



## OPTIMUM DESIGN OF HOLLOW FLANGE BEAMS

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### SUMMARY

A hollow flange beam (HFB) consists of a straight web and two closed flanges of triangular shape. HFB-s are optimized considering constraints on normal stress due to bending and on local buckling of web and flange. The comparison between optimized HFB and welded I-beams shows the advantages of HFB as follows: the mass and deflection are smaller and the load-carrying capacity against lateral-torsional buckling is larger.

### 1. INTRODUCTION

A hollow flange beam (HFB) (Fig.1) consists of a straight web and two hollow flanges of triangular shape. HFB is cold-rolled from a single strip of steel passing through a rolling mill. The triangular flanges are closed by two electric-resistance welds. This new type of thin-walled beams, called also "dogbone", is developed by the Palmer Tube Technologies Pty Ltd Australia [1].

They use a higher-strength steel of ultimate tensile strength 520 MPa and yield strength 450 MPa. The depth of beams varies between 200-450 mm, the flange width is 90 mm, the thickness is 2.3-3.8 mm. Australian standards has been used for design.

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The advantages of dogbone over the conventional welded or rolled I-beams are as follows: 1) the cross-sectional area or the mass of a HFB is smaller; 2) the moment of inertia is larger, thus, the beam deflection is smaller; 3) the torsional stiffness is much larger, thus, the critical bending moment for lateral-torsional buckling is also larger.

These advantages can be verified by comparison of optimized sections, thus we have worked out an optimum design procedure for dogbone and compared it with the optimized welded I-beams. In the optimization the objective function to be optimized is the cross-sectional area and the constraints on maximum normal stress due to bending and on the local buckling of web and flange are considered according to Eurocode 3 (EC3)[2]. The simply supported uniformly loaded beams are checked also for shear. It should be mentioned that the Australian beams have been designed also for flange crushing due to concentrated normal forces.

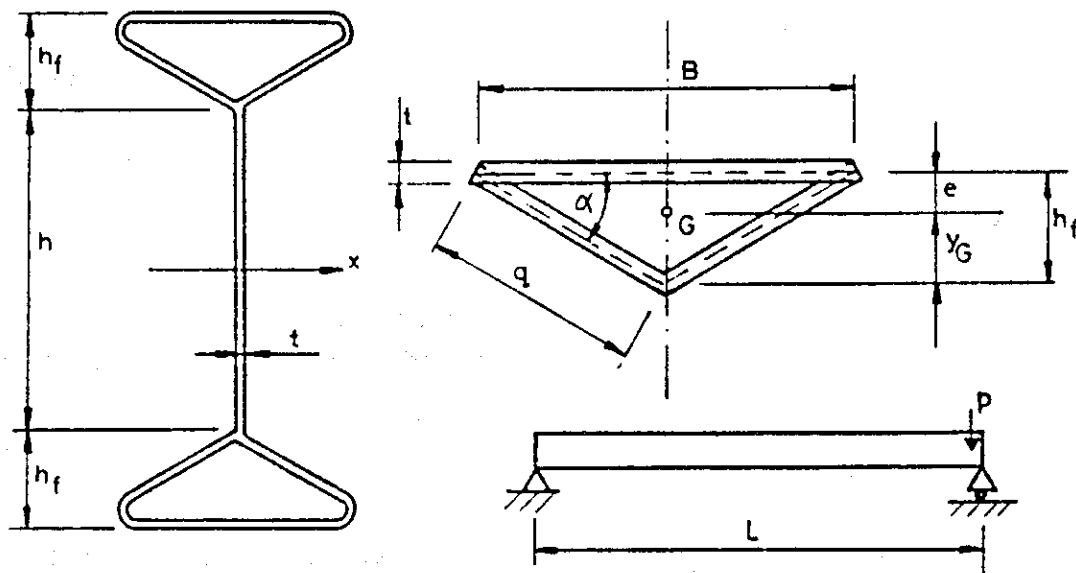


Fig.1. Hollow flange beam, flange details

## 2. SECTIONAL PROPERTIES

The cross-sectional area of a triangular hollow flange is (Fig.1)

$$A_f = t(B + 2q) = t \left( \frac{2h_f}{\tan \alpha} + \frac{2h_f}{\sin \alpha} \right) = \frac{2h_f t (1 + \cos \alpha)}{\sin \alpha} \quad (1)$$

Introducing the ratio  $\zeta = h_f / (h/2)$  as the main variable and the slenderness  $\beta = t/h$ , the flange height can be expressed as  $h_f = \zeta h/2$ , the thickness  $t = \beta h$  and

$$A_f = \beta \zeta h^2 (1 + \cos \alpha) / \sin \alpha = c_2 h_f^2, \quad c_2 = \frac{4\beta(1 + \cos \alpha)}{\zeta \sin \alpha} \quad (2)$$

