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Optimum design of a welded stringer-stiffened steel cylindrical shell subject to axial compression and bending

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Abstract

A column fixed at the bottom and free on the top is made of stringer-stiffened cylindrical shell and loaded by axial compression as well as by a horizontal force acting on the top. Halved rolled I-section stringers are welded outside of the shell by longitudinal fillet welds. The aim is to study the economy of stiffened shells. Therefore both the stiffened and the unstiffened version are optimized and their costs are compared to each other. The stiffening is economic when the shell thickness can be decreased in such a measure that the cost savings caused by this decreasing is higher than the additional cost of stiffening material and welding.

The cost function to be minimized includes the costs of material, forming of shell elements into the cylindrical shape, assembly, welding and painting. The constraints relate to the shell buckling, stringer panel buckling and limitation of the horizontal displacement of the column top. The cost comparison shows that the cost of stiffened version is lower than that of the unstiffened one only in those cases, when the constraint on horizontal displacement is active.

Key words: *stiffened cylindrical shells, shell buckling, fabrication cost, economy of welded structures, structural optimization, minimum cost design*

1. Introduction

An important requirement from modern welded structures is the economy, since the cost of welding is high. Therefore the basis of comparison of different structural versions is the cost. Since only the optimum versions can be realistically compared to each other, the minimum cost design should be performed for each structural version.

The economy of stiffened cylindrical shells depends on several parameters as follows: load (axial compression, bending, external pressure or combined load), type of stiffening (ring-, stringer-stiffeners or orthogonal stiffening), stiffener profile (flat, rolled I, halved rolled I, L-, hollow section or trapezoidal).

It has been shown that ring-stiffening is economic in the case of external pressure [1], [2]. In the case of bending the ring-stiffening should be used to assure the sufficient cylindrical shape. In this case the cost of stiffened shell is higher than that of unstiffened one, since the shell thickness cannot be decreased by ring-stiffeners [3].

Stiffening is economic only in those cases, when the thickness can be decreased in such a measure that the cost savings caused by this decreasing is higher that the additional cost of stiffening material and welding.

As a part of our systematic research relating to stiffened cylindrical shells, in the present study a column is investigated subject to an axial compression and a horizontal force acting on the top of the column (Fig.1). The column is fixed at the bottom and free on the top. It is shown that a shell stiffened outside with stringers can be economic, when a constraint on horizontal displacement of the column top is active. In order to decrease the welding cost of stiffeners, their cross-sectional area is increased, i.e. halved rolled I-section (UB) stiffeners are used instead of flat ones. The halved I-sections are advantageous, since the web can be easier welded to the shell than the flange. It should be mentioned that stringer-stiffening can also be economic in those cases, when the corresponding unstiffened version needs too thick shell (more than 40 mm).

The cross-section of the stiffened shell is constant along the whole height. Constraints on local shell buckling, on stringer panel buckling and on horizontal displacement are taken into account. The buckling constraints are formulated according to the DNV design rules [4]. The cost function to be minimized includes the cost of material, forming of shell elements into cylindrical shape, assembly, welding and painting.

In order to demonstrate the economy of the stiffened shell, the unstiffened version is also optimized. The results show that the cost savings depends on the active displacement constraint.

2. Problem formulation

The investigated structure is a supporting column loaded by an axial and horizontal force (Fig.1). The horizontal displacement of the top is limited by the reasons of serviceability of the supported structure. Both the stiffened and unstiffened shell version is optimized and their cost is compared to each other. In the stiffened shell outside longitudinal stiffeners of halved rolled I-section (UB) are used. The cost function is formulated according the fabrication sequence.

