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MINIMUM COST DESIGN OF A CONICAL SHELL

External pressure, non-equidistant ring-stiffening

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INTRODUCTION

Conical shells are applied in numerous structures e.g. in submarine and offshore structures, aircraft, tubular structures, towers, tanks, etc. Their structural characteristics are as follows.

- Material: steels, Al-alloys, fibre-reinforced plastics,
- Geometry: slightly conical (transition parts between two circular shells), strongly conical (storage tank roofs), truncated,
- Stiffening: ring-stiffeners, stringers, combined, equidistant, non-equidistant,
- Stiffener profile: flat, box, T-, L-, Z-shape,
- Loads: external pressure, axial compression, torsion, combined,
- Fabrication technology: welding, riveting, bolting, gluing.

Klöppel and Motzel [1] have carried out buckling experiments with truncated unstiffened and ring-stiffened steel conical shell specimens and proposed simple formulae for critical buckling stress.

Rao and Reddy [2] have worked out an optimization procedure for minimum weight of truncated conical shells. Rectangular ring-stiffeners and stringers are used and constraints on shell buckling as well as on natural frequency are considered.

In the book written by Ellinas et al. [3] experimental results and design of stiffened conical shells are treated.

Spagnoli has written a PhD thesis on buckling behaviour and design of stiffened conical shells under axial compression [4]. Rectangular stringers are considered. Later Spagnoli also with co-authors [5,6,7] has published other articles in this field.

Chryssanthopoulos et al. [8] have used finite element method for buckling analysis of stringer-stiffened conical shells in compression.

Singer et al. [9] have given a detailed description of experiments carried out with stiffened conical shell models.

Minimum cost design has been worked out for ring-stiffened circular cylindrical shell in our study [10,11].

In the present study we select the following structural characteristics: steel, slightly conical shell, ring-stiffeners of welded square box section to avoid tripping, non-equidistant stiffening, external pressure, welding. Design rules of Det Norske Veritas [12, 13] are applied for shell and stiffener buckling constraints.

The variables to be optimized are as follows: length of shell segments for a given shell thickness (Fig. 1), dimensions of ring-stiffeners (h_i , t_{ri}). Stiffeners should be used at the ends of the shell, thus, two stiffeners are used in the first shell segment. The ring stiffeners are placed in a small distance from the circumferential welds connecting two segments to allow the inspection of welded joints, this is marked in Fig. 1 by dotted lines. The cost function includes the cost of material, assembly, welding and painting and is formulated according to the fabrication sequence.

The optimization process has the following parts:

- (a) design of each shell segment length for a given shell thickness using the shell buckling constraint,
- (b) design of ring-stiffeners for each shell segment using the stiffener buckling constraint,
- (c) cost calculation for each shell segment and for the whole shell structure.

These design steps should be carried out for a series of shell thicknesses. On the basis of calculated costs the optimum solution corresponding to the minimum cost can be determined.

1 DESIGN OF SHELL SEGMENT LENGTHS

According to DNV rules [13], for shell segments between two ring-stiffeners of radii R_i and R_{i+1} , the buckling constraint valid for circular cylindrical shells with equivalent radius

$$R_{ei} = \frac{R_{i+1} + R_i}{2 \cos \alpha}, \cos \alpha = \frac{1}{\sqrt{\tan^2 \alpha + 1}} \quad (1)$$

$$\tan \alpha = \frac{R_{n+1} - R_1}{L_0}, R_{i+1} = L_i \tan \alpha + R_i \quad (2)$$

and equivalent thickness

$$t_{ei} = t_i \cos \alpha \quad (3)$$

The normal stress due to external pressure in a shell segment should be smaller than the critical buckling stress

$$\sigma_i = \frac{\gamma_b p R_i}{t_{ei}} \leq \sigma_{cri} = \frac{f_{y1}}{\sqrt{1 + \lambda_i^4}}, \quad \lambda_i = \sqrt{\frac{f_{y1}}{\sigma_{Ei}}} \quad (4)$$

$$\sigma_{Ei} = \frac{C_i \pi^2 E}{12(1 - \nu^2)} \left(\frac{t_{ei}}{L_{ei}} \right)^2, L_{ei} = \frac{L_i}{\cos \alpha} \quad (5)$$

where

$$C_i = 4 \sqrt{1 + \left(\frac{0.6 \xi_i}{4} \right)^2}, \xi_i = 1.04 \sqrt{Z_i}, Z_i = \frac{L_{ei}^2}{R_{ei} t_{ei}} \sqrt{1 - \nu^2} \quad (6)$$

Using Eqs (5), Eq.(6) can be written in the form of

$$C_i = 4 \sqrt{1 + 0.023214 \frac{L_{ei}^2}{R_{ei} t_{ei}}} \quad (7)$$

From the shell buckling constraint Eq.(4) the unknown L_i can be calculated using a Mathcad program.

2 DESIGN OF A RING-STIFFENER FOR EACH SHELL SEGMENT

For ring-stiffeners a square box section welded from 3 parts is selected to avoid tripping, which is dangerous failure mode for open-section stiffeners (Fig. 1).