According to the Australian sections we use here  $\alpha = 30^\circ$  thus

$$c_2 = 14.92820\beta / \zeta \quad (3)$$

The location of the gravity center G of the flange is described by the distance

$$e = \frac{2qth_f / 2}{A_f} = \frac{h_f}{2(1 + \cos \alpha)} \quad (4)$$

$$\text{and } y_G = h_f - e = \frac{h_f(1 + 2\cos \alpha)}{2(1 + \cos \alpha)} = c_3 h_f \quad (5)$$

$$c_3 = 0.73205 \quad (6)$$

The moment of inertia of the flange section is

$$I_{R_f} = Bte^2 + 2q^3 t \sin^2 \alpha / 12 + (h_f / 2 - e)^2 2qt = c_1 h_f^4 \quad (7)$$

$$c_1 = \beta \left[ \cos \alpha + \cos^2 \alpha + (1 + \cos \alpha)^2 / 3 \right] / \left[ \zeta \sin \alpha (1 + \cos \alpha)^2 \right] = 159487\beta / \zeta \quad (8)$$

The cross-sectional area of the whole dogbone section is

$$A = ht + 2A_f = \beta h^2 + c_2 \zeta^2 h^2 / 2 = h^2 P_1(\zeta) \quad (9)$$

and the whole moment of inertia can be written as

$$I_x = h^3 t / 12 + 2 \left[ I_{R_f} + A_f (h/2 + y_G)^2 \right] \quad (10)$$

$$\text{or } I_x = \beta h^4 / 12 + c_1 \zeta^4 h^4 / 8 + c_2 \zeta^2 h^4 (1 + c_3 \zeta)^2 / 8 \quad (11)$$

and the elastic section modulus is

$$W_{el,x} = \frac{I_x}{h_f + h/2} = \frac{2I_x}{h(1 + \zeta)} = \frac{\beta h^3}{6(1 + \zeta)} + \frac{\zeta^2 h^3 [c_1 \zeta^2 + c_2 (1 + c_3 \zeta)^2]}{4(1 + \zeta)} = h^3 P_2(\zeta) \quad (12)$$

Note that the sectional characteristics required for the calculation of the critical bending moment of lateral-torsional buckling are not detailed here.

## 3. DESIGN CONSTRAINTS

Constraint on maximum normal stress due to bending moment  $M_{max}$  according to EC3 is defined by

$$\sigma_{max} = M_{max} / W_{el,x} \leq f_{y1} = f_y / \gamma_{M1}, \quad \gamma_{M1} = 1.1 \quad (13)$$

or expressed by the required section modulus  $W_o$

$$W_{elx} \geq W_o = M_{\max} / f_{y1} \quad (14)$$

*Local buckling constraints*

for the web  $t \geq \beta_w h$  (15)

where  $1/\beta_w = 124\varepsilon\sqrt{1+\zeta}$ ,  $\varepsilon = \sqrt{235/f_{y1}}$  (16)

for the upper part of the flange  $t \geq \beta_{fo}B = \beta_{fo}gh / \tan \alpha = \beta_f h$  (17)

where  $1/\beta_{fo} = 42\varepsilon$ ,  $1/\beta_f = 42\varepsilon \tan \alpha / \zeta$  (18)

Since the web and flange have the same thickness, the larger value from  $\beta_w h$  and  $\beta_f h$  is governing. We use the symbol  $\beta$ , but we should perform the optimization either with  $\beta_w$  or with  $\beta_f$ .

*Constraint on shear buckling of web* is active only in the case of very short beams, so we do not treat it in details.

#### 4. OPTIMUM DESIGN

It is possible to use a mathematical programming method to find the optimum values of  $\zeta, h, t$  which minimize the objective function Eq.(9) and fulfil the design constraints Eqs. (14, 15, 17). On the other hand, it is possible to express  $A$  in function of  $\zeta$  as follows: considering the constraint Eq.(14) as active, expressing  $h^3$  from it and substituting it into Eq.(9) one obtains

$$h = (W_o / P_2)^{1/3} \quad (19)$$

and  $A / W_o^{2/3} = P_1 / P_2^{2/3}$  (20)

where  $P_1 = \beta + c_2 \zeta^2 / 2$  ( $\beta = \beta_w$  or  $\beta_f$ ) (21)

and  $P_2 = \frac{\beta}{6(1+\zeta)} + \frac{\zeta^2 [c_1 \zeta^2 + c_2 (1 + c_3 \zeta)^2]}{4(1+\zeta)}$  (22)

Investigating Eq.(20) numerically, the optimum  $\zeta$  value can be found, which minimizes  $A$ . The calculations should be performed either with  $\beta_w$  or with  $\beta_f$ .

