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Minimum cost design of a column-supported oil pipeline strengthened by a tubular truss

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ABSTRACT: In the case of a statically indetermined structure a systematic optimum design procedure should be performed, since the forces in structural parts depend on their dimensions and the cost function to be minimized is rather complicated. This is the case of a column supported oil pipeline, when it is strengthened by a tubular truss. The internal forces in the tubular truss are derived by a deflection equation. The strengthening tubular truss is optimized to fulfil the design constraints and minimize the cost. Design constraints relate to the stress in the original pipe and in the truss members as well as on strength and geometry the truss joints. The cost function includes the material cost as well as the cost of cutting of strut ends, assembly, welding and painting.

KEYWORDS: Oil pipeline, tubular truss, structural optimization, minimum cost design, fabrication cost, welded structures

1 INTRODUCTION

In the case, when the distance of supporting columns is in a special place larger than the other distances, a strengthening of the pipe is necessary. This strengthening can be realized by prestressed cables or by an upper or lower truss welded to the main transporting pipe. It should be noted that it is assumed that the larger distance is not too large and a special supporting bridge is not needed.

The aim of our study is to design a lower strengthening tubular truss (Fig.1). This simple truss consists of two diagonals and a vertical column. The diagonals are loaded by tension and have the same cross-sectional area. The vertical column is loaded by compression and bending and is designed against overall buckling.

This complex structural system is statically indeterminate and the unknown force in the column is calculated by force method using a deflection equation. The symmetric truss geometry has an unknown, the height H. This unknown as well as the dimensions of truss members are calculated from the condition that the material and fabrication costs of the strengthening tubular truss should be minimum.

Constraint on local buckling of circular hollow section truss members as well as the constraint on strength and geometry of the node are also considered. The advanced cost function, used in our previous study (Farkas & Jármai 2001, 2003), includes the material and fabrication costs. The fabrication costs relate to the cutting of strut ends, assembly, welding and painting. For the constrained function minimization the efficient mathematical computer method is used based on the Rosenbrock's hillclimb algorithm complemented by a discretization procedure to obtain available circular hollow sections.

2 DERIVATION OF THE COLUMN FORCE

The structure of the strengthened pipe is statically indetermined. The unknown column force X_1 can be derived from a deflection equation. The deflection at midspan of simply supported pipe without strengthening from the distributed load p is (Fig.1)

$$w_p = \frac{5pL^4}{384EI_x} \tag{1}$$

where L is the larger support distance, E is the elastic modulus, I_x is the moment of inertia of the original pipe. The deflection of the pipe without strengthening at the midspan from the column force X_1 is

