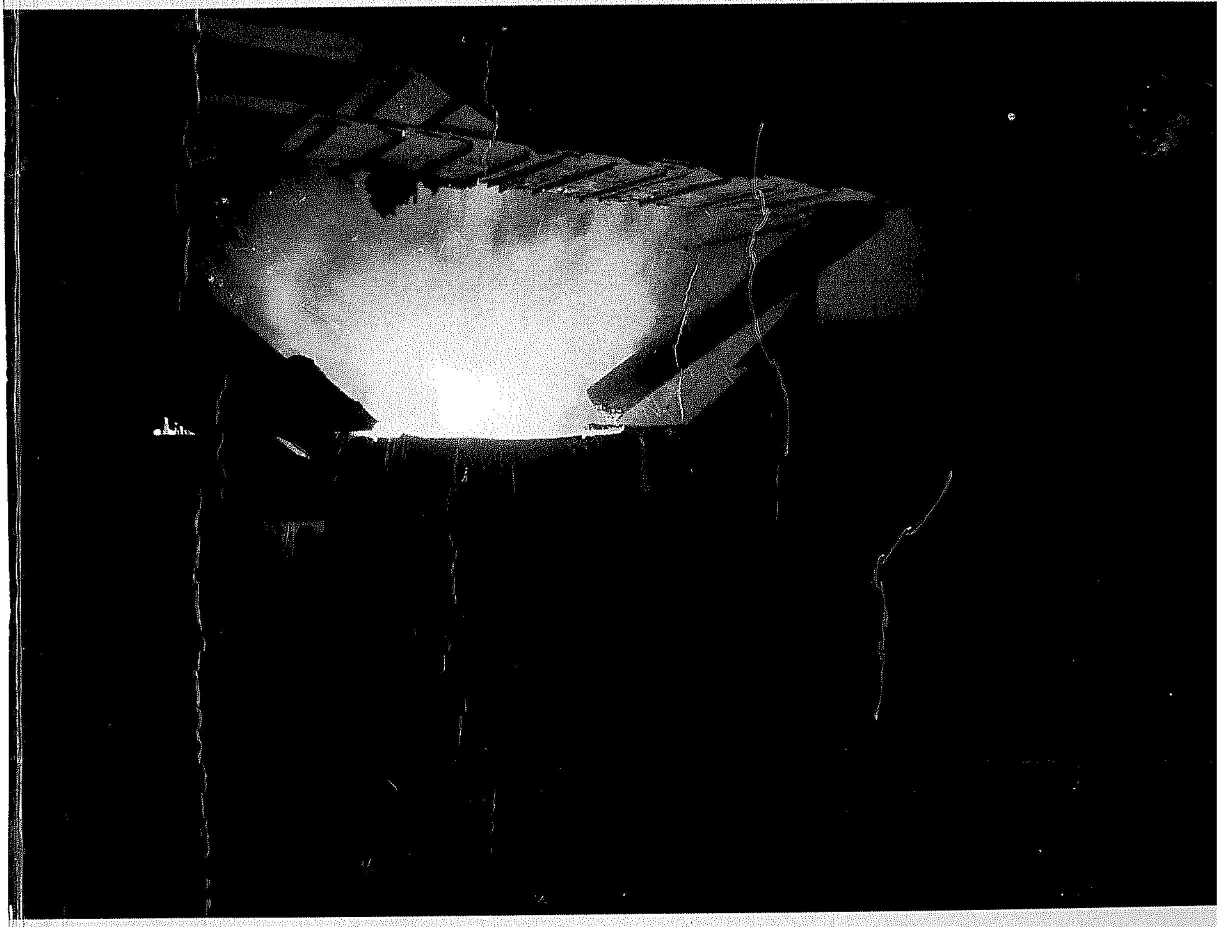
THE NINTH INTERNATIONAL CONFERENCE

METAL STRUCTURES

26-30 June 1995, Kraków (Poland)



Pig iron tapping in the Sendzimir Steelworks in Cracow

PRELIMINARY REPORT

VOLUME 2

Edited by Janusz Murzewski

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OPTIMUM DESIGN OF ALUMINIUM COMPRESSED STRUTS AND BOX BEAMS

1. INTRODUCTION

In recent years many results have been reached in the field of optimum design of steel structures, e.g. (Farkas 1984). Our aim is to extend this research to the optimization of simple aluminium structures.