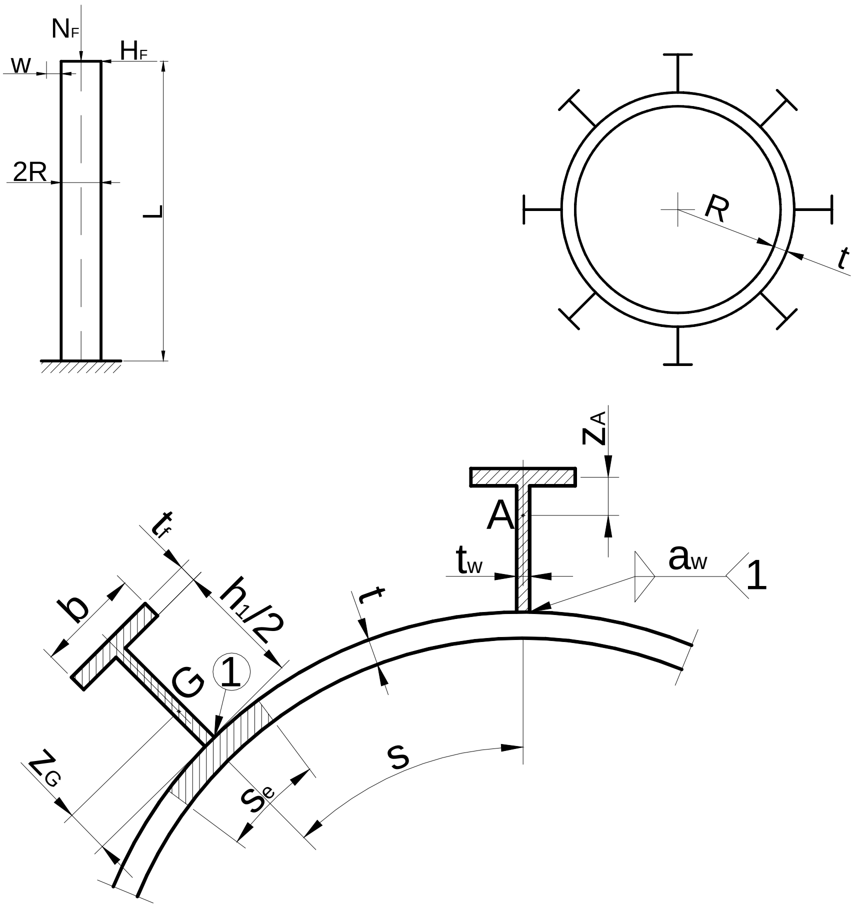

Figure 1. A column constructed as a stiffened cylindrical shell loaded by a compression force N_F and a horizontal force H_F. Cross-section and a detail of the cross-section with outside stiffeners of halved rolled I-section. The horizontal displacement of the top (w) is limited

Given data are as follows: column height L, shell radius R, factored axial compression force N_F, factored horizontal force H_F, yield stress of steel f_y, cost factors for material, fabrication and painting k_m, k_f, k_p. The unknowns are the shell thickness t as well as the height h and number n_s of halved rolled I-section stiffeners.

The characteristics of the selected UB profiles are given in Table 1.

Table 1. Characteristics of the selected rolled UB profiles (Profil Arbed [5])

UB Profile	h mm	b mm	t_w mm	t_f mm	A_S mm^2	I_y x10^{-4} mm^4
152x89x16	152.4	88.7	4.5	7.7	2032	834
168x102x19	177.8	101.2	4.8	7.9	2426	1356
203x133x25	203.2	133.2	5.7	7.8	3187	2340
254x102x25	257.2	101.9	6.0	8.4	3204	3415
305x102x28	308.7	101.8	6.0	8.8	3588	5366
356x127x39	353.4	126.0	6.6	10.7	4977	10172
406x140x46	403.2	142.2	6.8	11.2	5864	15685
457x152x60	454.6	152.9	8.1	13.3	7623	25500
533x210x92	533.1	209.3	10.1	15.6	11740	55230
610x229x113	607.6	228.2	11.1	17.3	14390	87320
686x254x140	683.5	253.7	12.4	19.0	17840	136300
762x267x173	762.2	266.7	14.3	21.6	22040	205300
838x292x194	840.7	292.4	14.7	21.7	24680	279200
914x305x224	910.4	304.1	15.9	23.9	28560	376400

In order to calculate with continuous values the geometric characteristics of an UB section (I_y, b, t_f) are approximated by curve-fitting functions as follows: h approximately equals to the first number of the profile name (Table Curve 2D [6]).

$$A_s = 1155.684135 + 0.034090823\, h^2 \tag{1}$$

$$t_f = \sqrt{33.20533808 + 0.0006701288\, h^2} \tag{2}$$

$$I_y = \exp\left[35.73636182 - \frac{156.07351689}{\ln(h)}\right] \mathrm{x} 10^4 \tag{3}$$

$$b = \sqrt{5851.784768098 + 0.01671843845 h^2 \ln(h)} \tag{4}$$

$$t_w = \sqrt{15.62577015376 + 4.358946969 \mathrm{x} 10^{-5} h^2 \ln(h)} \tag{5}$$

The surface to be painted is $A_L/2 = h + 2b$, $h_1 = h - 2t_f$. (6)

