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OPTIMUM DESIGN OF FIVE-LAYER SANDWICH BEAMS

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ABSTRACT

Welded aluminium box beams have very low vibration damping capacity. Regarding damping it is better to use a three-layer beam constructed from two rectangular hollow sections and a rubber layer glued between them. These three-layer beams have good damping capacity, but the dynamic deflection is large due to shear deformation of the rubber layer. To decrease this deflection two fiber-reinforced plastic layers are used. The minimum material cost design is worked out for such five-layer sandwich beams using the Rosenbrock Hillclimb mathematical programming method. Constraints on stress, deflection and local buckling are considered. Comparison is made between optimized versions of a simple welded aluminium beam, three-layer and five-layer beams. It is shown that the five-layer beam is the cheapest version when the deflection constraint is strict.

KEYWORDS

Sandwich beams, Structural optimization, Vibration damping, Minimum cost design, Fiber-reinforced plastics, Rubber layer

INTRODUCTION

A simple welded steel or aluminium-alloy box beam (Fig.1a.) has a very small vibration damping capacity, thus, in several industrial applications a sandwich beam may be used, which consists of two rectangular hollow sections (RHS) and a layer of high damping material (e.g. rubber) glued between them (Fig.1b.). The optimum material cost design of such sandwich beams has been treated in our recent study (Farkas and Jármai [3]).

It has been shown that a three-layer sandwich beam has better characteristics and lower material cost than a simple homogeneous welded box beam optimized with the same constraints. The only disadvantage of a three-layer beam is the relatively large deflection due to the shear deformation of the rubber layer.

This disadvantage can be eliminated by using fiber-reinforced plastic (FRP) layers which have a significant strengthening effect. Thus, a five-layer sandwich beam (Fig.1c.) has a high damping capacity and a large bending stiffness. These advantageous characteristics have been verified by experiments carried out in our laboratory (Bakk, Farkas and Jármai [1]).

The measurements showed that the FRP layers increase the bending stiffness but their damping capacity is very small, so, to obtain a high damping, it is necessary to use also a rubber layer. The FRP layer used in our experiments have had glass fibers. A 1.5 mm thick reinforcement contained 7 layers with the fiber orientation angles as follows: for 75% of fibers 0^0, for 25% 90^0. The elastic modulus in longitudinal direction is calculated according to the theory of composites (Hoa [4]). The investigations also showed that the deflections and stresses of a five-layer sandwich beam can be calculated with sufficient accuracy using the bending theory of sandwich beams with thick faces (Stamm and Witte [5]). In this case the faces can be treated as a thick face consisting of two elements and having a reduced tensile and bending stiffness as follows:

$$(EA)_{red} = E_{RHS}A_{RHS} + E_{FRP}A_{FRP} \text{ and similarly for } (EI)_{red}$$

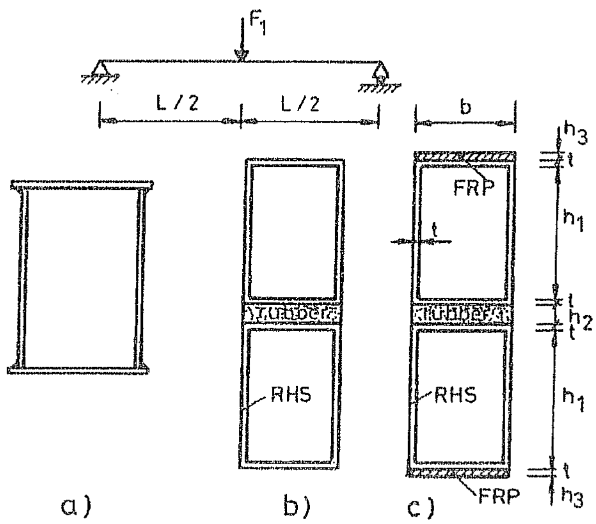

Fig. 1. a) aluminium welded box beam, b) a three-layer beam, c) a five-layer beam

Since the optimum design of such sandwich beams is not treated in the literature, our aim is to work out the structural synthesis of such beams to give designers useful aspects. Design aspects are usually based on comparisons of different structural versions, but only the optimized versions can be compared to each other realistically.

