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Optimization of a wind turbine tower structure

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ABSTRACT: A wind turbine tower is constructed as a slightly conical ring-stiffened welded steel shell. The 45 m high shell is approximated by three cylindrical shell parts of 15 m length, having an average constant diameter and thickness. The wind load is calculated according to Eurocode 1 Part 2–4. Design constraints on shell buckling and local buckling of flat ring-stiffeners are considered. To calculate fabrication costs, the process cost for forming of shells into near cylindrical shapes as well as the cost of assembly and welding are taken into account. The cost function to be minimized includes the material and fabrication costs. To prevent ovalization ring-stiffeners are necessary. The optimum shell thicknesses, dimensions and number of stiffeners are calculated using the Rosenbrock's direct search method for function minimization complemented by an additional discretization.

KEYWORDS: Ring-stiffened shells, shell buckling, wind load, structural optimization, welded structures, fabrication costs.

1 INTRODUCTION

Design optimization implies a search for better solutions, which minimize the objective function and fulfil the design requirements. The main requirements of up-to-date engineering structures are suitable load-carrying capacity (safety), producibility and economy. A structural optimization system has previously been developed (Farkas & Jármai 1997) in which the safety and producibility is guaranteed by design and fabrication constraints, while economy is achieved by minimization of a cost function. For the constrained function minimization, effective mathematical methods should be used.

This system has successfully been applied to several structural models (welded beams, layered sandwich beams, tubular trusses, frames, stiffened plates and shells) and industrial problems (silos, bunkers, bridge decks, a punching press table, an aluminium truck floor, a belt-conveyor bridge) (Farkas & Jármai 2003). This design system is now applied for the optimization of a wind turbine tower structure.

Wind turbines are becoming an important alternative to standard energy supply since wind energy costs are competitive to coal and nuclear on average kWh costs today (Krohn 2002). The most suitable load-carrying structure for a wind turbine is a welded steel shell tower. It can be constructed as a tower composed of cylindrical and conical shell parts (Bazeos et al. 2002).

2 METHODOLOGY

The aim of the present study is to formulate a design methodology to optimize a 45 m high, slightly conical ring-stiffened shell tower with linearly varying diameter and stepwise varying thickness. The shell is approximated by three cylindrical parts of 15 m length, having constant average diameter and thickness. A cost minimization procedure has already been developed (Farkas et al. 2003) to optimize the design of a ring-stiffened cylindrical shell loaded in bending. Design constraints on shell buckling and on local buckling of flat ring-stiffeners are formulated according to DNV design rules (Det Norske Veritas 1995) and API (2000).

The wind load acting on the shell tower is calculated according to Eurocode 1 Part 2–4 (1999) (EC1). The wind force and bending moment acting on the top of the 44 m high tower for a 1 MW wind turbine in Greece is given by Lavassas et al. (2003). To avoid shell ovalization a minimum number of 5 and a maximum number of 15 stiffeners is prescribed. In the constraint of shell buckling an imperfection factor is used, which expresses the effect of radial shell deformation due to shrinkage of circumferential welds as has been proposed by Farkas (2002).

The cost function includes the material and fabrication costs. The fabrication cost is formulated in terms of the production sequence and includes the cost of forming shell courses into near cylindrical shapes, the cost of cutting flat ring-stiffeners, as well as the cost of assembly and welding.

The unknowns in the optimization procedure are the average shell thicknesses and dimensions and number of ring-stiffeners. The minimization of the continuous cost function is complemented by a discrete neighbourhood search of available dimensions in the vicinity of the continuous minimum.

3 WIND LOAD ACTING ON THE TOWER

According to Eurocode 1 Part 2–4 (1999) (EC1) the wind force can be calculated as

$$F_W = q_{ref} c_e(z) c_d c_f A_{ref} \tag{1}$$

where $$q_{ref} = \frac{\rho_0}{2} v_{ref}^2 \tag{2}$$

with the air density $\rho_0 = 1.25\,\mathrm{kg/m^3} = 1.25\,\mathrm{Nm^{-4}s^2}$ and the wind velocity for Greece $v_{ref} = 36\,\mathrm{m/s}$, thus $q_{ref} = 810\,\mathrm{Nm^{-2}}$.