In optimum design structural versions are sought which minimiTe the objective function and fulfil the design constraints. The cross-sectional area as objective function is selected here. Design constraints express the safety of the structure against failure, overall and local buckling, large deformation, vibration, etc.

The buckling constraints play in the present calculations an important role. For steel structures several up-to-date standards are available. For aluminium structures there is no Eurocode standard. German DIN 4113 is based on very old DIN 4114 which is no longer valid for steel structures. The Mazzolani's method (Mazzolani 1985) is too complicated for optimization. Thus, we use here relatively simple buckling formulae given by (Sharp 1993) based on an ASCE article (Clark and Rolf 1966).

Circular and square hollow sections (CHS and SHS) are widely used in structures because of their high resistance against buckling and torsion. It has been shown (Farkas 1992a,b) that for steel CHS and SHS struts simple closed formulae can be derived. Unfortunately, it is not the case for aluminium struts, but some diagrams can be given to help designers.

We treat here only numerical examples, since it is impossible to give design diagrams valid for all types of aluminium alloy.

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2. MINIMUM CROSS-SECTIONAL AREA DESIGN OF CONCENTRICALLY COMPRESSED SHS STRUTS (Fig. 1)

According to (Sharp 1993) the overall and local buckling strength of compressed aluminium struts can be calculated in the form of $\pi^2 E/\lambda^2$ in elastic and B - $D\lambda$ in plastic range, where λ is the slenderness, B and D are constants depending on the type of aluminium alloy (Fig.2).

The slenderness of SHS struts is given by

$$\lambda_S = \frac{KL}{r} = \frac{KL\sqrt{6}}{b} = \frac{100K\sqrt{6}}{\vartheta_S}; \quad \vartheta_S = \frac{100b}{L} \tag{1}$$

where K is the end restraint factor, for pinned ends K = 1 is used here, $r=b/\sqrt{6}$ is the radius of gyration. Introducing the notation $\delta_S = b/t$ the cross-sectional area can be expressed as

$$A = 4bt = \frac{4b^2}{\delta_S} = \frac{4\vartheta_S^2 L^2}{10^4 \delta_S} \tag{2}$$

For our numerical examples we select the aluminium alloy 6061-T6 (AlMg1SiCu, solution heat-treated and then artificially aged). For not welded struts the following data can be used: the yield stress is f_y = 240 MPa, the elastic modulus is E = 7*10^4 MPa.

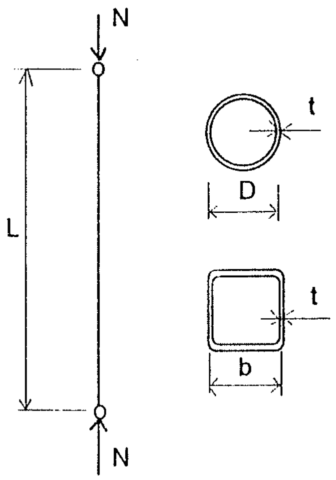

Fig. 1 Concentrically compressed SHS and CHS struts

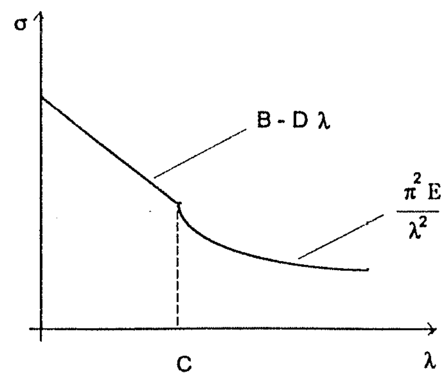

Fig.2 Overall buckling curve for SHS and CHS struts

The constants of the overall buckling curves are as follows:

$$B_S = f_y(1 + \sqrt{\frac{f_y}{15510}}) = 270 \text{ MPa} \tag{3}$$

$$D_S = \frac{B_S}{10}\sqrt{\frac{B_S}{E}} = 1.68 \text{ MPa}; \qquad C_S = \frac{0.41 B_S}{D_S} = 65.9 \tag{4}$$

For the constants of the local buckling curve the following formulae are given:

$$B_P = f_y(1 + \frac{\sqrt[3]{f_y}}{21.7}) = 308.7 \text{ MPa} \tag{5}$$

$$D_P = \frac{B_P}{10}\sqrt{\frac{B_P}{E}} = 2.05 \text{ MPa}; \qquad C_P = \frac{0.35 B_P}{D_P} = 52.7 \tag{6}$$