OBJECTIVE FUNCTION AND CONSTRAINTS

In the optimization procedure the five unknown dimensions (Fig.1c.) are sought to minimize the material cost function and to fulfil the following design constraints: constraint on maximum normal stress, shear stress in the core, maximum deflection, local buckling of the compression flange and webs of RHS. Our computer program based on the Rosenbrock's Hillclimb direct search constrained function minimization method is used to find the optimum continuous solution. The optimization procedure can be complemented by an additional program to obtain the discrete rounded optimum dimensions.

The stresses and deflection are calculated considering a dynamic force $F_2 = F_1/\eta$ acting at midspan (Fig.1.), where F_1 is a given static force and η is the damping factor of the sandwich beam calculated with the Ungar's formula (Ungar [6]). Structural versions of a homogeneous welded box beam, a three- and a five-layer sandwich beam (Fig.1a,b,c.), optimized with the same constraints, are compared to each other to show the effect of the rubber and FRP layers. For the optimization of the homogeneous welded box beam the first author's formulae are used (Farkas [2]).

The material cost function to be minimized is

$$K_m / L = 4t(b+h_1)k_{RHS} + bh_2 k_{rubber} + 2bh_3 k_{FRP} \tag{1}$$

where k_i are the cost factors.

Deflection constraint is given by

$$w_{max} = \frac{F_2 L^3}{48B} + \frac{F_2 L}{4B_q}\left(1-\frac{B_f}{B}\right)\left(1-\frac{\tanh\chi}{\chi}\right) \le w^* \tag{2}$$

where $B = B_f + B_S$,

$$B_f = 2(EI)_{red} = 2\left[E_{RHS}\left(\frac{h_1^3 t}{6} + \frac{bth_1^2}{2}\right) + E_{FRP} bh_3 \frac{(h_1+h_3)^2}{4}\right] \tag{3}$$

$$B_S = 2\left[E_{RHS} t(b+h_1)\frac{(h_1+h_2)^2}{2} + E_{FRP} bh_3\left(\frac{h_3}{2}+h_1+\frac{h_2}{2}\right)^2\right] \tag{4}$$

$$\chi = \frac{1}{2}\left[\frac{B_f}{B_q L^2}\left(1-\frac{B_f}{B}\right)\right]^{-1/2}; \qquad B_q = G_S b(h_1+h_2)^2/h_2 \tag{5}$$

G_S is the static shear modulus of the rubber layer, w^* is the admissible deflection. $F_2 = F_1/\eta$;

$$\eta = \eta_2 \frac{XY}{1+(2+Y)X+(1+Y)(1+\eta_2^2)X^2} \tag{6}$$

η_2 is the damping factor of rubber.

$$X = g_0 r_k^2; \qquad g_0 = \frac{2G_d b}{(EA)_{red} h_2}; \tag{7}$$

for a simply supported beam $r_k = L/\pi$,

$$Y = (EA)_{red}\frac{(h_1+h_2)^2}{2B_f}; \qquad (EA)_{red} = 2t(b+h_1)E_{RHS} + bh_3 E_{FRP} \tag{8}$$

G_d is the dynamic shear modulus of rubber. Normal stress constraint is expressed as

$$\sigma_{max} = E_{RHS}\frac{F_2 L}{4}\left[\frac{h_1+h_2/2}{B}\left(1-\frac{\tanh\chi}{\chi}\right)+\frac{h_1\tanh\chi}{2B_f\chi}\right] \le \sigma_{adm} \tag{9}$$

σ_{adm} is the admissible stress of the applied aluminium-alloy.

Local buckling constraints according to the BS 8118 (1991) for aluminium structures, for unwelded profiles

$$\frac{b}{t} \le 22\sqrt{\frac{250}{\sigma_{adm}}}; \qquad \frac{h_1}{t} \le \frac{22}{0.65}\sqrt{\frac{250}{\sigma_{adm}}}. \tag{10}$$

In addition the constraint on shear stress in the rubber layer and the constraint on shear buckling of RHS webs can be formulated for very short beams.

