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Parametric studies of uniaxially compressed and laterally loaded stiffened plates for minimum cost

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Keywords: stiffened plates, welded structures, stability, residual welding distortions, structural optimisation, minimum cost design

ABSTRACT: The elastic secondary deflection due to compression and lateral pressure is calculated using the Paik's solution (Paik et al. 1991) of the differential equation for orthotropic plates, and the self-weight is also taken into account. Besides this deflection some more deformations are caused by lateral pressure and the shrinkage of longitudinal welds. The stress constraint includes several effects as follows: the average compression stress and the stress caused by compression, lateral pressure and the shrinkage of longitudinal welds. The constraint on local buckling of the base plate strips is also included considering the effect of initial imperfections and residual welding stresses with formulae proposed by Mikami and Niwa (Mikami I., Niwa K. 1996). The unknowns are the thickness of the base plate as well as the dimensions and number of stiffeners. The optimum dimensions and number of stiffeners are determined by a mathematical programming method. The cost function to be minimized includes material and welding costs. The results show that the trapezoidal stiffener is the most economic one. The cost saving can be 69 % comparing with various ribs.

1 INTRODUCTION

Welded stiffened plates are widely used in various load-carrying structures, e.g. ships, bridges, bunkers, tank roofs, offshore structures, vehicles, etc. They are subject to various loadings, e.g. compression, bending, shear or combined load. The shape of plates can be square rectangular, circular, trapezoidal, etc. They can be stiffened in one or two directions with stiffeners of flat, L, trapezoidal or other shape.

Various types of plate geometry, loadings, stiffener shapes have been investigated. The strength and design in the case of compression and lateral pressure has been dealt with e.g. by Mansour (1971), Smith et al (1992), Davidson et al (1992), Bonello et al (1993), Mikami & Niwa (1996), Paik et al (2001) and Paik & Kim (2002).

Structural optimization of stiffened plates has been worked out by Farkas (1984), Farkas & Jármai (1997) and applied to uniaxially compressed plates with stiffeners of various shapes (Farkas & Jármai 2000), biaxially compressed plates (Farkas et al. 2001).

This paper contains the minimum cost design of longitudinally stiffened plates using the strength calculation methods of Mikami & Niwa (1996) and that of Paik et al (2001). Deflections due to lateral pressure, compression stress and shrinkage of longitudinal welds are taken into account in the stress constraint. The self-weight is added to the lateral pressure. The local buckling constraint of the base plate strips is formulated as well. The cost function includes material and welding costs. The unknowns are the thickness of the base plate as well as the dimensions and number of stiffeners. It should be mentioned that we have worked out this investigation for trapezoidal stiffeners (Jármai et al 2002). In this paper compare plates stiffened with ribs of flat, L and trapezoidal shapes.

2 GEOMETRIC CHARACTERISTICS

The stiffened plate is shown in Figure 1. Geometrical parameters of plates with flat-, L- and trapezoidal stiffeners can be seen in Figures 2-4.

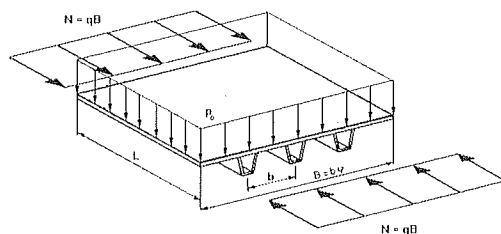


Figure 1. Longitudinally stiffened plate loaded by uniaxial compression and lateral pressure

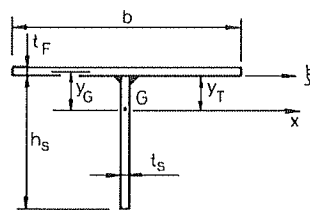


Figure 2. Dimensions of a flat stiffener

The calculations of geometrical parameters of the flat stiffener are

$$A_s = h_s t_s \quad (1)$$

$$h_s = 14 t_s \varepsilon \quad (2)$$

$$\varepsilon = \sqrt{235 / f_y} \quad (3)$$

$$y_G = \frac{h_s + t_f}{2} \frac{\delta_s}{1 + \delta_s} \quad (4)$$

$$\delta_s = \frac{A_s}{b t_f} \quad (5)$$

$$I_x = \frac{b t_f^3}{12} + b t_f y_G^2 + \frac{h_s^3 t_s}{12} + h_s t_s \left(\frac{h_s}{2} - y_G \right)^2 \quad (6)$$