3 The stiffened shell

3.1 Constraints

3.1.1 *Shell buckling (unstiffened curved panel buckling)*

The sum of the axial and bending stresses should be smaller than the critical buckling stress

$$\sigma_a + \sigma_b = \frac{N_F}{2R\pi t_e} + \frac{H_F L}{R^2 \pi t_e} \le \sigma_{cr} = \frac{f_y}{\sqrt{1+\lambda^4}} \tag{7}$$

where the reduced slenderness

$$\lambda^2 = \frac{f_y}{\sigma_a + \sigma_b}\left(\frac{\sigma_a}{\sigma_{Ea}} + \frac{\sigma_b}{\sigma_{Eb}}\right); t_e = t + \frac{A_s}{2s}; s = \frac{2R\pi}{n_s} \tag{8}$$

t_e is the equivalent thickness. The elastic buckling stress for the axial compression is

$$\sigma_{Ea} = C_a \left(1.5 - 50\beta\right) \frac{\pi^2 E}{10.92} \left(\frac{t}{s}\right)^2 \tag{9}$$

$$C_a = 4\sqrt{1 + \left(\frac{\rho_a \xi}{4}\right)^2}; Z = \frac{s^2}{Rt} 0.9539 \tag{10}$$

$$\rho_a = 0.5\left(1 + \frac{R}{150t}\right)^{-0.5}; \xi = 0.702Z \tag{11}$$

The elastic buckling stress for bending is

$$\sigma_{Eb} = C_b\left(1.5 - 50\beta\right)\frac{\pi^2 E}{10.92}\left(\frac{t}{s}\right)^2 \tag{12}$$

$$C_b = 4\sqrt{1 + \left(\frac{\rho_b \xi}{4}\right)^2} \tag{13}$$

$$\rho_b = 0.5\left(1 + \frac{R}{300t}\right)^{-0.5} \tag{14}$$

Note that the residual welding distortion factor $1.5 - 50\beta = 1$ when t>9 mm. The detailed derivation of it is treated in [7].

3.1.2 *Stringer panel buckling*

$$\sigma_a + \sigma_b \le \sigma_{crp} = \frac{f_y}{\sqrt{1 + \lambda_p^4}} \tag{15}$$

$$\lambda_p^2 = \frac{f_y}{\sigma_{Ep}}; \sigma_{Ep} = C_p \frac{\pi^2 E}{10.92}\left(\frac{t}{L}\right)^2 \tag{16}$$

$$C_p = \psi_p \sqrt{1 + \left(\frac{0.5\xi_p}{\psi_p}\right)^2}; Z_p = 0.9539\frac{L^2}{Rt} \tag{17}$$

$$\xi_p = 0.702 Z_p; \gamma_s = 10.92\frac{I_{sef}}{st^3} \tag{18}$$

$$\psi_p = \frac{1 + \gamma_s}{1 + \frac{A_s}{2s_e t}}; \tag{19}$$

Since the effective shell part s_e (Fig.1) is given by DNV with a complicate iteration procedure, we use here the simpler method of ECCS [8]

$$s_E = 1.9t\sqrt{\frac{E}{f_y}} \tag{20}$$

if $s_E < s$ $s_e = s_E$

if $s_E > s$ $s_e = s$

I_{sef} is the moment of inertia of a cross section containing the stiffener and a shell part of width s_e (Fig. 1). For a stiffener of halved rolled I-section it is

$$I_{sef} = s_e t z_G^2 + \frac{t_w}{12}\left(\frac{h_1}{2}\right)^3 + \frac{h_1 t_w}{2}\left(\frac{h_1}{4} - z_G\right)^2 + b t_f\left(\frac{h_1}{2} - z_G\right)^2 \tag{21}$$

$$z_G = \frac{h_1^2 t_w / 8 + h_1 b t_f / 2}{h_1 t_w / 2 + b t_f + s_e t} \tag{22}$$

3.1.3 *Horizontal displacement*

$$w_h = \frac{ML^2}{3EI_{x0}} \le w_{allow} = \frac{L}{\phi} \tag{23}$$

ϕ is the varied between 400 and 1000 (Table 2).

The exact calculation of the moment of inertia for the horizontal displacement uses the following formulae (Fig.1):

The distance of the center of gravity for the halved UB section is

$$z_A = \frac{h_1 t_w / 2\left(h_1 / 4 + t_f / 2\right)}{h_1 t_w / 2 + b t_f} \tag{24}$$

The moment of inertia of the halved UB section is expressed by

$$I_x = b t_f z_A^2 + \frac{t_w}{12}\left(\frac{h_1}{2}\right)^3 + \frac{h_1 t_w}{2}\left(\frac{h_1}{4} - z_A\right)^2 \tag{25}$$

The moment of inertia of the whole stiffened shell cross-section is

$$I_{x0} = \pi R^3 t + I_x \sum_{i=1}^{n_s} \sin^2\left(\frac{2\pi i}{n_s}\right) +$$

$$+\left(\frac{h_1 t_w}{2}+bt_f\right)\left(R+\frac{h_1+t_f}{2}-z_A\right)^2\sum_{i=1}^{n_s}\sin^2\left(\frac{2\pi i}{n_s}\right) \tag{26}$$

$$M = H_F L/\gamma_M ; \gamma_M = 1.5; H_F = 0.1N_F \tag{27}$$

Numerical data: N_F = 34000 kN, f_y = 355 MPa, R = 1850 mm, L = 15 m.