Also $$c_e(z) = c_r^2 c_t^2 (1 + 2gI_v), g = 3.5; \tag{3}$$

$$I_v = \frac{k_T}{c_r c_t}; k_T = 0.17; \tag{4}$$

$$c_r = k_T \ln\left(\frac{z}{z_0}\right); z_0 = 0.01 \ . \tag{5}$$

The values of k_T and z_0 are obtained from Table 8.1 (EC1) for sea or level area and $c_t = 1$ for level area. The calculated values of c_e for three characteristic heights are given in Table 1.

The dynamic factor for height $h = 45$ m and average diameter of $D = 3$ m is $c_d = 1.1$ (Figure 9.5 (EC1)).

The force factor is given by

$$c_f = c_{f0} \psi_\lambda \ . \tag{6}$$

c_{f0} is given in Figure 10.8.2 (EC1) as a function of the Reynolds number R_e and the ratio of k/D, where

$$R_e = \frac{D v_m(z)}{v}; D = 3\,\mathrm{m} \ ; \tag{7}$$

Table 1. Calculated values of c_e for different heights z.

z (m)	c_r	I_v	c_e
45	1.43	0.119	3.747
30	1.36	0.125	3.468
15	1.24	0.137	3.012

$$v_m = c_r v_{ref} = 1.43x36 = 51.48\,;$$
$$v = 15x10^{-6} m^2/s; R_e = 10.3x10^6\,.$$

From Table 10.8.1 (EC1) $k = 0.05$, for a steel surface, thus $k/D = 0.05/3 = 1.67 \times 10^{-2}$, so that $c_{f0} = 1.1$ from Figure 10.8.2 (EC1).

From Figure 10.14.1 (EC1) for a slenderness $l/D = 45/3 = 15$ and for the effective area $\varphi = 1$ one obtains $\psi_\lambda = 0.75$, thus $c_f = 1.1 \times 0.75 = 0.825$.

The uniformly distributed wind loads for the three shell parts are given in Figure 1 according to the formula

$$p_w = q_{ref} c_e c_d c_f D\,. \tag{8}$$

For the three shell parts the wind loads are as follows: $p_{w1} = 6.334$, $p_{w2} = 6.883$ and $p_{w3} = 6.864$ kN/m.

In Figure 1 the factored bending moments due to wind load are given, the safety factor being 1.5. The optimization is performed for the three shell parts using average diameters and with a bending moment acting in the middle of every shell part.

4 THE DESIGN CONSTRAINTS

4.1 *Local buckling of the flat ring-stiffeners*

The limitation of the height to thickness ratio of a flat ring-stiffener is (API 2000)

$$\frac{h_r}{t_r} \le 0.375\sqrt{\frac{E}{f_y}} \quad . \tag{9}$$

Considering this constraint as active, for $E = 2.1 \times 10^5$ MPa and yield stress $f_y = 355$ MPa, one obtains

$$h_r = 9t_r. \tag{10}$$

4.2 *Constraint on local shell buckling (as unstiffened)*

According to Det Norske Veritas (1995)

$$\sigma_{\max} = \frac{M}{\pi R^2 t} \le \sigma_{cr} = \frac{f_y}{\sqrt{1+\lambda^4}} \quad \text{where} \tag{11}$$