An equivalent plate slenderness can be calculated by equating the elastic plate buckling strength to $\pi^2 E / \lambda_P^2$

$$\frac{k\pi^2 E}{12(1-\nu^2)}(\frac{t}{b})^2 = \frac{\pi^2 E}{\lambda_P^2} \quad \text{from which one obtains} \quad \lambda_P = \sqrt{\frac{12(1-\nu^2)}{k}}\frac{b}{t}.$$

The Poisson's ratio for aluminium is ν = 1/3, for a uniformly compressed plate strip with simply supported edges k = 4, thus $\lambda_P = 1.63\delta_S$ (7)

The local buckling strength in elastic range is $\sigma_P = \frac{2.04\sqrt{B_P E}}{\lambda_P} = \frac{9483}{\lambda_P}$ (8)

In the optimization procedure the dimensionless unknowns δ_S and ϑ_S are sought which minimize $10^4 A/L^2$ and fulfil the design constraints. The factored compressive force N and the strut length L are known.

$$\frac{10^4 A}{L^2} = \frac{4\vartheta_S^2}{\delta_S} \rightarrow \min \tag{9}$$

The overall buckling constraint is

$$\frac{N}{A} = \frac{10^4 N}{L^2}\frac{\delta_S}{4\vartheta_S^2} \le \sigma_S \tag{10}$$

where, using Eqs. (3) and (4) $\sigma_S = 270 - 1.68\lambda_S$ for $\lambda_S \le 65.9$

$\sigma_S = \pi^2 * 7 * 10^4 / \lambda_S^2$ for $\lambda_S > 65.9$

The local buckling constraint is

$$\frac{N}{A} = \frac{10^4 N}{L^2}\frac{\delta_S}{4\vartheta_S^2} \le \sigma_P \tag{11}$$

where, using Eqs. (5), (6) and (8) $\sigma_P = 308.7 - 2.05\lambda_P$ for $\lambda_P \le 52.7$

$$\sigma_P = 9483/\lambda_P \qquad \text{for} \qquad \lambda_P > 52.7$$

It is known that the local buckling can decrease the overall buckling strength, thus a constraint is given for the interaction of overall and local buckling as follows:

$$\frac{N}{A} = \frac{10^4 N}{L^2}\frac{\delta_S}{4\vartheta_S^2} \le \sigma_S^{1/3}\sigma_P^{2/3} \qquad (12)$$

where σ_S and σ_P are given in Eqs (10) and (11). It has been verified for steel CHS struts (Farkas 1992a,b) that the overall and local buckling constraints are active, i.e. these inequalities can be treated as equalities.

The computation has been performed using the Rosenbrock's Hillclimb mathematical programming method (Rosenbrock 1960, Jármai 1989). The results are given in the form of a design diagram suitable for designers (Fig.3).

The optimal values are summarized in Table 1 as well. From these values the optimal b and t for a given $10^4N/L^2$ (in MPa) can be obtained using Eqs (1) and (9). It can be seen that, in the double-log coordinate system the curves consist of linear portions, so a linear interpolation can be used.

Table 1. Optimal values for compressed SHS struts

$10^4N/L^2$	0.1	1	10	100	1000
λ_S	242	166	113	77	42
δ_S	478	220	101	47	28
ϑ_S	1.01	1.48	2.2	3.2	5.9
$10^4A/L^2$	0.085	0.040	0.185	0.863	5.00

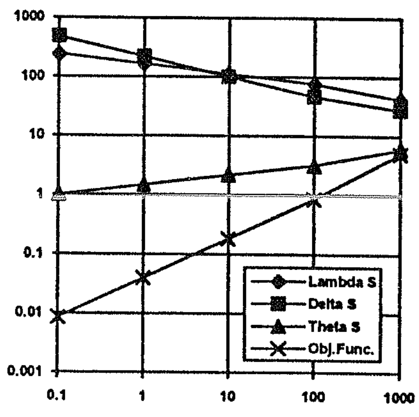

Fig. 3 Optimal values for compressed SHS struts as a function of $10^4N/L^2$ [MN/m^2],the objective function is $10^4A/L^2$