$$I_s = h_s^3 \frac{t_s}{3} \quad (7)$$

$$I_t = \frac{h_s t_s^3}{3} \quad (8)$$

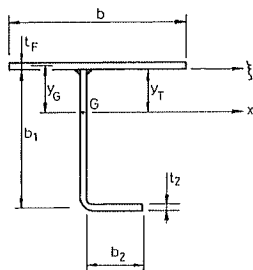


Figure 3. Dimensions of a L-stiffener

The calculations of geometrical parameters of the L-stiffener are

$$A_s = (b_1 + b_2) t_s \quad (9)$$

$$b_1 = 30 t_s \varepsilon \quad (10)$$

$$b_2 = 12.5 t_s \varepsilon \quad (11)$$

$$y_G = \frac{b_1 t_s \frac{b_1 + t_f}{2} + b_2 t_s \left(b_1 + \frac{t_f}{2} \right)}{b t_f + A_s} \quad (12)$$

$$I_x = \frac{b t_f^3}{12} + b t_f y_G^2 + \frac{b_1^3 t_s}{12} + b_1 t_s \left(\frac{b_1}{2} - y_G \right)^2 + b_2 t_s (b_1 - y_G)^2 \quad (13)$$

$$I_s = \frac{b_1^3 t_s}{3} + b_1^2 b_2 t_s \quad (14)$$

$$I_t = \frac{b_1 t_s^3}{3} + \frac{b_2 t_s^3}{3} \quad (15)$$

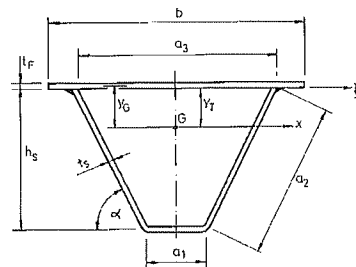


Figure 4. Dimensions of a trapezoidal stiffener

The calculations of geometrical parameters of the trapezoidal stiffener are

$$A_s = (a_1 + 2a_2) t_s \quad (16)$$

$$a_1 = 90, a_2 = 300 \text{ mm, thus} \quad (17)$$

$$h_s = (a_2^2 - 105^2)^{1/2} \quad (17)$$

$$\sin^2 \alpha = 1 - \left(\frac{105}{a_2} \right)^2 \quad (18)$$

$$y_G = \frac{a_1 t_s (h_s + t_f / 2) + 2 a_2 t_s (h_s + t_f) / 2}{b t_f + A_s} \quad (19)$$

$$I_x = \frac{b t_f^3}{12} + b t_f y_G^2 + a_1 t_s \left(h_s + \frac{t_f}{2} - y_G \right)^2 + \frac{1}{6} a_2^3 t_s \sin^2 \alpha + 2 a_2 t_s \left(\frac{h_s + t_f}{2} - y_G \right)^2 \quad (20)$$

$$I_s = a_1 h_s^3 t_s + \frac{2}{3} a_2^3 t_s \sin^2 \alpha \quad (21)$$

$$I_t = \frac{4A_p^2}{\sum b_i / t_i} \quad (22)$$

$$A_p = h_s \frac{a_1 + a_2}{2} = 195 h_s \quad (23)$$

3 CALCULATION OF THE DEFLECTION DUE TO COMPRESSION AND LATERAL PRESSURE

Since the self-weight is taken into account the lateral pressure is modified as

$$p = p_0 + \frac{\rho V g}{BL} \quad (24)$$

where g is the gravitation constant, $9.81 \text{ [m/s}^2\text{]}$.

Paik et al. (2001) have used the differential equations of large deflection orthotropic plate theory and the Galerkin method to derive the following cubic equation for the elastic deflection A_m of a stiffened plate loaded by uniaxial compression and lateral pressure

$$C_1 A_m^3 + C_2 A_m^2 + C_3 A_m + C_4 = 0 \quad (25)$$

where

$$C_1 = \frac{\pi^2}{16} \left(E_x \frac{m^4 B}{L^3} + E \frac{L}{B^3} \right) \quad (26)$$

$$C_2 = \frac{3\pi^2 A_{om}}{16} \left(E_x \frac{m^4 B}{L^3} + E \frac{L}{B^3} \right) \quad (27)$$

$$C_3 = \frac{\pi^2 A_{om}^2}{8} \left(E_x \frac{m^4 B}{L^3} + E \frac{L}{B^3} \right) + \frac{m^2 B}{L} \sigma_{sav} + \frac{\pi^2}{t_F} \left(D_x \frac{m^4 B}{L^3} + 2H \frac{m^2}{LB} + D \frac{L}{B^3} \right) \quad (28)$$

$$C_4 = A_{om} \frac{m^2 B}{L} \sigma_{sav} - \frac{16LB}{\pi^4 t_F} p \quad (29)$$

$$E_x = E \left(1 + \frac{n A_s}{B t_F} \right); \quad E_y = E \quad (30)$$

The number of stiffeners is $n = \varphi - 1$.