$$w_1 = \frac{X_1L^3}{48EI_x} \tag{2}$$

The deflection caused by the axial deformation of the tubular truss is

$$w_2 = \sum_i \frac{S_i s_i L_i}{EA_i} \tag{3}$$

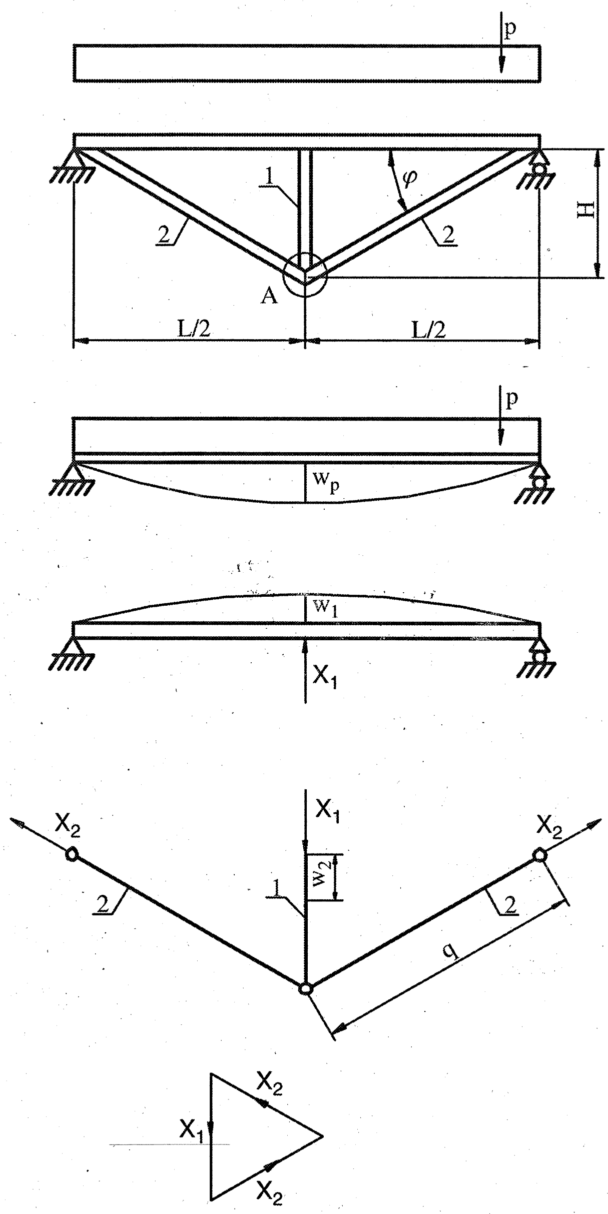

Figure 1. The simply supported pipe strengthened by a tubular truss. The deflections w_p, w_1 and w_2 are used to derive the unknown internal force X_1.

where S_i is the normal force in the i-th strut caused by X_1, s_i is the normal force due to $X_1 = 1$, L_i is the strut length and A_i is the cross-sectional area of the strut.

Introducing the length of diagonals as

$$q = \sqrt{\frac{L^2}{4} + H^2} \tag{4}$$

the normal force in the diagonals is

$$X_2 = \frac{X_1 q}{2H} \tag{5}$$

and

$$w_2 = \frac{X_1 H}{EA_1} + \frac{X_1 q^3}{2EA_2 H^2} \tag{6}$$

From the deflection equation of

$$w_p - w_I = w_2 \tag{7}$$

one obtains

$$X_1 = \frac{5pL^4}{384 I_x Q}, \tag{8}$$

$$Q = \frac{L^3}{48 I_x} + \frac{H}{A_1} + \frac{q^3}{2A_2 H^2} \tag{9}$$

3 DESIGN OF THE ORIGINAL PIPE

Take the original pipe span length as $L_0 = 12$ m. Consider a simply supported pipe. Loads: self mass, oil in the pipe and internal pressure. We select for the original pipe according to DIN 2458 (1981) a profile of 219.1 × 6.3, self mass 33.06 kg/m. The cross-sectional area for the oil filling is $A_F = \pi 206.5^2/4 = 33491\,\text{mm}^2$. Oil density is 0.8 × 10^{-6} kg/mm^3. According to the standard EN 1594 (2001) the partial safety factor for self mass is 1.5, for oil 1.39 and the yield strength should be multiplied by 0.72. The intensity of the factored uniformly distributed normal load is $p = 1.5 \times 33.06 + 1.39 \times 0.8 \times 33.491 = 86.83\,\text{kg/m} = 0.8683\,\text{N/mm}$.

The maximum bending moment is:

$$M_{\max} = pL_0^2/8 = 15.63x10^6\,\text{Nmm}.$$

The section modulus is

$$W_x = \pi 212.8^2 x6.3/4 = 224065\,\text{mm}^3$$

and the maximum normal stress due to bending is

$$\sigma_{\max} = M_{\max}/W_x = 69.8\,\text{MPa}.$$

Stress due to an internal pressure of 64 bar = 6.4 MPa is

$$\sigma_P = p_P D/(2t) = 6.4x212.8/(2x6.3) = 108.1\,\text{MPa},$$

the factored stress is 1.39 × 108.1 = 150.2 MPa, the reduced stress is

$$\sigma_{red} = \sqrt{\sigma_{\max}^2 + \sigma_P^2} = \sqrt{69.8^2 + 150.2^2}$$
$$= 165.6 < 0.72x235 = 169.2\ \text{MPa, OK.}$$

Take the larger span length of $L = 17$ m. For this span length a larger pipe is needed, we select the profile of 355.6 × 12.5 with a self mass of 106 kg/m.