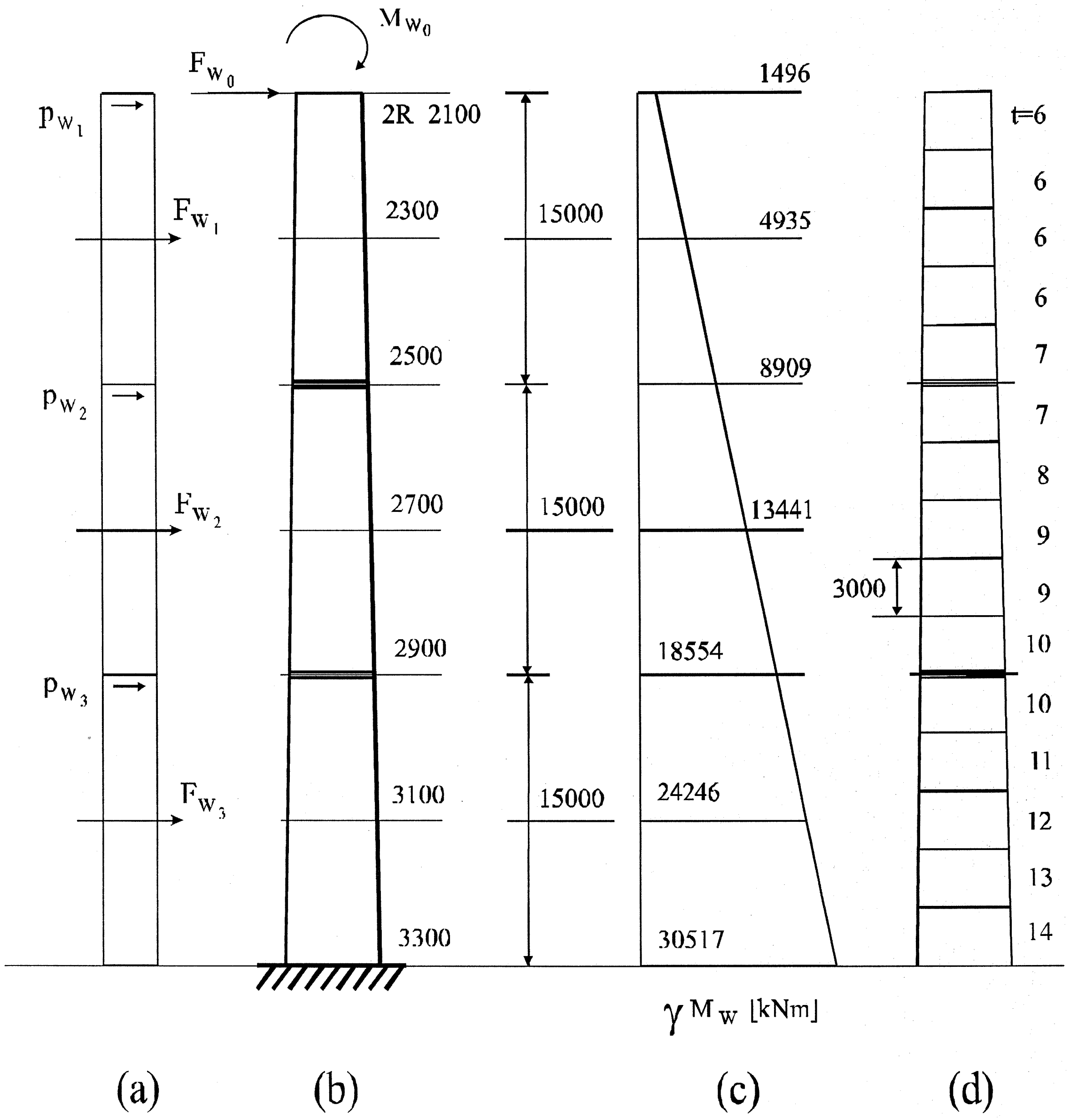

Figure 1. (a) Wind load, (b) diameters of the shell tower, (c) factored bending moment due to wind load (safety factor = 1.5), (d) thicknesses of the shell courses in mm.

$$\sigma_E = (1.5 - 50\beta) C \frac{\pi^2 E}{10.92} \left(\frac{t}{L_r} \right)^2 \quad \text{and} \tag{12}$$

$$L_r = \frac{L}{n+1}; \quad \lambda^2 = \frac{f_y}{\sigma_E} \quad . \tag{13}$$

where the length of one shell segment $L = 15$ m, the number of ring stiffeners in one shell segment is n. The factor of $(1.5–50\beta)$ in Equation 12 expresses the effect of initial radial shell deformation caused by the shrinkage of circumferential welds and can be calculated as follows (Farkas 2002):

The maximum radial deformation of the shell caused by the shrinkage of a circumferential weld is

$$u_{\max} = 0.64 A_T \sqrt{R/t} \tag{14}$$

where $A_T t$ is the area of specific strains near the weld, R and t are the radius and thickness of the shell respectively. According to previous results (Farkas & Jármai 1998)

$$A_T t = \frac{0.3355 Q_T \alpha_0}{c_0 \rho} \tag{15}$$

where c_0 is the specific heat, ρ is the material density and α_0 is the coefficient of thermal expansion.