3. MINIMUM CROSS-SECTIONAL AREA DESIGN OF CONCENTRICALLY COMPRESSED CHS STRUTS (Fig.1)

The overall buckling curve is the same as for SHS struts. The constants for the local buckling strength formulae are as follows:

$$B_C = f_y(1+\frac{\sqrt[5]{f_y}}{12.8}) = 296 \text{ MPa} \tag{13}$$

$$D_C = \frac{B_C}{15}\sqrt[3]{\frac{B_C}{E}} = 10.64 \text{ MPa} \tag{14}$$

The cross-sectional area is given by $A = \pi Dt = \pi D^2 / \delta_C,\ \delta_C = D/t$ (15)

The objective function to be minimized is $\frac{10^4 A}{L^2} = \frac{\pi \vartheta_C^2}{\delta_C} \rightarrow \min,\ \vartheta_C = \frac{100D}{L}$ (16)

The overall buckling constraint is defined by $\frac{N}{A} = \frac{10^4 N}{L^2}\frac{\delta_C}{\pi \vartheta_C^2} \le \sigma_S$ (17)

where $\sigma_S = 270 - 1.68\lambda_C;\ \lambda_C = \frac{100K\sqrt{8}}{\vartheta_C}$ for $\lambda_C \le 65.9$

$\sigma_S = \pi^2 * 7 * 10^4 / \lambda_C^2$ for $\lambda_C > 65.9$

The local buckling constraint is expressed by $\frac{N}{A} = \frac{10^4 N}{L^2}\frac{\delta_C}{\pi \vartheta_C^2} \le \sigma_C$ (18)

Table 2. Optimal values for compressed CHS struts

$10^4 N/L^2$	1	10	100	1000
λ_C	112	81	48	26
δ_C	770	410	203	85
ϑ_C	2.3	3.5	5.8	10.9
$10^4 A/L^2$	0.022	0.094	0.530	4.41

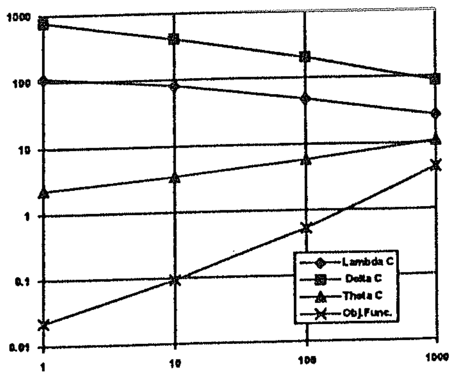

Fig. 4. Optimal values for compressed CHS struts (see Fig. 3.)

where $\sigma_C = 296 - 10.64\sqrt{\frac{\delta_C}{2}}$ for $\sqrt{\frac{\delta_C}{2}} \le 12$

$\sigma_C = \pi^2 * 7 * 10^4 / \lambda^2$ for $\sqrt{\frac{\delta_C}{2}} > 12$

$$\lambda = 4\sqrt{\frac{\delta_C}{2}}\left(1 + \frac{1}{35}\sqrt{\frac{\delta_C}{2}}\right)$$

Note that the interaction of overall and local buckling in the case of CHS struts is not treated in Sharp's book. According to (Ellinas et al. 1984) the interaction is not active for steel CHS struts, thus, it can be neglected.

The results of optimization are given in Fig. 4 and Table 2 similarly to results for SHS struts. Note that the δ_{opt} values are for small $10^4 N/L^2$ values so large (Tables 1, 2), that the small thicknesses cannot be fabricated. Then the thickness should be limited: $t = \vartheta L / (100\delta) \ge t_{min}$.

4. MINIMUM CROSS-SECTIONAL AREA DESIGN OF WELDED BOX BEAMS LOADED IN BENDING AND SHEAR (Fig.5)

The cross-sectional area to be minimized is $A = ht_w + 2bt_f \rightarrow \min$ (19)

$$I_{eff} = I_x - 4*30\, t_f\left(\frac{h+t_f}{2}\right)^2 - 4*30\frac{t_w}{2}\left(\frac{h}{2} - 15\right)^2 \qquad (20)$$

where
$$I_x = \frac{h^3 t_w}{12} + 2bt_f\left(\frac{h+t_f}{2}\right)^2 \qquad (21)$$

The section modulus is
$$W_x = \frac{2I_{eff}}{h+t_f} \qquad (22)$$

The maximal bending moment for a simply supported beam subject to a uniformly distributed normal load is $M_{max} = pL^2/8$, where p is the factored intensity of the load.