The section modulus is $W_x = 1.117 \times 10^6 \text{mm}^3$. The cross-sectional area for oil filling is $A_F = 85841 \text{ mm}^2$. The load intensity is $p = 1.5 \times 106 + 1.39 \times 0.8 \times 85.841 = 2.5445$ N/mm. The bending moment is $M = 91.92 \times 10^6$ Nmm. The bending stress is $\sigma_b = 82.29$ MPa. The factored stress from internal pressure is $1.39\sigma_p = 122.1$ MPa and the reduced stress is 147.2 < 169.2 MPa, OK. For the sake of comparison, we calculate the cost of this larger pipe according to Table 1: $1.3642 \times 106 \times 17 = 2458$ \$.

The cost of the original pipe for a larger span length is (Table 1) $K_0 = 1.2922 \times 17 \times 33.06 = 726$ \$.

Now we calculate a strengthening for the original pipe in the case of span length $L = 17$ m. In the design of the strengthening we perform a minimum cost design procedure to achieve a maximum cost savings against the larger pipe without strengthening.

The unknown variables are as follows: outer diameters d_1, d_2, thicknesses t_1, t_2, geometric dimension H (Fig. 1).

4 OPTIMIZATION OF THE STRENGTHENING TUBULAR TRUSS

4.1 *Design constraints*

4.1.1 *Stress constraint for the original pipe*

$$\sqrt{\left(\frac{\frac{pL^2}{8} - \frac{X_1 L}{4}}{W_x}\right)^2 + 150.2^2} \leq 169.2 \text{MPa}; \quad (10)$$

$W_x = 2.2406\text{x}10^5 \text{ mm}^3$;

$\frac{pL^2}{8} = 31.37x10^6$ Nmm

4.1.2 *Size limitation for tension members for fabrication reasons*

$$d_2 \geq 1.08 d_1 \quad (11)$$

4.1.3 *Stress constraint for tension member*

$$\frac{X_1 q / H}{A_2} \leq 213 \text{ MPa}; \quad A_2 = \pi(d_2 - t_2)t_2; \quad (12)$$

4.1.4 *Stress constraint for the column subject to compression and bending (cross section of class 3 according to EC3)*

In order to avoid the column buckling in lateral direction, a lateral force F_u acting on the truss node should be considered. This force can be calculated using the formulae of the BS 5400 Part 3: 1982: for design of U-frames of bridges with unbraced compression chords.

$$F_u = \frac{P_C}{P_E - P_C} \cdot \frac{l_E}{667\delta}; \quad (13)$$

$$\delta = \frac{H^3}{3EI_{x1}}; \quad P_E = \frac{\pi^2 EI_{x2}}{l_E^2}; \quad (14)$$

$$I_{x2} = \frac{\pi(d_2 - t_2)^3 t_2}{8}; \quad P_C = X_1 q / H; \quad (15)$$

$$l_E = 2.5\left(EI_{x2}\delta L / 2\right)^{0.25} \quad (16)$$

This force causes bending of the column, thus it should be checked for compression and bending according to Eurocode 3 (2002).

$$\frac{X_1}{\chi A_1 f_y / \gamma_{M1}} + \frac{kF_u H}{W_{x1} f_y / \gamma_{M1}} \leq 1; \quad (17)$$

$$A_1 = \pi(d_1 - t_1)t_1; \quad W_{x1} = \frac{\pi(d_1 - t_1)^2 t_1}{4}; \quad (18)$$

$$I_{x1} = \frac{\pi(d_1 - t_1)^3 t_1}{8} \quad (19)$$

$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}}; \quad (20)$$

$$\phi = 0.5\left[1 + 0.34(\bar{\lambda} - 0.2) + \bar{\lambda}^2\right]; \quad (21)$$

$$\bar{\lambda} = \frac{2H}{r_1 \lambda_E}; \quad r_1 = \sqrt{\frac{I_{x1}}{A_1}}; \quad (22)$$

$$\lambda_E = \pi\sqrt{\frac{E}{f_y}} = 93.91; \quad (23)$$

$$k = \frac{0.79 - 0.1188\frac{X_1}{A_1 \chi f_y / \gamma_{M1}}}{1 - \frac{X_1}{A_1 f_y / \gamma_{M1}}} \quad (24)$$

