# Tubular Structures VIII

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### Optimum design of a statically indeterminate tubular truss

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ABSTRACT: The minimum weight design of a two-span truss is treated. The truss is welded from square hollow section rods and the height at the middle support is linearly increased. The cross-sectional areas of all chords are the same and two different cross-sectional areas are considered for bracing, since the bracing forces near the middle support are higher than those in other parts. Thus, in the optimization procedure three cross-sectional areas and two factors for height are sought, which minimize the structural volume and fulfil the stress and buckling constraints. For the calculation of the flexural buckling strength of compression members closed formulae are applied, which can be derived from the buckling curves of the Japanese Road Association. First the cross-sectional areas are calculated using an iterative process for constant height factors, then these factors are varied. Finally the strength of overlapped nodes are checked according to CIDECT design formulae. The illustrative numerical example shows that the optimum truss geometry depends on the number of different cross-sectional areas.

#### LINTRODUCTION

Tubular trusses are widely applied in various structures. Their main advantage is the economy in weight because of high flexural buckling strength of compressed rods. The authors have published a lot of articles dealing with the optimum design of tubular trusses. The economy of higher strength steels in trusses has been treated in Farkas (1984).

The role of fatigue constraints has been studied in Farkas (1987a,b). The minimum cost design has been worked out taking into account buckling and fatigue constraints in Farkas (1990). The absorbed

energy of tubular braces has been determined in Farkas (1993).

In the article of Farkas and Jármai (1994) roof trusses welded from circular (CHS) or square (SHS) hollow section as well as double-angle section rods have been compared to each other and it has been shown that the structural weight of CHS or SHS trusses is much smaller than that of double-angle trusses. In this article, it has been verified that the optimum truss geometry depends on the cross-sectional shape of compressed members.

The optimum truss topology of a simply supported belt-conveyor bridge structure has been determined in Jármai and Farkas (1994).

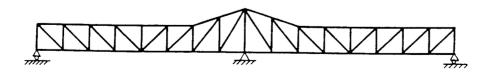


Figure 1. A two-span truss with increased height at the middle support

The optimum height of a statically determinate tubular truss with parallel chords has been calculated taking into consideration also the CIDECT rules for the strength of nodes in Farkas and Jármai (1995).

The minimum cost design of a Vierendeel truss welded from SHS rods has been worked out in Farkas and Jármai (1996). In the book Farkas and Jármai (1997) the optimum design of tubular trusses is treated in a separate chapter.

In the present study the minimum weight design of a statically indeterminate tubular truss shown in Figure 1 is treated. It is well-known that the strut forces in this case depend on the cross-sectional area of members, thus, the design of struts should be performed by an iterative process. Another problem is that, in a two-span truss the strut forces are the largest near the middle support, thus, the height should be increased here (Fig.1).

In the optimum design of this representative truss the following unknowns are sought, which minimize the weight of the structure and fulfil the design constraints: the cross-sectional areas  $A_C$  of all chords and two different profiles  $A_{BI}$  and  $A_{B2}$  for bracing as well as the two height factors  $\Omega = II/a$  and  $\omega = h/a$ .

#### 2 CALCULATION OF ROD FORCES

The unknown force X acting on the statically determinate basic structure (Fig.2) can be calculated from the displacement equation expressing that the horizontal displacement of node 17 should be zero

$$u_0 - X u_1 = 0 X = u_0 / u_1 (1$$

where

$$u_0 = \sum_{i} \frac{S_{0i} S_i L_i}{E A_i}$$
 and  $u_1 = \sum_{i} \frac{S_i^2 L_i}{E A_i}$  (2)

 $S_{0i}$  and  $s_i$  are the strut forces from external load and from X = 1, respectively,  $L_i$  and  $A_i$  are the length and the cross-sectional area of *i*-th strut, E is the modulus of elasticity.