NUMERICAL DATA AND RESULTS

Data and results of the computation are as follows: data: $L = 4$m, $F_1 = 475$ N, elastic moduli in MPa: $E_{RHS} = 7*10^4$, $E_{FRP} = 4.6*10^4$, $G_S = 2.36$, $G_d = 7.0$; $\eta_2 = 0.18$; $\sigma_{adm} = 100$ MPa, w^* is varied; $k_{RHS} = 20000$, $k_{rubber} = 5000$, $k_{FRP} = 70000$ \$/m^3.
Since the deflection constraint is active, it is advantageous to show the results in function of the allowable deflection. Tables 1 and 2 show the results for three- and five-layer beams, respectively, in the case of span length $L = 4$ m. To illustrate the dependence of the results on span length, Table 4 and Fig.3 show the costs for three- and five-layer beams in the case of span length of 3 and 4 m, respectively. It can be seen that the five-layer beam is cheaper than the three-layer one, when the allowable deflection $w^*<4$ mm for $L = 3$ m, and $w^*<3$ mm for $L = 4$ m.
Figs 2a and b show the optimum dimensions of five-layer beams in function of the allowable deflection. It can be seen that the w^* affects mostly the height h_1.

Table 1. Optimum results for three-layer beams in the function of the allowable deflection. Dimensions in mm

h_1	h_2	b	t	Cost, \$	Damping	Allowable deflection, mm
368.30	5.01	40.01	7.07	924.16	0.05483	1
280.00	5.01	67.24	6.06	679.86	0.04654	2
267.11	5.01	40.08	5.11	506.05	0.05143	3
246.59	5.01	40.70	4.73	438.71	0.04919	4
232.66	5.09	40.01	4.45	392.25	0.04860	5
221.86	5.00	40.31	4.24	359.92	0.04607	6
212.49	5.94	40.02	4.06	333.06	0.04768	7
203.52	8.08	42.95	3.91	315.64	0.04997	8
199.40	11.98	41.02	3.81	303.07	0.05417	9
192.78	7.01	40.01	3.69	280.21	0.04739	10

Table 2. Optimum results for five-layer beams in the function of the allowable deflection, Dimensions in mm

h_1	h_2	h_2	b	t	Cost, \$	Damping	Allowable deflection, mm
349.76	5.01	1.32	59.75	6.83	845.03	0.05183	1
294.01	5.00	1.0	48.56	5.62	648.52	0.05082	2
263.34	5.00	1.01	47.56	5.04	532.51	0.04839	3
241.25	5.87	1.00	55.79	4.61	476.24	0.04565	4
231.61	5.12	1.00	40.01	4.43	411.45	0.04763	5
219.3	5.57	1.00	44.19	4.19	383.25	0.04579	6
211.21	8.88	1.05	39.99	4.04	353.73	0.04942	7
203.61	7.19	1.00	40.05	3.89	331.75	0.04950	8
197.36	9.73	1.01	39.98	3.78	317.33	0.05204	9
191.39	7.57	1.01	40.01	3.66	299.56	0.04818	10

In order to compare the sandwich beams with the simple welded aluminium box beams, optimum dimensions of the box beam are calculated with formulae as follows:

The required moment of inertia, considering the deflection constraint, is

$$I_0 = \frac{F_2 L^3}{48 E w^*} \tag{11}$$

The cross-sectional area of the beam optimized for deflection is (Farkas [2])

$$A = \sqrt{64 \beta I_0 / 3} \tag{12}$$

where the local buckling constraint of the welded aluminium web plate is

$$\frac{h}{t_w / 2} \le \frac{18}{0.35} \sqrt{\frac{250}{\sigma_{\max}}} = 81 = \frac{1}{\beta} \tag{13}$$

The results are summarized in Table 3 for the case of span length 4 m. It can be seen that the box beam is the most expensive structural version for all allowable deflections.