The constraint on local buckling of the compressed stiffener flange according to Eurocode 3 [14] is expressed by

$$t_{ri} \geq \delta h_i, 1/\delta = 42\varepsilon, \varepsilon = \sqrt{235/f_y} \quad (8)$$

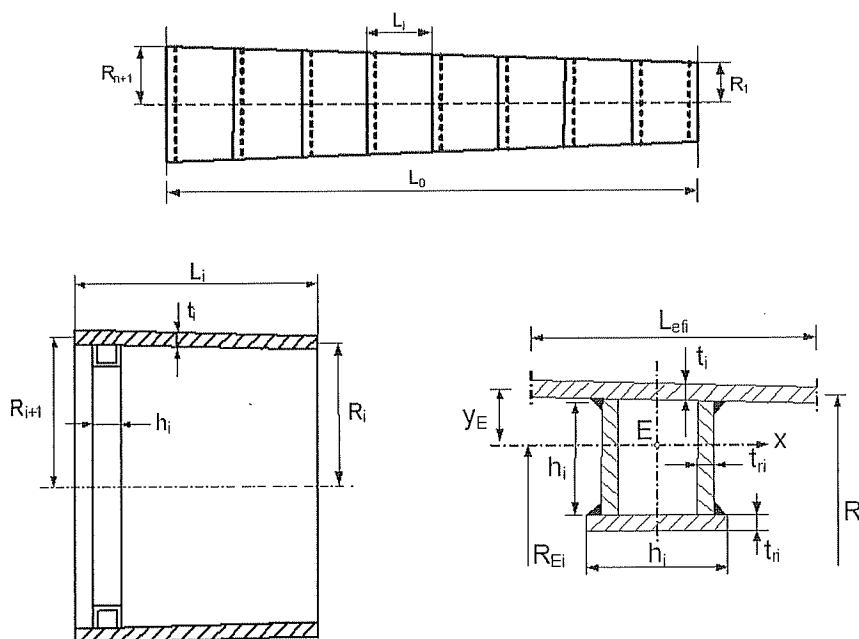


Fig. 1. The main dimensions of the conical shell – a shell segment with the ring-stiffener of welded square box section

for $f_y = 355 \text{ MPa}$ $1/\delta = 34$.

Calculating with Eq.(8) as equality, the only unknown for a square ring-stiffener is the height h_i . This dimension can be determined from the stiffener buckling constraint relating to the required moment of inertia of a stiffener section about the axis x of the point E , which is the gravity centre of the cross-section including the 3 stiffener parts and the effective part of the shell (Fig.1)

$$I_{xi} \geq I_{reqi} = \frac{\gamma_b p R_i R_{Ei}^2 L_{efi}}{3E} \left[2 + \frac{3E y_{Ei} 0.005 R_i}{R_{Ei}^2 (f_{y1} / 2 - \sigma_i)} \right] \quad (9)$$

where

$$I_{xi} = \frac{\delta h_i^4}{6} + 2\delta h_i^2 \left(\frac{h_i}{2} - y_{Ei} \right)^2 + \delta h_i^2 (h_i - y_{Ei})^2 + \frac{L_{efi} t_i^3}{12} + L_{efi} t_i y_{Ei}^2 \quad (10)$$

$$y_{Ei} = \frac{2\delta h_i^3}{3\delta h_i^2 + L_{efi} t_i} \quad (11)$$

$$L_{efi} = \min(L_i, L_{ef0i}), L_{ef0i} = 1.56 \sqrt{R_i t_i} \quad (12)$$

$$R_{Ei} = R_i - \left(h_i + \frac{t_i}{2} + \frac{\delta h_i}{2} - y_{Ei} \right) \quad (13)$$

The required h_i can be calculated from Eq.(9).

3 THE COST FUNCTION

The cost function is formulated according to the fabrication sequence as follows [11].

- (1) Forming of 3 plate elements for shell segments into slightly conical shape (K_{F0}).
- (2) Welding 3 curved shell elements into a shell segment with GMAW-C (gas metal arc welding with CO₂) butt welds (K_{F1}).
- (3) Welding of $n+1$ ring-stiffeners each from 3 elements with 2 GMAW-C fillet welds (K_{F2}).
- (4) Welding of a ring-stiffener into each shell segment with 2 GMAW-C fillet welds (K_{F3}).
- (5) Assembly of the whole stiffened shell structure from n shell segments (K_{F4A}).
- (6) Welding of n shell segments to form the whole shell structure with $n-1$ circumferential GMAW-C butt welds (K_{F4W}).
- (7) Painting of the whole shell structure from inside and outside (K_p).