For the structure shown in Figure 2 the horizontal displacements are as follows:

$$u_0 = \frac{Fa\Omega}{8\omega^2 A_C} \left[ 251 + \frac{98(1+\varsigma_1^3)\omega^2}{(\Omega+\omega)^2} + 36\varsigma_1^3 \right] +$$

$$+\frac{Fa\Omega}{A_{B1}}\left(\frac{7.5\omega}{8}+\frac{\omega_1^3}{\omega^2}\right)+\frac{Fa\Omega}{A_{B2}}\left[-\frac{3.5\Omega}{8}-\frac{\omega_1^3}{4\omega^2}+\left(\frac{3\Omega}{\omega}-4\right)*\right]$$

$$*\frac{6\Omega - 3\omega}{16} + \frac{6\Omega - \omega}{\Omega + \omega} \frac{0.75\Omega - \omega}{2} - \frac{\varsigma_2^3 \left(6\Omega - \omega\right)\left(4 - \frac{3\Omega}{\omega}\right)}{4\omega(\Omega + \omega)^2} -$$

$$-\frac{7\Omega_1^3}{\left(\Omega+\omega\right)^2\Omega}\left(-0.75\Omega+\omega\right)$$

$$u_1 = \frac{\Omega^2 a}{64\omega^2 A_c} \left[ 146 + \frac{196\omega^2}{\left(\Omega + \omega\right)} \left(1 + \zeta_1^3\right) + \frac{64\omega^2}{\Omega^2} + \frac{64\omega^2}{\Omega^2} \right]$$

$$+36\varsigma_{1}^{3}$$
  $+\frac{\Omega^{2}a}{64\omega^{2}A_{n3}}\left(5\omega^{3}+4\omega_{1}^{3}\right)+\frac{\Omega^{2}a}{64\omega^{2}A_{n3}}\left[\left(-4+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$ 

$$+\frac{3\Omega}{\omega}^{3}$$
  $\left(-\frac{3\Omega}{\omega^{3}} + 2\omega_{1}^{3} + \frac{32(0.75\Omega - \omega)^{2}\omega^{2}}{\Omega + \omega} + \Omega\omega^{2} + \frac{3\Omega}{\omega^{3}}\right)$ 

$$+\frac{4\varsigma^{3}\omega^{2}}{\left(\Omega+\omega\right)^{2}}\left(-4+\frac{3\Omega}{\omega}\right)^{2}+64\frac{\left(0.75\Omega-\omega\right)^{2}\Omega^{3}\omega^{2}}{\left(\Omega+\omega\right)^{2}\Omega^{2}}\right]$$

(4)

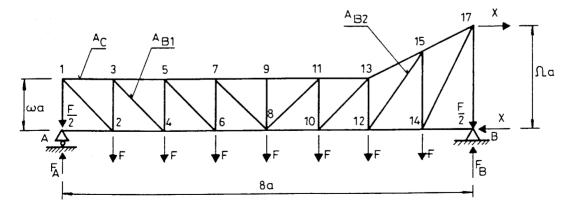


Figure 2. Forces acting on the half structure of the numerical example

The actual rod forces are given by

$$S_i = S_{0i} + Xs_i$$
.

It can be seen that these displacements depend on cross-sectional areas.

It should be mentioned that for parallel chords  $(\Omega = \omega)$  and only two different cross-sectional areas  $(A_{BI} = A_{B2})$  the formulae (3) and (4) take the following form:

$$u_0 = 336F/\omega \tag{3a}$$

$$u_1 = (\omega^3 + \omega_1^3) A_C / A_B + 43$$
 (4a)

In the above formulae the following symbols are used:  $\omega_1 = \sqrt{1 + \omega^2}$ ;  $\Omega_1 = \sqrt{1 + \Omega^2}$ ;

$$\varsigma_1 = \sqrt{1 + \frac{(\Omega - \omega)^2}{4}}; \qquad \varsigma_2 = \sqrt{1 + \frac{(\Omega + \omega)^2}{4}}$$

The rods of bracing are separated to two groups, the cross-sectional area  $A_{B2}$  is considered for rods 8-11, 10-13, 12-13, 12-15, 14-15, 14-17 and B-17 (Fig.2), the cross-sectional area for other rods is  $A_{B1}$ .

