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## NOVÉ TRENDY V STROJÁRSTVE NA PRAHU TRETIEHO TISÍCROČIA

## 2. sekcia
## VÝPOČTOVÉ A EXPERIMENTÁLNE METÓDY PRI STAVBE STROJOV A ZARIADENÍ

# Optimum design for bending of welded box beams with longitudinal stiffeners

by

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## Introduction

Welded box beams are widely used because of their large bending and torsional stiffness. When there are no limitations in beam height, longitudinal stiffeners can be used to decrease the weight and to allow for thinner web plates. In this case the transverse diaphragms have important role to support the stiffeners against overall buckling.
Our aim is to work out the minimum cross-sectional area design for box beams subject to bending

## Objective function and design constraints

The cross-sectional area to be minimized is (Fig.1)

$$A = 2ht_w + 2bt_f + 2t_s(b_1 + b_2) \tag{1}$$

The stress constraint can be expressed as

$$\sigma_{max} = M_{max} / W_x \le f_y \tag{2}$$

where $M_{max}$ is the maximum bending moment, $W_x$ is the section modulus, $f_y$ is the yield stress. In the moment of inertia the effect of stiffeners is neglected, thus

$$I_x \cong h^3 t_w / 6 + 2bt_f (h/2)^2 \tag{3}$$

The stress constraint (2) can be written in the following form

$$W_x \cong I_x / (h/2) = h^2 t_w / 3 + bt_f h \ge W_0 = M_{max} / f_y \tag{4}$$

From (1) we obtain

$$bt_f = A/2 - ht_w - A_s \tag{5}$$

Substituting (5) into (4) we get

$$W_x = Ah/2 - 2h^2 t_w / 3 - A_s h \ge W_0 \tag{6}$$

and from (6) one obtains

$$A \ge 2W_0 / h + 4h^2 t_w / 3 + 2A_s \tag{7}$$

The local buckling constraint for the compression flange according to Eurocode 3 (1990) can be expressed as

$$b/t_f \leq 1/\delta = 42\varepsilon, \qquad \varepsilon = \sqrt{235/f_y} \tag{8}$$

The local buckling constraint for the upper part of webs is

$$0.2h/t_w \leq 42\varepsilon/(0.67+0.33\psi) \tag{9}$$

for $\psi = 0.6$ we get $h/t_w \leq 242\varepsilon = 1/\beta$ (10)