3.2 The cost function

Fabrication sequence:

1. Fabrication of 5 shell elements of length 3 m without stiffeners. For one shell element 2 axial butt welds are needed (GMAW-C) (K_{F1}). The cost of forming of a shell element into the cylindrical shape is also included (K_{F0}).
2. Welding of the whole unstiffened shell from 5 elements with 4 circumferential butt welds (K_{F2}).
3. Welding of n_s stiffeners to the shell with double-sided GMAW-C fillet welds. Number of fillet welds is $2n_s$. (K_{F3}).

The material cost is $K_M = k_{M1} 5 \rho V_1 + k_{M2} \rho n_s A_s L/2$ (28)

$V_1 = 3000x2R\pi t; \rho = 7.85x10^{-6}$ kgmm^{-3}. k_F = 1.0 \$/min, k_{M1} = 1.0 \$/kg. (29)

The cost of forming of a shell element into the cylindrical shape according to [3] is

$$K_{F0} = k_F \Theta e^{\mu}; \mu = 6.8582513 - 4.527217 t^{-0.5} + 0.009541996 (2R)^{0.5} \tag{30}$$

$$K_{F1} = k_F \left[\Theta\sqrt{\kappa \rho V_1} + 1.3x0.1520x10^{-3} t^{1.9358} (2x3000) \right] \tag{31}$$

where Θ is a difficulty factor expressing the complexity of the assembly and κ is the number of elements to be assembled

$$\kappa = 2; V_1 = 2R\pi t x3000; \Theta = 2 \tag{32}$$

$$K_{F2} = k_F \left(\Theta\sqrt{25 \rho V_1} + 1.3x0.1520x10^{-3} t^{1.9358} x4x2R\pi \right) \tag{33}$$

$$K_{F3} = k_F \left(\Theta \sqrt{\left(n_s + 1\right) \rho V_2} + 1.3x0.3394x10^{-3} a_W^2 \, 2Ln_s \right) \quad (34)$$

The fillet weld size $a_w = 0.3t_w$, $a_{wmin} = 3$ mm.

$$V_2 = 5V_1 + n_s A_s L / 2 \quad (35)$$

The cost of painting is

$$K_P = k_P \left(4R\pi L + n_S A_L L / 2 \right); k_P = 14.4x10^{-6} \,\$/\mathrm{mm}^2. \quad (36)$$

The total cost is

$$K = K_M + 5K_{F1} + 5K_{F0} + K_{F2} + K_{F3} + K_P \quad (37)$$

4 The unstiffened shell

4.1 Constraints

4.1.1 *Shell buckling*

$$\sigma_a + \sigma_b = \frac{N_F}{2R\pi t} + \frac{H_F L}{R^2 \pi t} \le \sigma_{cr} = \frac{f_y}{\sqrt{1 + \lambda^4}} \quad (38)$$

$$\lambda^2 = \frac{f_y}{\sigma_a + \sigma_b} \left(\frac{\sigma_a}{\sigma_{Ea}} + \frac{\sigma_b}{\sigma_{Eb}} \right) \quad (39)$$

$$\sigma_{Ea} = C_a \left(1.5 - 50\beta \right) \frac{\pi^2 E}{10.92} \left(\frac{t}{L} \right)^2 \quad (40)$$

$$C_a = \sqrt{1 + \left(\rho_a \xi \right)^2}; Z = \frac{L^2}{Rt} 0.9539 \quad (41)$$

$$\rho_a = 0.5 \left(1 + \frac{R}{150t} \right)^{-0.5}; \xi = 0.702Z \quad (42)$$

$$\sigma_{Eb} = C_b\left(1.5 - 50\beta\right)\frac{\pi^2 E}{10.92}\left(\frac{t}{L}\right)^2 \tag{43}$$

$$C_b = \sqrt{1 + \left(\rho_b \xi\right)^2} \tag{44}$$

$$\rho_b = 0.5\left(1 + \frac{R}{300t}\right)^{-0.5} \tag{45}$$

4.1.2 *Horizontal displacement*

$$w_h = \frac{ML^2}{3E\pi R^3 t} \le w_{allow} = \frac{L}{\phi} \tag{46}$$

$$M = H_F L/\gamma_M ; \gamma_M = 1.5; H_F = 0.1N_F \tag{47}$$

4.2 The cost function

Fabrication sequence:

(1) Fabrication of 5 shell elements of length 3 m without stiffeners. For one shell element 2 axial butt welds are needed (GMAW-C) (K_{F1}). The cost of forming of a shell element into the cylindrical shape is also included (K_{F0}).