## 3 THE OBJECTIVE FUNCTION AND THE DESIGN CONSTRAINTS

The objective function is defined by

$$V/a = 2A_{\epsilon} \left(7 + \varsigma_{1}\right) + A_{n_{1}} (6\omega + 4\omega_{1}) +$$

$$+A_{n2}\left[2\omega_{\perp}+1.5(\Omega+\omega)+\varsigma_{2}+\Omega_{\perp}\right] \tag{5}$$

where V is the volume of the half structure.

To simplify the design of compression members for flexural buckling, the formulae of the Japanese Road Association (JRA) are applied (Hasegawa et al. 1985), which give near the same buckling factors as the Eurocode 3 (EC3) curve "b". From JRA formulae closed expressions can be derived for the required cross-sectional area as detailed below.

The flexural buckling constraint for the compression members subject to compressive force N is expressed as

$$N/A \le \chi f_{x1} \qquad f_{x1} = f_{x}/\gamma_{x1} \tag{6}$$

 $f_y$  is the yield stress,  $\gamma_{M1} = 1.1$  is the partial safety factor according to EC3. For the buckling factor JRA formulae are as follows:

$$\chi = 1$$
 for  $\overline{\lambda} \le 0.2$  (7a)

$$\chi = 1.109 - 0.545\overline{\lambda}$$
 for  $0.2 \le \overline{\lambda} \le 1$  (7b)

$$\chi = 1/(0.773 + \overline{\lambda}^2)$$
 for  $\overline{\lambda} \ge 1$  (7c)

where  $\overline{\lambda} = KL/r\lambda_E$ , K is the effective length factor. According to Packer et al (1992), in tubular trusses, for chords K=0.9, for bracing K=0.75.

The radius of gyration for SHS of width b is

$$r = h / \sqrt{6}$$
.  $\lambda_E = \pi \sqrt{E / f_v}$ , for the modulus of

elasticity of steels  $E = 2.1*10^{\circ}$  MPa and yield stress  $f_V = 355$  MPa  $\lambda_E = 76.41$ .

Introducing symbols  $\theta = 100b/L$ , the limiting

local stenderness 
$$\delta_L = (b/t)_t$$
,  $c = 100 K \sqrt{6} / \lambda_t$ 

and 
$$v = 10^4 N\delta_L / (4L^2 f_{11})$$
 and solving (6) with

(7b) one obtains (Farkas and Jármai 1997)

$$\theta = 0.24572c \left( 1 + \sqrt{1 + \frac{14.93475\nu}{c^2}} \right) \text{ for } \theta \ge c$$
 (8)

and for  $9 \le c$  with (7c)

$$\mathcal{G} = \left[ 0.3865 \nu \left( 1 + \sqrt{1 + \frac{6.69424 c^2}{\nu}} \right) \right]^{\nu/2}$$
 (9)

Considering the relationship

$$A = 4bt = \frac{4b^2}{\delta_L} = \frac{4\theta^2 L}{10^4 \delta_L} \tag{10}$$

with (8) we get

$$A = \frac{0.4830 L^2}{10^4 \delta_L} \left( c^2 + 7.4644 \nu + c^2 \sqrt{1 + \frac{14.93475 \nu}{c^2}} \right)$$
(11)

It should be noted that, for our numerical example (7b) is valid for all compression members. When  $9 \le c$  another formula for A should be derived.

The design constraint for tension force is given by

$$N/A \le f_{vi} \tag{12}$$

#### **4 NUMERICAL EXAMPLE**

Data: a = 3000 mm,  $f_y = 355$  MPa, the static concentrated forces F = 450 kN act on lower nodes representing the factored load of a belt-conveyor bridge.