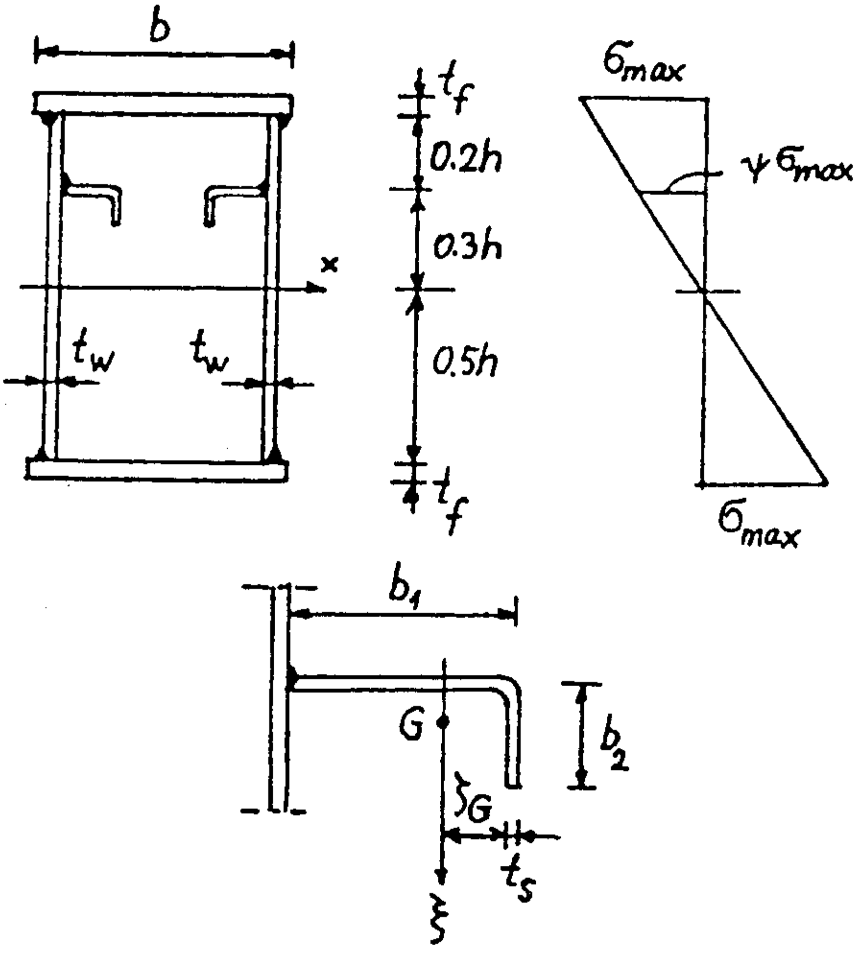


Fig. 1. Stiffened box beam and the detail of the stiffener

For the lower part of webs it is $\psi = -5/3 = -1.667$ and

$$0.8h/t_w \leq 62\varepsilon(1-\psi)\sqrt{-\psi} \quad \text{or} \quad h/t_w \leq 267\varepsilon \tag{11}$$

thus, the limiting slenderness defined by (10) is governing.

The local buckling constraints for the cold-formed stiffeners according to DASt Richtlinie 016 (1986) $b_1 / t_s \le 1.33\sqrt{E / f_y}$ and $b_2 / t_s \le 0.43\sqrt{E / f_y}$ or in another form $b_1 / t_s \le 30\varepsilon = \delta_1$ and $b_2 / t_s \le 12.5\varepsilon = \delta_2$ (12)

The overall buckling constraint for compressed stiffeners according to API design rules

$$I_\xi \ge 4at_w^3 \tag{13}$$

where $a$ is the distance of diaphragms. The location of the center of gravity for a stiffener (Fig.1), calculating with $b_1 = \delta_1 t_s$ and $b_2 = \delta_2 t_s$ is

$$\zeta_G = \frac{t_s\delta_1^2}{2(\delta_1 + \delta_2)} \tag{14}$$

and the moment of inertia is given by

$$I_\xi = C_s t_s^4; \qquad C_s = \frac{1}{12}\left(\delta_1^3 + \frac{3\delta_1^2\delta_2^2}{\delta_1 + \delta_2}\right) \tag{15}$$

Expressing $a$ in terms of $h$ $\quad a = C_a h$ (16)

and using (15), from (13) one obtains $\quad t_s = h\sqrt[4]{4C_a\beta^3 / C_s}$ (17)

and (7) can be written as

$$A = 2W_0 / h + 4\beta h^2 / 3 + 2h^2 C_A; \quad C_A = (\delta_1 + \delta_2)\sqrt{4C_a\beta^3 / C_s} \tag{18}$$

The condition of $dA/dh = 0$ gives the optimum beam height

$$h_{opt} = \sqrt[3]{\frac{W_0}{4\beta / 3 + 2C_A}} \tag{19}$$

Without longitudinal stiffeners we obtain

$$h_{0opt} = \sqrt[3]{\frac{3W_0}{4\beta_0}} \quad \text{with} \quad 1/\beta_0 = 124\varepsilon \tag{20}$$

already derived in the book of Farkas (1984).

Expressing $W_0$ from (19) and substituting it into (18) we get

$$A_{min} = \sqrt[3]{18W_0^2(2\beta + 3C_A)} \tag{21}$$

and without stiffeners

$$A_{0min} = \sqrt[3]{36\beta_0 W_0^2} \tag{22}$$

## Comparison of beams with and without stiffeners

Taking $f_y = 235$ MPa, we calculate with the following values $1/\beta = 242$; $1/\beta_0 = 124$, $\delta_1 = 30$; $\delta_2 = 12.5$; in (15) $C_t = 3077.2$, taking $C_a = 1.5$ we get $C_A = 1/2006$.

With (19) we obtain $h_{opt} = 5.36\sqrt[3]{W_0}$ and with (20) $h_{0opt} = 4.53\sqrt[3]{W_0}$ i.e. the beam with longitudinal stiffeners has 15% higher webs than that without stiffeners.

According to (21) $A_{min} = 0.5601\sqrt[3]{W_0^2}$ and with (22) $A_{0min} = 0.6622\sqrt[3]{W_0^2}$ i.e. the beam with stiffeners has 18% smaller weight than that without stiffeners

## Conclusion

The simple design formulae derived for box beams in bending are generalized for the case of box beams with longitudinal stiffeners placed at 1/5 of the webs height The box beams with longitudinal stiffeners are advantageous, since their weight is 18% smaller than that of beams without stiffeners.

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