4.1.5 *Constraint on local buckling of tubular members*

$$\frac{d_i - t_i}{t_i} \leq 50; i = 1,2 \quad (25)$$

4.1.6 *Constraint on chord plastification at the joint (Fig.2)*

According to Wardenier et al. (1991)

$$N_1^* = \frac{f_y t_2^2}{\sin\theta_1}\left[2.8 + 14.2\left(\frac{d_1}{d_2}\right)^2\right]\left(\frac{d_2}{2t_2}\right)^{0.2} \geq X_1 \quad (26)$$

$$\sin\theta_1 = \frac{L}{2q}$$

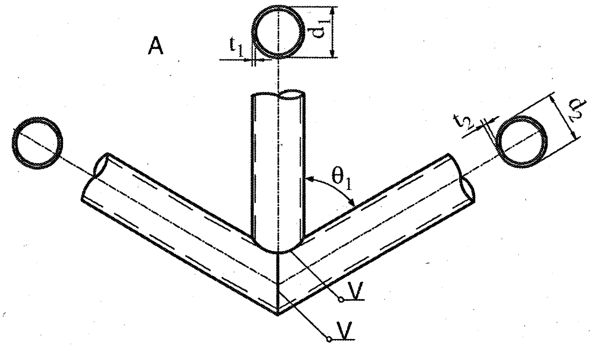

Figure 2. Details of the truss node A in Figure 1.

Table 1. Material cost factors for circular hollow sections (CHS).

d (mm)	k_M (\$/kg)
88.9, 101.6, 114.3	1.0553
139.7, 168.3, 177.8, 193.7	1.1294
219.1, 244.5, 273.0, 323.9	1.2922
355.6, 406.4	1.3642
457.0, 508.0	1.4081

4.1.7 *Geometric constraint (range of validity) (Fig.1)*

According to Wardenier et al. (1991)

$$\varphi \geq 30^0 \text{ i.e. } H \geq 4900 \text{ mm} \tag{27}$$

4.2 *The cost function*

Material cost is calculated as:

$$K_M = k_{M1}\rho A_1 H + 2k_{M2}A_2\rho q; \quad \rho = 7850 \text{ kg/m}^3 \tag{28}$$

The material cost factors are given in Table 1 in function of the outer diameter according to the Price list of the British Steel (1995).

The cost of cutting and grinding of strut ends (Jármai & Farkas 1999, Farkas & Jármai 2003) (Fig.2)

$$K_C = k_F\Theta_C\pi\left[\left(d_1 + \frac{d_1}{\cos\varphi}\right)\left(4.54 + 0.4229t_1^2\right)\right] + k_F\Theta_C\pi\left[\left(\frac{2d_2}{\sin\varphi} + \frac{2d_2}{\cos\varphi}\right)\left(4.54 + 0.4229t_2^2\right)\right]$$

$$\sin\varphi = \frac{H}{q}; \cos\varphi = \frac{L}{2q} \tag{29}$$

fabrication cost factor $k_F = 0.667$ \$/min, $\Theta_C = 3$.

Cost of assembly of the original pipe with the strengthening:

$$K_A = k_F\Theta_A\sqrt{\kappa\rho V}; \kappa = 4; \Theta_A = 3 \tag{30}$$

$$V = \pi DtL + H\pi(d_1 - t_1)t_1 + +2q\pi(d_2 - t_2)t_2 \tag{31}$$

$$\pi DtL = 60.6767x10^6 \text{ mm}^3$$

Cost of welding and additional works: SMAW butt and fillet welds

$$K_W = 1.3k_F\left[3.13x10^{-3}t_2\frac{\pi(d_2 - t_2)}{\cos\varphi}\right] + 1.3k_F\left[2x0.7889x10^{-3}t_2^2\frac{\pi(d_2 - t_2)}{\sin\varphi}\right] + 1.3k_F\left[4x0.7889x10^{-3}t_1^2\pi(d_1 - t_1)\right] \tag{32}$$

The cost of painting is given by

$$K_P = k_P\left[H\pi d_1 + 2q\pi d_2\right] \tag{33}$$

$k_P = 14.4x10^{-6}$ \$/mm^2.