On the basis of preliminary calculations the following rods are selected for each rod groups:

Table 1. Results of the numerical example for three different profiles (A and V/a in mm<sup>2</sup>)

Ω	ω	$A_C$	$A_{BI}$	$A_{B2}$	10 <sup>-1</sup> V/a
2.2	1.0	7580	3446	8780	2.687
2.2	0.9	7631	3347	8795	2.632
2.2	0.8	7655	3258	8795	2.574
2.2	0.7	7658	3177	8795	2.517
2.2	0.6	8177	3099	8795	2.598
2.0	0.9	8284	3356	8504	2.649
2.0	0.8	8357	3255	8523	2.601
2.0	0.7	8397	3165	8534	2.551
2.0	0.6	8409	3084	8534	2.500
2.0	0.5	9907	3005	9004	2.743
1.8	0.9	8990	3385	8200	2.680
1.8	0.8	9144	3268	8237	2.646
1.8	0.7	9247	3164	8261	2.607
1.8	0.6	9306	3072	8275	2.564
1.8	0.5	9805	2988	9043	2.668
1.6	0.9	9732	3442	7877	2.722
1.6	0.8	10012	3302	7935	2.708
1.6	0.7	10221	3180	7979	2.686
1.6	0.6	10365	3073	8009	2.658
1.6	0.5	10451	2979	9066	2.715
1.4	1.4	7539	4536	7001	2.675
1.4	1.3	8121	4330	7099	2.683
1.4	1.2	8729	4123	7208	2.700
1.3	1.3	8132	4424	6892	2.657
1.3	1.2	8788	4216	6995	2.680
1.2	1.2	8815	4316	6777	2.658
1.2	1.1	9567	4104	6892	2.701
1.1	1.1	9621	4209	6678	2.685
1.1	1.0	10488	3993	6793	2.749
1.0	1.0	10604	4102	6694	2.754
1.0	0.9	11674	3875	7072	2.875

(a) for all rods of chords two compressed rods are selected: rod 14-B subject to  $N = S_{IJ-B} = X$  and rod 5-7 with force

$$N = S_{5-7} = \frac{7.5F}{\omega} - \frac{3\Omega}{8\omega} X$$
.

The required  $A_C$  is calculated for larger compressive force using (11) with values of c = 2.8851 and v with L = a and, according to CIDECT rules (Packer et al 1992),  $\delta_L = 30$ ;

(b) for rods of bracing excluding the rods near the middle support (mentioned at the end of Section 2): rod A-1, subject to  $N = S_{A-1} = 3.5F - \lambda\Omega/8$  compressive force,  $A_{BI}$  is calculated with (11) considering c = 2.4043 and  $L = \omega a$ ;

Table 2. Results for two different profiles

Ω	ω	$A_C$	$A_B$	10 V/a
2.0	1.2	7523	8302	3.332
2.0	1.1	7762	8365	3.285
2.0	1.0	7951	8415	3.228
2.0	0.9	8093	8453	3.162
2.0	0.8	8191	8479	3.088
2.0	0.7	8251	8495	3,008
2.0	0.6	8559	8509	2.970
2.0	0.5	10102	8933	3.212
1.8	0.9	8749	8143	3.148
1.8	0.8	8933	8187	3.094
1.8	0.7	9062	8217	3.031
1.8	0.6	9244	8237	2.961
1.8	0.5	10009	8968	3.141
1.6	1.2	8106	7538	3.138
1.6	1.1	8592	7639	3.147
1.6	1.0	9037	7732	3.148
1.4	1.4	7078	6916	2.956
1.4	1.3	7652	7018	2.986
1.4	1.2	8261	7127	3.021
1.3	1.3	7652	6810	2.915
1.3	1.2	8301	6917	2.958
1.3	1.1	8988	7031	3.008
1.2	1.2	8316	6706	2.893
1.2	1.1	9056	6818	2.953
1.1	1.1	9092	6602	2.893
1.1	1.0	9949	6721	2.975
·1.0	1.0	10033	6587	2.943
1.0	0.9	11091	6999	3.113