For steels the relation thus reduces to

$$A_T t = 0.844x10^{-3} Q_T \ (A_T t \text{ in mm}^2, Q_T \text{ in J/mm}) \quad (16)$$

$$Q_T = \eta_0 \frac{UI}{v_W} = C_A A_W \quad (17)$$

where η_0 is the coefficient of thermal efficiency, U is the arc voltage, I is the arc current, v_W is the speed of welding, A_W is the cross-sectional area of the weld.

For butt welds

$$Q_T = 60.7 A_W \ (A_W \text{ in mm}^2). \quad (18)$$

When $t \le 10$ mm, $A_W = 10t$. (19a)

When $t > 10$ mm, $A_W \cong 3.05t^{1.45}$. (19b)

Introducing a reduction factor of β for which

$$0.01 \le \beta = \frac{u_{max}}{4\sqrt{Rt}} \le 0.02 \quad , \quad (20)$$

$\beta = 0.01$, for $\beta \le 0.01$ and $\beta = 0.02$ for $\beta \ge 0.02$, the shell buckling strength should be multiplied by the imperfection factor (1.5–50β).

Furthermore

$$C = \psi \sqrt{1 + \left(\frac{\rho_1 \xi}{\psi}\right)^2}, Z = 0.9539 \frac{L_r^2}{Rt} \quad (21)$$

$$\psi = 1, \rho_1 = 0.5\left(1 + \frac{R}{300t}\right)^{-0.5} \quad \text{and} \quad (22)$$

$$\xi = 0.702Z.$$

It can be seen that σ_E does not depend on L_r , since in Equation 12 L_r^2 is in denominator and in C (Eq.21) is in the numerator. The fact that the buckling strength does not depend on the shell length was first derived by Timoshenko & Gere (1961). Note that this dependence of σ_E on L_r is very small according to API design rules (American Petroleum Institute 2000). It has however been determined that in the case of external pressure the distance between ring-stiffeners plays an important role (Farkas et al. 2002, Jármai et al. 2003).

4.3 *Constraint on panel ring buckling*

Requirements for a ring stiffener are as follows:

$$A_r = h_r t_r \ge \left(\frac{2}{Z^2} + 0.06\right) L_r t \quad (23)$$

$$I_r = \frac{h_r^3 t_r}{12} \cdot \frac{1+4\omega}{1+\omega} \ge \frac{\sigma_{max} t R_0^4}{500 E L_r} \quad (24)$$

$$R_0 = R - y_G; y_G = \frac{h_r}{2(1+\omega)}; \quad (25a)$$

$$\omega = \frac{L_e t}{h_r t_r} \quad (25b)$$

$$L_e = \min\left(L_r, L_{e0} = 1.5\sqrt{Rt}\right). \quad (26)$$

5 THE COST FUNCTION

The possible fabrication sequence is as follows: (1) Fabricate five shell elements of length 3 m without rings. For one shell element 2 axial butt welds are needed (GMAW-C). The cost for forming a shell element to a slightly conical, near cylindrical shape is also included (K_{F0}). According to the time data obtained from a Hungarian production company (Jászberényi Aprítógépgyár, Crushing Machine Factory, Jászberény) for plate elements of 3 m width, the times for forming and for reducing the initial imperfections due to forming can be approximated by the following function of the plate thickness and the diameter

$$\ln T = 6.85825 - 4.5272t^{-0.5} + +0.0095419D^{0.5} \quad (27)$$

$K_{F0} = k_F \Theta_F T$; $\Theta_F = 3$ is the difficulty factor expressing the complexity of fabrication.

The welding cost of a shell element is

$$K_{F1} = k_F\left(\Theta_W \sqrt{\kappa \rho V_1}\right) +$$

$$+ k_F\left[1.3x0.2245x10^{-3} t^2 (2x3000)\right] \quad (28)$$

where Θ_W is a difficulty factor expressing the complexity of the assembly and κ is the number of elements to be assembled

$\kappa = 2$; $V_1 = 2R\pi t x3000$; $\Theta_W = 2$.

