27.79 and y_0 = 23.20 mm. With Eqs (6) and (7) one obtains H_F = 12.0 and S_F = 23.2 kN. According to Eqs. (9) and (10), with L_C = 5176.4 mm, it is $\Delta_T = 0.93, y_0 \sin\alpha = 6.00$ mm.

The creep effect is calculated for a year i.e. for 525600 minimization for a stress level $f_C/\gamma = 1000$ MPa, it is approx. 75% of 1370 MPa, thus from Table 1 c_1 = 0.0717165 and c_2 = 0.169366,

$\varepsilon_C = c_1 \exp(5.7206 c_2) = 0.1890\%, \Delta_C = 9.78$ mm. The required prestressing is (Eq.12) $\Delta_p = 2*16.71 = 33.42$ mm. Eq.(15) gives A_C = 45 mm^2.

Using an approximate formula valid for steel wire ropes $A_C = 0.79\pi d_C^2/4$ one obtains d_C = 8.5mm. According to the data of the German standard DIN 3052 (for helical ropes of one stranding) the mass of a cable of diameter 9 mm is m = 0.407 kg/m and the cost of this cable (with zinc covering of wires) is approx. 1.25 \$/kg.

With A_C = 50.3 mm^2 Eq.(13) yields

$$F_p = 30A_C(10.35 + 23.2\sin\alpha\cos\alpha) = 24.35\,\text{kN} \quad (29)$$

With Eq.(2) F_T = 578 kN, thus the guess of 590 kN was good and an iteration is not needed.

The material cost of cables is (Eq.17)

$$K_C = 6k_C\rho_C A_C L_C \quad (30)$$

and the material cost of tubular supporting bars is (Eq.17)

$$K_S = 3k_{CHS}\rho_{CHS} A_S L_S \quad (31)$$

To find the optimal angle α, the total cost is calculated for several angles between 10 and 30^0 and the calculated values are summarized in Table 2. It can be seen that the cost of cables decreases and the cost of supporting bars increases with the increase of the angle. Furthermore the total cost slightly increases when the angle increases, so that the optimal angle may be selected in the range of 10-20^0.

4.2 *The case of three supports*

We calculate with the angle $\alpha_1 = 20^0$. For an approx. compressive force F_T = 520 kN and an effective buckling length $L/4$ = 2.5 m one obtains for the core tube a CHS of 193.7*3.2 mm with A = 1920 mm^2, r = 67.4 mm, I_x = 8.69*10^6 mm^4. With Eq.(19) we get F_E = 2.8817*10^3 kN. Eq.(4) gives a_0 = 28.20 mm, $a_0 + y_0$ = 34.41 mm. Eqs (21a,b) give H_0 = 4.19 and H_1=2.96 kN. Eq.(23) yields $\alpha_0 = 8.6^0$ Eqs (22) give S_1 = 14.78 and S_0 = 14.01 kN. Furthermore L_1 =

Table 2. Cost calculation as a function of the cable slope angle

α ^{0}	10	15	20	25	30
A_C (mm^2)Eq(15)	60.73	45.03	37.76	33.65	30.99
rounded d_C(mm)	10	9	8	8	7
A_C for rounded d_C	62.05	50.26	39.71	39.71	30.40
F_p (kN) Eq.(29)	26.65	24.35	21.21	22.92	18.60
F_T (kN) Eq.(2)	588	578	565	565	549
K_C (\$) Eq.(30)	18.6	15.3	12.4	12.9	10.3
F_S (kN) Eq.(16)	17.36	24.67	30.32	38.88	41.67
CHS supporting bars	51*1.2	63.5*1.4	82.5*1.6	101.6*2	108*2
K_S (\$) Eq.(31)	3.9	8.6	17.4	34.3	45.3
K (\$) Eq.(17)	234.5	235.9	241.8	259.2	267.6

2660 and $L_0 = 2528$ mm, $\Delta_{T1} = 0.48; \Delta_{T0} = 0.46$ mm; $y_1 = y_0 \sin(\pi/4) = 4.39; y_0 - y_1 = 1.82$ mm. As in Section 4.1 $\varepsilon_C = 0.189\%; \Delta_{C1} = 5.03, \Delta_{C0} = 4.78$ mm. From Eqs (27) we get $A_{C1} = 24.44$ and $A_{C0} = 20.82$ mm^2.

For all ties we select wire ropes of $d_C = 7$ mm, $A_C = 30.4$ mm^2, $m = 0.246$ kg/m, $k_C = 1.25$ \$/kg. Eqs (25) give $\Delta_{p1} = 14.0; \Delta_{p0} = 11.0$ mm. With Eqs (26) one obtains $F_{p1} = 12.02$ and $F_{p0} = 9.94$ kN, with Eqs (28) $F_{S0} = 7.16$ and $F_{S1} = 5.59$ kN.

For a supporting bar at the midpoint we use a CHS profile designed for a compressive force $F_{S0} = 7.16$ kN and an effective buckling length 2*1288 = 2576 mm. The selected CHS profile for all supporting bars is 42.4*1.2 mm with $A_S = 155$ mm^2. The total length of supporting bars is 3*1288+6*910=9324 mm.

The total material cost of the prestressed structure, with $k_{CHS} = 1.0$ \$/kg, $L_C = 31.128$ m, is

$$K = K_{core} + K_C + K_S = 150.7+9.6+11.4=171.7\$.$$

For the column without prestressing the CHS profile of 273*4.5 mm is selected with the material cost of 298.3\$, thus, it can be concluded that, in this numerical example, the material cost savings achieved by using one support with $\alpha = 10^0$ is 21% and with three supports is 42%.

5 CONCLUSIONS

The weight of a compressed tubular column can be significantly decreased by prestressing it with ties. The larger the number of supports the smaller the weight with the same compressive strength.

The necessary prestressing can be calculated by relatively simple closed-form equations based on the overall buckling relations considering the initial crookedness.

In the calculation of prestress the temperature change and the creep of cable ties is taken into account. The supporting CHS bars are designed for overall buckling.

The weight of these bars and the cable ties is small compared with the core tube and the change of their weight with the varied angle between the core tube and ties does not significantly affect the total weight or total material cost. The material cost calculations in the case of one support show that the optimal value of this angle is about 10-20^0.

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