(c) for rods of bracing near the middle support: rod B-17 of compressive force

 $N = S_{B-17} = 3.5F + \lambda\Omega/8$ ,  $A_{B2}$  is calculated with (11) considering c = 2.4043 and  $L = \Omega a$ ; rod 12-15 of tension force

$$S_{12-15} = F\varsigma_2 \frac{6\Omega - \omega}{\omega(\Omega + \omega)} + X\left(4 - \frac{3\Omega}{\omega}\right) \frac{\varsigma_2\Omega}{4(\Omega + \omega)}$$

calculated with (12), rod 10-13 of tension force

÷. \*

$$S_{10-13} = 1.5F\omega_1/\omega + X\Omega\omega_1/(8\omega).$$

The results are summarized for three different profiles in Table 1, for two different profiles in Table 2.

#### 5 THE OPTIMIZATION METHOD

This method is a direct search one without derivatives. Rosenbrock's (1960) method is an iterative procedure that bears some correspondence to the exploratory search of Hooke and Jeeves in that small steps are taken during the search in However, instead orthogonal coordinates. the co-ordinates continually searching corresponding to the directions of the independent variables, an improvement can be made after one cycle of co-ordinate search by lining the search directions up into an orthogonal system, with the overall step on the previous stage as the first building block for the new search coordinates. Rosenbrock's method locates  $x^{(k+1)}$  by successive unidimensional searches from an initial point  $x^{(k)}$ along a set of orthonormal directions.

The method is executed as follows:

Minimize the objective function  $f(x_i) \to \min$ .

Design constraints are:

explicit 
$$x_i^L \le x_i \le x_i^U$$
  $(i = 1, 2, ..., N),$   
implicit  $g_L(x_i) \ge 0$   $(j = 1; 2, ..., M).$  (13)

- (i) Before starting the minimization process, define a set of 'initial' step lengths  $S_i$ , to be taken along the search directions  $M_i$ , i = 1,2,...,N. The starting point must satisfy the constraints and should not lie in the boundary zones.
- (ii) After each function evaluation, the following steps are carried out: Define by  $f^{\,0}$  the current best

objective function value for a point where the constraints are satisfied, and f(x) where in addition to this the boundary zones are not violated.  $f^0$  and f(x) are initially set equal to the objective function value at the starting point.

- (iii) The first variable  $x_I$  is stepped a distance  $S_I$  parallel to the axis and the function evaluated. If the current point objective function value, f, is worse (greater or less) than f or if the constraints are violated, the trial point is a failure and  $S_I$  decreased by a factor  $\beta$ ,  $0 < \beta \le 1$ , and the direction of movement reversed. If the move is termed a success,  $S_I$  increased by a factor  $\alpha$ ,  $\alpha \ge 1$ . The new point is retained, and a success is recorded. The values of  $\alpha$  and  $\beta$  are usually taken as 3.0 and 0.5 respectively.
- Continue the search sequentially stepping the variables,  $x_i$ , a distance  $S_i$  parallel to the axis. The same acceleration or deceleration and reversal procedure is followed for all variables, until at least one step has been successful and one step has failed in each of the N directions. Perturbations are continued sequentially in the search directions until a success is followed by a failure in every direction, at which time the kth stage is terminated. Since an equal value of a function counts as a success, a success is eventually reached in each direction as the multipliers of reduce the magnitude of the step length. The final point obtained becomes the initial point for the succeeding stage  $x^{(k+1)} = x^{(k)}$ . The normalized direction  $S_i^{(k+1)}$  is chosen parallel to  $x_0^{(k+1)} - x_0^{(k)}$ , and the remaining directions are chosen orthonormal to each other and to  $S_i^{(k+1)}$ .
- (v) Compute the new set of directions rotating the axes. In general, the orthogonal search directions can be expressed as combinations of all the coordinates of the independent variables.
- (vi) Search is made in each of the x directions using the new co-ordinate axes. In each x direction the variables are stepped a distance  $S_i$  parallel to the axis and the function is evaluated.
- (vii) If the current point lies within a boundary zone, the objective function is modified as follows;