The stress constraint is $\sigma_{max} = M_{max} / W_x \le f_y$ (23)

In a numerical example $f_y = 240$ MPa is taken for 6061-T6 alloy.

The local buckling constraint for the compression flange can be calculated using Eqs. (5),(6),(7),(8) and (11): $\sigma_{max} \le \sigma_{Pf}$ (24)

where $\sigma_{Pf} = 308.7 - 2.05\lambda_{Pf}$; $\lambda_{Pf} = 1.63b/t_f$ for $\lambda_{Pf} \le 52.7$

$$\sigma_{Pf} = 9483 / \lambda_{Pf}; \qquad \text{for } \lambda_{Pf} > 52.7$$

The local buckling constraint for the webs loaded in bending with constants

$$B_w = 1.3 f_y (1 + \frac{\sqrt[3]{f_y}}{13.3}) = 457.8 \text{ MPa} \tag{25}$$

$$D_w = \frac{B_w}{20}\sqrt{\frac{6B_w}{E}} = 4.53 \text{ MPa}; \quad C_w = \frac{0.5B_w}{D_w} = 50.5 \tag{26}$$

is expressed as $\quad \sigma_{max} \le \sigma_w \qquad (27)$

where $\quad \sigma_w = 457.8 - 4.53\lambda_{Pw}; \quad \lambda_{Pw} = 0.67 * 2h / t_w \quad$ for $\lambda_{Pw} \le 50.5$

and $\quad \sigma_{Pw} = \dfrac{2.04\sqrt{B_w E}}{\lambda_{Pw}} = \dfrac{11548}{\lambda_{Pw}}; \qquad$ for $\lambda_{Pw} > 50.5$

The shear stress constraint is $\tau_{max} = \dfrac{Q_{max}}{(h-60)t_w} \le \tau_y = 140$ MPa; $Q_{max} = pL/2$ (28)

where τ_y is the shear yield stress, Q is the shear force.

The constants for the shear buckling constraint of webs are calculated as follows:

$$B_Q = \tau_y (1 + \frac{\sqrt[3]{\tau_y}}{17.7}) = 181 \text{ MPa} \tag{29}$$

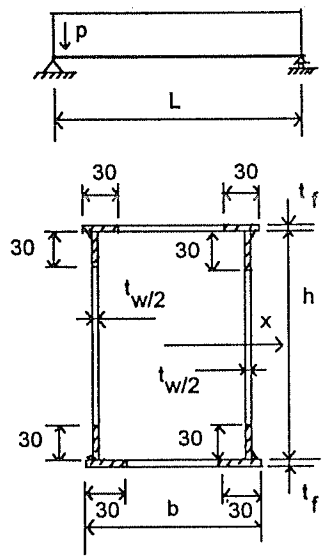

Fig. 5. The effective cross section of a welded box beam

The fusion welding causes a partial annealing of the material in the vicinity of the weld. This effect of welding should be considered by substracting plate strips from the working cross-section area. The width of the reduced-strength zone in both sides of the weld may be taken as 25 mm according to Sharp or 30 mm prescribed in DIN 4113 Part 2 (1993). We use here 30 mm (Fig.5). The effective moment of inertia is calculated using the method proposed in DIN 4113 as follows:

$$D_Q = \frac{B_Q}{10}\sqrt{\frac{B_Q}{E}} = 0.9204 \text{ MPa}; \; C_Q = \frac{0.41 B_Q}{D_Q} = 80.6 \tag{30}$$

The elastic shear buckling strength is

$$\tau_{cr} = \frac{5.34\pi^2 E}{12(1-\nu^2)}\left(\frac{t_w}{2h}\right)^2 = \frac{68.77}{\lambda_w^2}; \; \lambda_w = 1.41\frac{2h}{t_w} \tag{31}$$

According to Sharp the actual shear buckling strength can be calculated as

$$\tau_{el} = \tau_{cr} + 0.866(\tau_y - \tau_{cr}) = 0.134\tau_{cr} + 121.2 \tag{32}$$