The total cost includes the cost of material, assembly, welding and painting

$$K = K_M + K_{F0} + K_{F1} + K_{F2} + K_{F3} + K_{F4} + K_p \quad (13)$$

$$K_M = k_M \rho V, k_M = 1.0 \$ / \text{kg} \quad (14)$$

The volume of the whole structure includes the volume of shell segments (V_{li}) and ring-stiffeners (V_{ri})

$$V = \sum_{i=1}^n V_{li} + \sum_{i=1}^{n+1} V_{ri} \quad (15)$$

$$K_{F0i} = k_F \Theta e^\mu, \mu = 6.8582513 - 4.527217t_i^{-0.5} + 0.009541996(2R_{ei})^{0.5},$$

$$K_{F0} = \sum_{i=1}^n K_{F0i} \quad (16)$$

where the factor of fabrication difficulty is taken as $\Theta = 3$ and the steel density is $\rho = 7.85 \times 10^{-6} \text{ kg/mm}^3$.

$$K_{F1i} = k_F \left[\Theta \sqrt{3\rho V_{li}} + 1.3 \times 0.152 \times 10^{-3} t_i^{1.9358} \times 3L_{ei} \right], K_{F1} = \sum_{i=1}^n K_{F1i} \quad (17)$$

$$V_{li} = 2\pi R_{ei} L_{ei} t_i \quad (18)$$

$$K_{F2i} = k_F \left[\Theta \sqrt{3\rho V_{ri}} + 1.3 \times 0.3394 \times 10^{-3} a_{wi}^2 \times 4\pi (R_i - h_i) \right] \quad (19)$$

where

$$V_{ri} = 4\pi t_{ri} h_i (R_i - h_i / 2) + 2\pi t_{ri} h_i (R_i - h_i)$$

and the fillet weld size $a_{wi} = 0.7\delta h_i$.

$$K_{F3i} = k_F \left[\Theta \sqrt{2\rho V_{3i}} + 1.3 \times 0.3394 \times 10^{-3} a_{wi}^2 \times 4\pi R_i \right], V_{3i} = V_{li} + V_{ri} \quad (20)$$

$$K_{F4} = K_{F4A} + K_{F4W}, K_{F4A} = k_F \Theta \sqrt{n\rho V}, K_{F4W} = \sum_{i=2}^n K_{F4Wi} \quad (21)$$

$$K_{F4Wi} = 1.3 k_F \times 0.152 \times 10^{-3} t_i^{1.9358} \times 2\pi R_i \quad (22)$$

$$K_p = K_{p1} + \sum_{i=1}^{n+1} K_{pi}, K_{p1} = k_p 4\pi \frac{R_{\max} + R_1}{2} L_0 \quad (23)$$

$$K_{pi} = k_p 4\pi h_i (R_i - h_i / 2) \quad (24)$$

$$k_p = 2 \times 14.4 \times 10^{-6} \$ / \text{mm}^2.$$

4 NUMERICAL DATA

Total shell length $L = 15000$, side radii $R_{\min} = R_l = 1850$ and $R_{\max} = R_{n+1} = 2850$ mm, yield stress of steel $f_y = 355$ MPa, with a safety factor for yield stress $f_{y1} = f_y / 1.1$, external pressure intensity

$p = 0.5$ MPa, safety factor for loading $\gamma_b = 1.5$, Poisson ratio $\nu = 0.3$, elastic modulus $E = 2.1 \times 10^5$ MPa (Fig. 1).

5 RESULTS OF THE OPTIMIZATION

The detailed calculations are carried out for shell thicknesses $t_i = 14$ -20 mm.. The corresponding material and total costs are summarized in Table 1.

Table 1. The material and total costs in \$ for investigated shell thicknesses. The optima are marked by bold letters

t_i mm	K_M	K
14	28490	82280
16	29620	76150
18	32390	75040
20	38170	80120

It can be seen that the optimum shell thickness for material cost is 14 and for total cost 18 mm. This difference is caused by the fact that the fabrication (assembly, welding and painting) cost represents a large amount of total cost. The cost data show that, in the fabrication cost a significant part have the forming of plate elements into shell shape, welding and painting.

In order to characterize the dimensions of the optimum structure, the main data are given in Table 2.

Table 2. Main dimensions (in mm) of the optimum shell structure ($t = 18$ mm)

R_i	L_i	h_i	t_{ri}
1850	2630	121	4
2025	2376	134	4
2183	2189	146	5
2329	2044	158	5
2465	1927	170	5
2593	1831	182	6
2715	1750	194	6
2832	(1680)	207	7

6 SUMMARY

The optimum design problem is solved for a slightly conical shell loaded in external pressure with non-equidistant ring-stiffeners of welded square box section. The optimum shell thickness is found, which minimizes the cost function and fulfils the design constraints.

The length of each shell segment is calculated from the shell buckling constraint. This constraint is similar to that for circular cylindrical shells, but equivalent thickness and segment length is used according to the DNV design rules [13].

The dimensions of ring-stiffeners for each shell segment are determined on the basis of the ring buckling constraint. This constraint is expressed by the required moment of inertia of the ring-stiffener cross-section.

The cost function includes the cost of material, forming of plate elements into shell shape, assembly, welding and painting. The fabrication cost function is formulated according to the fabrication sequence. The forming, welding and painting costs play an important role in the total cost.

The cost difference between the maximum and minimum cost in the investigated range of is $(82280 - 75040) / 82280 \times 100 = 9\%$, thus, a significant cost savings can be achieved by optimization.

The ring-stiffening is very effective, since the unstiffened shell needs a thickness of 42 mm, which is unrealistic for fabrication.

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