$$f(new) = f(old) - (f(old) - f^*)(3\lambda - 4\lambda^2 + 2\lambda^3)$$
(14)

where

The boundary zones are defined as follows:

$$\lambda = \frac{\text{distance into boundary zone}}{\text{width of boundary zone}}$$
 (15)

At the inner edge of the zone,  $\lambda = 0$ , i.e., the function is unaltered (f(new) = f(old)). At the constraints,  $\lambda = 1$ , and thus  $f(\text{new}) = f^*$ .

For a function which improves as the constraint is approached, the modified function has an optimum in the boundary zone.

- (viii)  $f^*$  is set equal to  $f^0$  if an improvement in the objective function has been obtained without violating the boundary zones or constraints.
- (ix) The search procedure to find the continuous values of the variables is terminated when the convergence criterion is satisfied.
- (x) The procedure was modified by a secondary search to find the discrete values of the variables.

The procedure stops if the convergence criterion or the iteration limit is reached. The procedure is very quick, but it gives usually local optima, so it is advisable to use more starting points. The Turbo/Borland C version of Hillclimb technique can be found in Farkas & Jármai (1997).

6 CHECK OF THE OPTIMUM VERSION  $(\Omega = 2.0, \omega = 0.6, \text{ three different profiles})$ 

From Table 1 we get  $A_C = 8409$ ,  $A_{BI} = 3084$  and  $A_{B2} = 8534$  mm<sup>2</sup>. According to (1), (3) and (4) one obtains X = 2501 kN. Using European Standard prEN 10219 (1996) SHS 250\*10 with A = 9260 mm<sup>2</sup> and radius of gyration r = 97 mm is selected for all rods of chords. According to Section 4  $S_{I4-B} = X = 2501$  kN,

 $S_{5.7} = 2449$  kN, thus, the rod 14-B is governing and should be checked for flexural buckling with formulae (6) and (7).

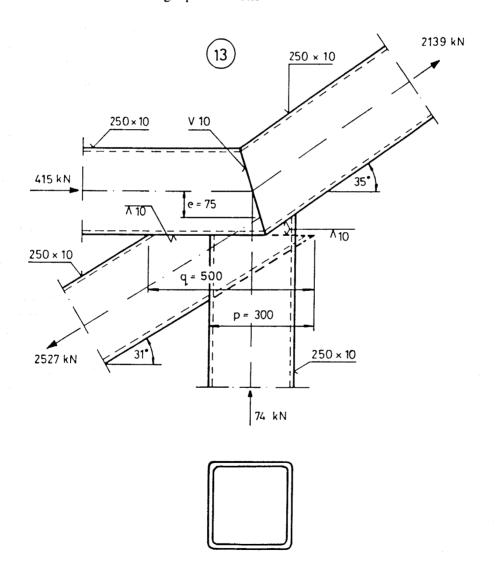


Figure 3. A characteristic node of the optimum structure ( $\Omega = 2.0, \omega = 0.6$ )

$$\bar{\lambda} = \frac{0.9 * 3000}{76.41 * 97} = 0.3643, \quad \chi = 0.9105$$

N/A = 2501000/9260 = 270 < 0.9105\*323 = 294 MPa, OK.

For smaller rods of bracing the section of 150\*6 is selected with  $A = 3360 \text{ mm}^2$  and r = 58.4 mm. According to Section 4  $S_{A-1} = 950 \text{ kN}$ ,

$$\bar{\lambda} = \frac{0.75 * 0.6 * 3000}{76.41 * 58.4} = 0.3025, \quad \chi = 0.9441.$$

 $N/A = 283 \le 0.9441*323 = 305 \text{ MPa. OK}$ .