The objective function to be minimized is

$$K = K_M + K_C + K_A + K_W + K_P \tag{34}$$

4.3 *The optimization procedure and results*

In the optimization procedure the optimum values of H, d_1, t_1, d_2 and t_2 are sought, which fulfil the design constraints (Eqs 10, 11, 12, 17, 25 and 27) and minimize the cost function (Eq. 34). For this purpose the Rosenbrock hillclimb algorithm is used. This is a direct search method (Farkas & Jármai 1997) which does not use derivatives and results in continuous optimum values. It is complemented by a discretization process to find the corresponding available CHS profiles.

The available CHS dimensions in mm according to DIN 2458 (1981) are as follows:

d: 88.9; 101.6; 114.3; 139.7; 168.3; 177.8; 193,7 (outer diameter)

t: 1.8; 2.0; 2.3; 2.6; 2.9; 3.2; 3.6; 4.0; 4.5; 5.0; 5.6; 6.3(thickness).

The results are given in Table 2.

It can be seen that the value of $H = 4900$ mm gives the minimum cost, thus the optimum solution is determined by the geometric constraint prescribing the minimum inclination angle of diagonals (Eq. 27).

Comparing the minimum cost of the original pipe and the strengthening tubular truss $K_{min} = 726 + 458 = 1184$ \$ with the cost of the larger pipe $K = 2458$ \$, it can be concluded that the strengthened pipe is much cheaper than the larger one.

Table 2. Discretized optimization results for different values of H.

H (mm)	d_1, t_1 (mm)	d_2, t_2 (mm)	K ($)
4900	101.6 × 3.2	114.3 × 2.0	458
5000	108.0 × 2.0	127.0 × 2.6	505
5500	108.0 × 2.0	127.0 × 2.6	517
6000	114.3 × 2.6	127.0 × 2.6	542

5 CONCLUSIONS

The optimum dimensions of a welded tubular truss are determined, which strengthen a column-supported oil pipeline for a larger (17 m) span length. The pipe is designed for an original span length (12 m). The truss column is designed for compression and bending. Bending is caused by a transverse force considered to avoid the lateral buckling. Since the strengthened structure is statically indetermined, the unknown force in the truss column is calculated from a deflection equation.

Design constraints relate to the member stresses as well as to strength and geometry of truss nodes. The cost function includes the costs of material, cutting and grinding of strut ends, assembly, welding and painting.

In the optimization process the unknowns are the truss height H and the diameter and thickness of truss members. The optimization shows that the optimal H is determined by the geometric constraint prescribing the minimum inclination angle of diagonals. The active constraints are defined by Equations 10, 11, 17, 25.

The cost comparison shows that the cost of the strengthened pipe is much lower than that of the larger pipe without strengthening.

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REFERENCES

British Steel 1995. *Price list 20. Steel tubes, pipes and hollow sections. Part 1b. Structural hollow sections*. British Steel, Corby, UK.

BS 5400: Part 3: 1982. *Steel, concrete and composite bridges. Code of practice for design of steel bridges*. British Standards Institution, London, UK.

DIN 2458: 1981. *Geschweisste Stahlrohre. Masse*. Berlin, Deutsches Institut für Normung.

Eurocode 3: 2002. *Design of steel structures*. CEN Brussels.

EN 1594: 2001. *Gas supply systems. Pipelines for maximum operating pressure over 16 bar. Functional requirements*. CEN Brussels.

Farkas, J. & Jármai, K. 1997. *Analysis and optimum design of metal structures*. Balkema, Rotterdam-Brookfield.

Farkas, J. & Jármai, K. 2001. Height optimization of a triangular CHS truss using an improved cost function. Tubular Structures IX. Proceedings of the *9th International Symposium on Tubular Structures*, Düsseldorf, 2001. Swets & Zeitlinger, Lisse, 429–435.

Farkas, J. & Jármai, K. 2003. *Economic design of metal structures*. Rotterdam, Millpress.

Jármai, K. & Farkas, J. 1999. Cost calculation and optimization of welded steel structures. *Journal of Constructional Steel Research* 50: 115–135.

Wardenier, J., Kurobane, Y., Packer, J.A., Dutta, D. & Yeomans, N. 1991. *Design guide for circular hollow section (CHS) joints under predominantly static loading*. Köln, Verlag TÜV Rheinland.