The flexural and torsional stiffnesses of the orthotropic plate are as follows:

$$D_x = \frac{E t_F^3}{12(1-\nu_{xy}^2)} + \frac{E t_F y_G^2}{1-\nu_{xy}^2} + \frac{E I_x}{b} \quad (31)$$

$$D_y = \frac{E t_F^3}{12(1-\nu_{xy}^2)}$$

$$\nu_x = \frac{\nu}{0.86} \sqrt{\frac{\frac{E}{E_x} \left(\frac{E t_F^3}{12} + E t_F y_G^2 + \frac{E I_x}{b} \right) - \frac{E t_F^3}{12}}{\frac{E I_x}{b} \left(\frac{E}{E_x} \right)^2}} \quad (32)$$

$$\nu_y = \frac{E}{E_x} \nu_x; \quad \nu_{xy} = \sqrt{\nu_x \nu_y} \quad (33)$$

$$H = \frac{G_{xy} I_t}{b}; \quad G_{xy} = \frac{E}{2(1+\nu_{xy})} \quad (34)$$

$$\sum \frac{b_i}{t_i} = \frac{a_1 + 2a_2}{t_s} + \frac{a_3}{t_F} \quad (35)$$

The deflection due to lateral pressure is

$$A_{om} = \frac{5qL^4}{384EI_x}; \quad q = pb; \quad b = B/\varphi \quad (36)$$

The average compression stress is

$$\sigma_{sav} = \frac{N}{B t_F + (\varphi - 1) A_s} \quad (37)$$

It is assumed that, for a simply supported plate, the largest value of A_m is given by the smallest number of half buckling length $m = 1$.

The solution of Equation 25 is

$$A_m = -\frac{C_2}{3C_1} + k_1 + k_2 \quad (38)$$

where

$$k_1 = \sqrt[3]{-\frac{Y}{2} + \sqrt{\frac{Y^2}{4} + \frac{X^3}{27}}} \quad (39)$$

$$k_2 = \sqrt[3]{-\frac{Y}{2} - \sqrt{\frac{Y^2}{4} + \frac{X^3}{27}}}$$

$$X = \frac{C_3}{C_1} - \frac{C_2^2}{3C_1^2}; \quad Y = \frac{2C_2^3}{27C_1^3} - \frac{C_2 C_3}{3C_1^2} + \frac{C_4}{C_1} \quad (40)$$

4 DEFLECTION DUE TO SHRINKAGE OF LONGITUDINAL WELDS

According to Farkas & Jármai (1997) or Jármai & Farkas (1999) the deflection of the plate due to longitudinal welds is as follows

$$f_{\max} = CL^2/8 \quad (41)$$

where the curvature for steels is

$$C = 0.844 \times 10^{-3} Q_T y_T / I_x \quad (42)$$

Q_T is the heat input, y_T is the weld eccentricity

$$y_T = y_G - t_F / 2 \quad (43)$$

and I_x is the moment of inertia of the cross-section containing a stiffener and the base plate strip of width b .

The heat input for a stiffener is proportional to the weld size

$$Q_T = 2 \times 59.5 a_w^2 \quad (44)$$

5 THE STRESS CONSTRAINT

The stress constraint includes several effects as follows: the average compression stress and the bending stress caused by deflections due to compression, lateral pressure and the shrinkage of longitudinal welds.

$$\sigma_{\max} = \sigma_{\text{av}} + \frac{M}{I_x} y_G \leq \sigma_{UP} \quad (45)$$

where

$$M = \sigma_{\text{av}} (A_{0m} + A_m + f_{\max}) + \frac{qL^2}{8} \quad (46)$$

According to Mikami & Niwa (1996), the calculation of the local buckling strength of a face plate strip of width

$$b_I = \max(a_3, b - a_3) \quad (47)$$

is performed taking into account the effect of initial imperfections and residual welding stresses

$$\sigma_{UP} = f_y \quad \text{when} \quad \lambda_p \leq 0.526 \quad (48a)$$

$$\sigma_{UP} = \left(\frac{0.526}{\lambda_p} \right)^{0.7} \quad \text{when} \quad \lambda_p \geq 0.526 \quad (48b)$$

where

$$\lambda_p = \left(\frac{4\pi^2 E}{10.92 f_y} \right)^{1/2} \frac{b_I}{t_F} = \frac{b_I / t_F}{56.8 \varepsilon} \quad (49)$$