For larger rods of bracing the section of 250\*10 is selected with  $A = 9260 \text{ mm}^2$  and r = 97 mm.

 $S_{B-17} = 2200$  kN,  $\overline{\lambda} = 0.6071$ ,  $\chi = 0.7781$ , NA = 238 < 251 MPa, OK. Since this section is valid only for the half truss, rod B-17 should be designed for double force 4400 kN and should have a cross-sectional area 2\*8534 = 17068 mm<sup>2</sup>. Section 300\*16 can be used with A = 17100 mm<sup>2</sup>

and r = 114 mm.  $\lambda = 0.5166$ ,  $\chi = 0.8275$ ,

 $N/A = 440000/17100 = 257 \times 0.8275*323 = 267$  MPa, OK. To enable the fabrication of node 17, the width of chords 15-17 and 14-B should be increased to 300 mm.

The formula given in Section 4 yields for this case a tension force  $S_{12-15} = 660 \text{ kN}$ ,  $N/A = 660000/9260 \pm 71 \pm 323 \text{ MPa}$ , OK.

To illustrate the SHS nodes in the optimum structure the drawing of node 13 is given in Figure 3.

Check the node 13 for effective width according to Packer et al (1992). For the rod 10-13 a section of 250\*10 should be used ( $A=9260 \text{ mm}^2$ ). The tension force in this rod is

$$S_{10/13} = 1.5 F \omega_1 / \omega + X \Omega \omega_1 / (8\omega) = 2527 \text{ kN},$$

N/A = 2527000/9260 = 273 < 323 MPa, OK. According to the Figure 3 the overlapping factor is Ov = (p/q)100 = (300/500)100 = 60%,

$$h_{e} = h_{eov} = \frac{10}{h_{0} / t_{0}} \frac{t_{0}}{t_{1}} h_{1} = \frac{10}{250 / 10} \frac{10}{10} 250 = 100 \text{ mm},$$

$$N_{1} = f_{v} t_{1} \left[ \frac{Ov}{50} (2h_{1} - 4t_{1}) + h_{e} + h_{eov} \right] =$$

$$= 355 * 10 \left[ \frac{60}{50} 460 + 2 * 100 \right] = 2670 > 2527 \text{ kN, OK.}$$

#### 7 CONCLUSIONS

The evaluation of the results of the numerical example gives the following conclusions.

- (a) designing the truss with three different cross-sectional areas, i.e. all chords with the same profile and bracing divided into two profile groups, the structure of increased height at the middle support gives the minimum structural weight for  $\Omega = 2.0$  and  $\omega = 0.6$  ( $V/a = 2.500*10^5$  mm<sup>2</sup>);
- (b) designing the truss with three different profiles, for the optimum version of parallel chords is the solution of  $\Omega = \omega = 1.3$  ( $V/a = 2.657*10^{\circ}$  mm<sup>2</sup>). 6% larger than that for the version with increased height;
- (c) designing the truss with only two different profiles, i.e. one profile for all chords and other for all bracing rods, the best non-parallel solution is  $\Omega = 2, \omega = 0.6$  ( $V/a = 2.970*10^5 \text{ mm}^2$ ) and the parallel chord solution of  $\Omega = \omega = 1.2$  gives the minimum weight ( $V/a = 2.893*10^5 \text{ mm}^2$ ). The non-parallel chord solution gives 19% larger weight than that of three different profiles.

The main conclusion is that the optimum truss geometry depends on the number of different rod profiles.

It should be noted that the optima are not very sensitive to the change of height parameters, although the weight difference between the best and worst ( $\Omega = 2.0, \omega = 1.2$ , V/a = 3.332\*10°) structural version is 33%.

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