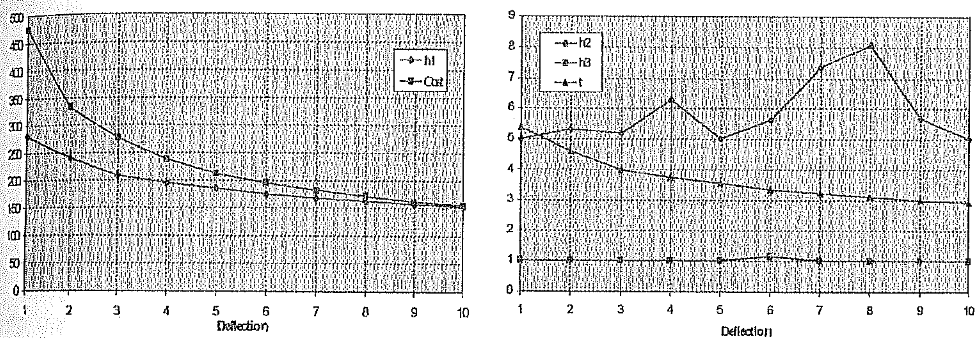

Fig. 2a, 2b. The effect of allowable deflection on h_1, h_2, h_3 and t optimum dimensions of 5 layers beams

Table 3. Costs in $ of the three structural versions for span length of 4 m

w* mm	10	9	8	7	6	5	4	3	2	1
a)	451	475	504	539	582	638	713	823	1008	1426
b)	280	303	315	333	360	392	439	506	680	924
c)	300	317	332	354	383	411	476	532	648	845

Table 4. Comparison of the five and three-layer beams in the function of the allowable deflection

Cost, 5 Layers, 3m, $	Cost, 5 Layers, 4m, $	Cost, 3 Layers, 3m, $	Cost, 3 Layers, 4m, $	Allowable deflection, mm
475.45	845.03	554.94	924.16	1
332.12	648.52	313.40	679.86	2
276.23	532.51	279.20	506.05	3
238.27	476.24	218.81	438.71	4
211.03	411.45	196.35	392.25	5
195.41	383.25	179.07	359.92	6
182.44	353.73	165.69	333.06	7
172.07	331.75	155.54	315.64	8
161.45	317.33	146.96	303.07	9
154.57	299.56	139.79	280.21	10

Table 3 gives the costs of the three structural versions as shown in Fig.1a,b,c for different allowable deflections. It can be seen that the welded box beam is the most expensive version. The five-layer sandwich beam is the cheapest version when the deflection constraint is strict, i.e. when $w^* < 3$ mm.

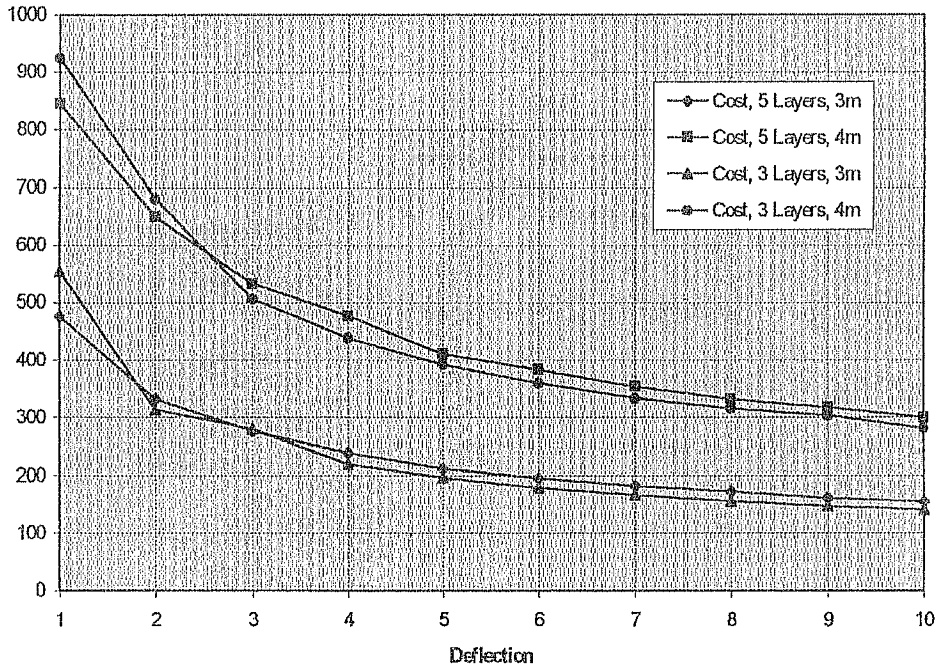

Fig. 3. Comparison of the five and three-layer beams in the function of the allowable deflection

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