(2) Welding the 5 units together with 4 circumferential butt welds (K_{F2}).

The material cost is $K_M = k_{M1} 5 \rho_1 V_1$ (48)

$$V_1 = 3000x2R\pi t \tag{49}$$

$$K_{F0} = k_F \Theta e^{\mu}; \mu = 6.8582513 - 4.527217t^{-0.5} + 0.009541996\left(2R\right)^{0.5} \tag{50}$$

$$K_{F1} = k_F\left(\Theta\sqrt{\kappa\rho_1 V_1} + 1.3x0.152x10^{-3}t^{1.9358}x6000\right) \tag{51}$$

$$\Theta = 2; \kappa = 2; \rho_1 = 7.85x10^{-6}\ \text{kg/mm}^3$$

$$K_{F2} = k_F \left(\Theta\sqrt{5x5\rho_1 V_1} + 1.3x0.152x10^{-3} t^{1.9358} 8R\pi \right) \tag{52}$$

k_F = 1.0 \$/min, k_{M1} = 1.0 \$/kg.

The cost of painting is

$$K_P = k_P \left(4R\pi L \right); k_P = 14.4x10^{-6} \text{ \$/mm}^2. \tag{53}$$

The total cost is

$$K = K_M + 5K_{F1} + 5K_{F0} + K_{F2} + K_P \tag{54}$$

5 Optimization and results

The optimization is performed using the Particle Swarm mathematical algorithm [1]. The results are summarized in Table 2.

Table 2. Results of the optimization for stiffened and unstiffened shell. The positive cost difference means savings due to stiffening.

Stiffened							Un-stiffened				
ϕ	h mm	n_S	t mm	w_h<w_{allow} mm	$\sigma < \sigma_{cr}$ MPa	K \$	t mm	w_h <w_{allow} mm	$\sigma < \sigma_{cr}$ MPa	K \$	cost difference %
400	203	5	24	25<37.5	314<317	56310	22	27.7<37.5	349<351	49480	-14
500	610	5	22	24<30	307<311	56082	22	27.7<30	349<351	49480	-13
600	406	5	23	24.8<25	313<314	55760	25	24.4<25	307<352	55800	0
700	686	14	16	21<21.4	293<294	57751	29	21<21.4	264<353	64440	12
800	914	10	16	18.2<18.7	268<282	62294	33	18.5<18.7	232<354	73370	18
900	914	15	12	16<16.7	248<254	66545	37	16.5<16.7	207<354	82580	24
1000	914	18	11	14.4<15	227<253	70571	41	14.9<15	187<354	92100	30

It can be seen that the buckling (stress) constraint is active when the allowable horizontal displacement is L/400 – L/500 and for these cases the unstiffened shell is cheaper than the stiffened one. On the other hand, for L/700-L/1000 the displacement constraint is active and the stringer-stiffened shell is cheaper than the unstiffened one. The cost savings achieved by stiffening is 12-30%.

Comparison of the costs for unstiffened and stiffened shells

This comparison is shown in Table 3.

Table 3. Summary of costs (positive difference means cost savings) (Costs in $)

Cost	Unstiffened shell	Stiffened shell	Difference %
Material K_M	56117	45321	24
Forming $5K_{F0}$	8385	4342	93
Welding $5K_{F1}+K_{F2}$	22577		122
Welding $5K_{F1}+K_{F2}+K_{F3}$		10169	
Painting K_P	5021	10739	-114
Total	92100	70571	30

It can be seen that the cost savings caused by stringer stiffening are significant in forming and welding costs, but the painting for unstiffened shell is 114% cheaper than that for stiffened one. It can be concluded that the cost factors of fabrication and painting play an important role in the achievable cost savings.

Conclusions

Cylindrical shells stiffened outside by stringers are economic for axial compression and bending with an active deflection constraint, but without a deflection constraint they are uneconomic. In order to decrease the welding cost, the stiffeners should have cross-sectional area as large as possible and should welded to shell with welds as small as possible, thus the outside halved rolled I-section stringers are advantageous for this purpose. In the investigated numerical problem 12-30 % cost savings can be achieved using this stiffening in the case of displacement limit of L/700-L/1000. It should be noted that cost savings cannot be achieved by stringers welded inside of the shell.

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