(2) Weld the complete unstiffened shell from 5 elements with 4 circumferential butt welds

$$K_{F2} = k_F\left(\Theta_W \sqrt{5\rho V_1}\right) +$$

$$+ k_F 1.3x0.2245x10^{-3} t^2 x19x2R\pi \quad (29)$$

(3) Cut n flat plate rings with acetylene gas (Farkas & Jármai 2003)

$$K_{F3} = k_F \Theta_c C_c t_r^{0.25} L_c \quad (30)$$

where Θ_c, C_c and L_c is the difficulty factor for cutting, the cutting parameter and the cutting length respectively, $\Theta_c = 3$, $C_c = 1.1388 \times 10^{-3}$ and

$$L_c \approx 2R\pi n + 2(R - h_r)\pi n.$$

(4) Weld n rings into the shell with double-sided GMAW-C fillet welds. ($2n$ fillet welds):

$$K_{F4} = k_F\left(\Theta_W\sqrt{(n+1)\rho V_2}\right) +$$

$$+k_F 1.3x0.3394x10^{-3} a_W^2 x4R\pi n \quad (31)$$

$a_W = 0.5t_r$, but $a_{Wmin} = 3$ mm.

$$V_2 = 5V_1 + 2\left(R - \frac{h_r}{2}\right)\pi h_r t_r n \quad (32)$$

The total material cost is

$$K_M = k_M \rho V_2 \quad (33)$$

The total cost is

$$K = K_M + 5(K_{F0} + K_{F1}) + K_{F2} + K_{F3} + K_{F4} \quad (34)$$

The material cost factor is $k_M = 1$ \$/kg; the labour cost factor is $k_F = 1$ \$/min.

6 OPTIMIZATION AND RESULTS

The optimization is performed using the Rosenbrock's Hillclimb algorithm (Farkas & Jármai 1997). The optimal values of the shell thicknesses (t) as well as the thicknesses (t_r) and the number of ring-stiffeners (n), which comply with the design constraints and minimize the cost function are summarized in Table 2.

It can be seen that the cost increases with an increase in the number of stiffeners. Thus, the minimum number of stiffeners ($n = 5$) should be used. The stiffener thickness decreases when the number of stiffeners increases. It should be noted that the cost difference in the given domain ($n = 5$–15) is not very large (1.5–3.6%). The cost difference is most apparent in the top part.

The calculation of the first eigenfrequency of the optimized tower structure is made using the Eurocode 1 Part 2–4 (1999b) and results in 0.53 Hz, which is larger than the frequency of the rotor excitation 0.37 Hz given by Lavassas et al. (2003).

Fatigue calculations for fillet welded joints of the shell on the bottom and on the intermediate diaphragms are performed using the wind spectrum data (Lavassas et al. 2003). According to the Eurocode 3 Part 1–9 (2002) the fatigue stress range for toe failure and for 2×10^6 cycles in the case of T-joints is 63–71 MPa depending on the diaphragm thickness. The calculated safety factor is in all cases of the spectrum larger than the prescribed value of 1.35.

Table 2. The optimal thicknesses and the corresponding costs for different numbers of ring-stiffeners.

Shell part	n	t (mm)	t_r (mm)	K (\$)
Bottom	5	12	15	103200
	10	12	11	104405
	15	12	9	105258
Middle	5	9	15	78315
	10	9	10	78851
	15	9	8	79512
Top	5	6	11	52038
	10	6	8	52937
	15	6	7	53934

7 CONCLUSIONS

Since the shell tower is slightly conical it can be divided into three parts and each part optimized as a cylindrical shell. The shell is predominantly loaded by bending due to wind forces and designed against buckling. Since the shell thickness does not depend on the number of ring-stiffeners (n), the optimal shell thickness and stiffener thickness are calculated for different numbers of stiffeners ranging from 5 to 15 to prevent shell ovalization. The calculations show that the minimum cost solution corresponds to the minimum number of ring stiffeners.

In the shell buckling constraint the significant effect of the radial shell deformations due to the shrinkage of circumferential welds is taken into account. The cost function includes the material and fabrication costs. In the latter the forming of shell courses into near cylindrical shapes has been considered . It has also been verified that the structure meets the fatigue requirements specified in Eurocode 3 Part 1–9 (2002) and that the natural frequency of the tower is well beyond the rotor excitation frequency.

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