With known  $\zeta_{opt}$ ,  $h_{opt}$  can be calculated using Eq.(19), then

$$h_f = gh / 2, B = 2h_f / \tan \alpha, t = \beta h.$$

#### 5. OPTIMUM DIMENSIONS OF WELDED I-SECTIONS

The optimum dimensions of a welded I-section, considering the constraints on normal stress due to bending and on local buckling of web and flange, have been derived by the first author [3] as follows:

$$h_{opt} = (1.5W_o / \beta_f)^{1/3}; t_w = \beta_f h; b = h\sqrt{\beta_f / 2\delta_f}$$

$$A_{min} = (18\beta_f W_o^2)^{1/3}; 1/\beta_f = 124\varepsilon / \delta_f = 28\varepsilon; \varepsilon = \sqrt{235 / f_{yl}} \quad (23)$$

$h$  is the web depth,  $t_w$  is the web thickness,  $b$  is the flange width,  $t_f$  is the flange thickness.

## 6. NUMERICAL COMPARISON OF DOGBONE AND WELDED I-SECTION

We take  $f_y = 355$  MPa, thus  $f_{yl} = 355/1.1 = 323$  MPa,  $\varepsilon = 0.8529$ . Fig.2 shows the values of Eq.(20) in function of  $\zeta$  for  $\beta_w$  and  $\beta_f$ . It can be seen that the optimum value is given by the intersection point of the two curves,  $\zeta_{opt} = 0.18$ . The calculated dimensions and sectional properties are given in Table 1. The main characteristics show the advantages of dogbone over welded I-sections as follows: 1) the cross-sectional areas are 5% smaller, 2) the moments of inertia are 10% larger, 3) the critical lateral-torsional buckling lengths  $L_{cr}$  are 5-13% larger. Note that the  $L_{cr}$ -values are calculated according to EC3.

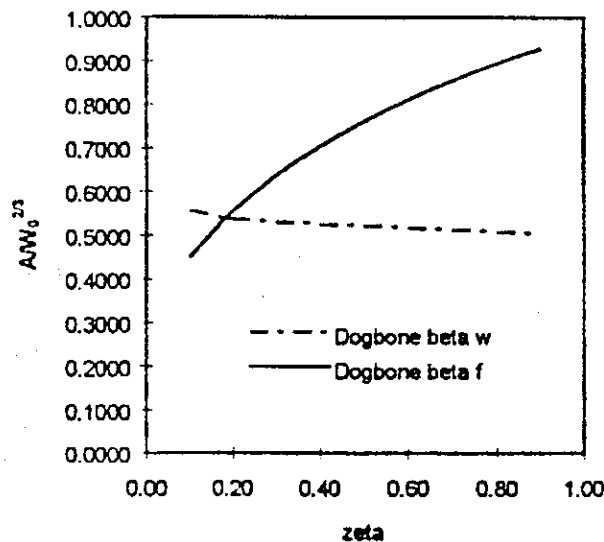


Fig.2.  $A/W_o^{2/3}$ - values in function of  $\zeta$ ,  $\zeta_{opt} = 0.18$

Table 1. Properties of optimum dogbone and welded I-section beams for some bending moments  $M$  (kNm). The cross-sectional area  $A$  is in  $\text{mm}^2$ , the moment of inertia  $I_x$  is in  $\text{mm}^4$ , other values in mm.

$M$	Dogbone						I - section						
	$A$	$h$	$B$	$t$	$10^{-7}I_x$	$L_{cr}$	$A$	$h$	$t_w$	$b$	$t_f$	$10^{-7}I_x$	$L_{cr}$
80	2116	314.6	98.1	2.87	4.6000	5720	2221	334.7	3.32	137.8	4.03	4.1490	5420
120	2773	360.1	112.3	3.28	7.8999	5660	2911	383.2	3.80	157.7	4.61	7.1242	5340
160	3359	396.4	123.6	3.61	11.593	5600	3526	427.8	4.20	173.6	5.10	10.455	5250
200	3898	426.9	133.1	3.90	15.611	5530	4092	454.3	4.50	186.9	5.47	14.078	5150
240	4402	453.7	141.5	4.14	19.907	5450	4621	482.8	4.80	198.7	5.81	17.952	5020
280	4878	477.6	148.9	4.36	24.449	5340	5121	508.3	5.04	209.2	6.12	22.048	4870
320	5353	499.4	155.7	4.56	29.214	5220	5598	531.4	5.30	218.7	6.40	26.345	4680
360	5769	519.4	161.9	4.74	34.181	5030	6055	552.7	5.50	227.4	6.70	30.825	4430

## REFERENCES

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