The value of 121.2 is modified to have a continuous diagram, thus

$$\tau_{el} = \frac{9.27*10^4}{\lambda_w^2} + 98.7 \tag{33}$$

Using Eqs (29), (30) and (33) the shear buckling constraint for webs is defined by

$$\tau_{max} \le \tau_Q \tag{34}$$

where
$$\tau_Q = 181 - 0.9204\lambda_w; \quad \text{for } \lambda_w \le 80.6$$
$$\tau_Q = \tau_{el} = \frac{9.27*10^4}{\lambda_w^2} + 98.7 \quad \text{for } \lambda_w > 80.6$$

In the design of structures the deflection is often restricted. The deflection constraint can be formulated for a beam shown in Fig.5 as follows:

$$w_{max} = \frac{5 p_o L^4}{384 E I_x} \le w^* \tag{35}$$

where $p_o = p/\gamma$ with a safety factor of $\gamma = 1.5$. The allowable deflection is taken as $w^* = L/300$.

Table 3. Optimal values of welded box beams

$10^6 p/8L$ (N/mm^2)	50	100	150	200	250	300	350
L (mm)	25000	12500	8333	6250	5000	4167	3571
h (mm)	1216	634	422	331	294	234	205
$t_w/2$ (mm)	6.6	3.9	2.7	2.4	2.8	2.2	1.8
b (mm)	565	391	326	253	166	169	172
t_f (mm)	9.1	7.1	6.2	5.4	4.7	4.7	4.4
$10^4 A/L^2$	0.4199	0.6634	0.8980	1.096	1.286	1.484	1.717

In a numerical example, with the value of p = 10 kN/m = 10 N/mm and $E = 7*10^4$ MPa, the optimal dimensions of h, $t_w/2$, b and t_f are computed and given in Table 3 with the corresponding minimal values of $10^4 A/L^2$ as a function of $10^6 M_{max}/L^3 = 10^6 p/(8L)$ or L. Diagrams for these optimal values show that the optimal values for another values of L can be calculated using a linear interpolation between the given values.

5. CONCLUSIONS

In the optimum design of compressed SHS and CHS struts the optimal b and t as well as D and t values are computed which minimize the cross-sectional area and fulfil the design constraints relating to the overall and local buckling. The buckling constraints are defined using the formulae given in Sharp's book. The computation has been performed by using the software developed on the basis of the Rosenbrock's Hillclimb mathematical programming method. The results are shown in diagrams and tables suitable for designers.

In the optimum design of welded box beams the effect of welding is considered by substracting the annealed cross-sectional parts in the calculation of the moment of inertia. The optimal dimensions are calculated considering the constraints on normal and shear stress, local buckling of the compression flange, local buckling of webs due to bending and shear as well as on deflection. The results of a numerical example are given in a table.

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OPTYMALIZACJA ALUMINIOWYCH PRĘTÓW ŚCISKANYCH I BELEK SKRZYNKOWYCH

Streszczenie

Wyznaczono minimalny przekrój poprzeczny ściskanych aluminiowych prętów rurowych (kwadratowych i okrągłych) oraz spawanych aluminiowych belek skrzynkowych przy zginaniu i ścinaniu. Obliczone optymalne wymiary minimalizują powierzchnię przekroju poprzecznego i spełniają ograniczenia konstrukcyjne. W przypadku prętów ściskanych warunki wyboczenia ogólnego i lokalnego (Eqs 10, 11, 12 oraz 17, 18) określono na podstawie formuł Sharpa (Sharp, 1993).

W spawanych belkach skrzynkowych rozpatrzono następujące przypadki: naprężenia normalne i ścinające, wyboczenie miejscowe pasa ściskanego, wyboczenie miejscowe środników przy zginaniu i ścinaniu, jak również ze względu na ograniczenia wygięć (Eqs 23, 24, 27, 33, 34 oraz 35). Efekt spawania uwzględniono poprzez pominięcie przyległych stref do spoin podłużnych (po 30 mm zkażdej stronę) przy obliczaniu momentu bezwładności.

Wprzykładach obliczeniowych przyjęto stop aluminium 6061-T6 o granicy plastyczności 240 MPa. Zadanie optymalizacyjne rozwiązano przy pomocy metody programowania matematycznego Rosenbrocka (Rosenbrock, 1960).

Wyniki obliczeń przedstawiono na Fig. 3 i 4 oraz w Tabl. 1, 2 i 3, które pozwalają wybrać optymalne wymiary, poprzez interpolację liniową podanych wielkości.