6 THE COST FUNCTION

The objective function to be minimized is defined as the sum of material and fabrication costs

$$K = K_m + K_f = k_m \rho V + k_f \sum T_i \quad (50)$$

or in another form

$$\frac{K}{k_m} = \rho V + \frac{k_f}{k_m} (T_1 + T_2 + T_3) \quad (51)$$

where ρ is the material density, V is the volume of the structure, K_m and K_f as well as k_m and k_f are the material and fabrication costs as well as cost factors, respectively, T_i are the fabrication times as follows:
- time for preparation, tacking and assembly

$$T_1 = \Theta_d \sqrt{\kappa \rho V} \quad (52)$$

where Θ_d is a difficulty factor expressing the complexity of the welded structure, κ is the number of structural parts to be assembled;
- T_2 is time of welding, and T_3 is time of additional works such as changing of electrode, deslagging and chipping. $T_3 \approx 0.3T_2$, thus,

$$T_2 + T_3 = 1.3 \sum C_{2i} a_w'' L_{wi} \quad (53)$$

where L_{wi} is the length of welds, the value of C_2 is a constant concerning welding procedures, a_w is the weld dimension. The fillet weld size is $a_w = 0.5t_S$, but $a_{wmin} = 4$ mm.

In our case, the volume of the structure is

$$V = BLt_F + (\varphi - 1)A_3L \quad (54)$$

the weld length is

$$L_w = 2(\varphi - 1)L \quad (55)$$

and for GMAW-M (Gas-Metal Arc Welding with Mixed Gas) fillet welds it is

$$C_w a_w'' = 0.3258 \times 10^{-3} a_w^2 \quad (56)$$

The optima are calculated for $k_F/k_M = 0$ and 1.5 kg/min corresponding to minimum mass design and higher fabrication cost design respectively.

7 THE OPTIMIZING METHOD

Rosenbrock's Hillclimb (Rosenbrock, H.H. 1960) mathematical method is used to minimize cost function. This is a direct search mathematical programming method without derivatives. The iterative algorithm based on Hooke & Jeeves searching method. It starts with a given initial value, and it takes small steps in direction of orthogonal coordinates during the search. The algorithm is modified that secondary searching is carried out to determine discrete values. The procedure finishes in case of convergence criterion is satisfied or the iterative number reaches its limit.

8 NUMERICAL DATA

The given data are width $B = 4000$ [mm], length $L = 6000$ [mm], compression force $N = 1.974 \times 10^7$ [N], Young modulus $E = 2.1 \times 10^5$ [MPa], density $\rho = 7.85 \times 10^{-6}$ [kg/mm³]. There are three values of lateral pressures $p_0 = 0.05, 0.1, 0.2$ [MPa] and two values of yield stresses $f_y = 255, 355$ [MPa]. At the higher strength steel the material cost is 10 % higher than that of the normal steel.

9 RESULTS AND CONCLUSIONS

The plate is simply supported on four edges. The unknowns – the thicknesses of the base plate and the stiffener and the number of the ribs – are limited in size as follows:

$$\begin{aligned} 3 \leq t_f &\leq 40 \text{ [mm]} \\ 3 \leq t_s &\leq 12 \text{ [mm]} \\ 3 \leq \varphi &\leq 10 \end{aligned} \quad (57)$$

The results are shown in Tables 1-6. The optima results are marked by bold letters.

Table 1. Optimum dimensions with *flat* stiffener for $k_F/k_m=0$, the material minima

| f_y [MPa] | p_0 [MPa] | t_f [mm] | t_s [mm] | φ | K/k_m [kg] $k_F/k_m=0$ $k_F/k_m=1.5$ | |
|----------------|----------------|---------------|---------------|-----------|---|-------|
| 235 | 0,2 | - | - | - | - | - |
| 235 | 0,1 | 38 | 12 | 10 | 8014 | 11758 |
| 235 | 0,05 | 30 | 12 | 6 | 6127 | 8362 |
| 355 | 0,2 | - | - | - | - | - |
| 355 | 0,1 | 28 | 12 | 10 | 6568 | 10137 |
| 355 | 0,05 | 20 | 12 | 9 | 4825 | 7914 |

Table 2. Optimum dimensions with *flat* stiffener for $k_F/k_m=1.5$, the cost minima

| f_y [MPa] | p_0 [MPa] | t_f [mm] | t_s [mm] | φ | K/k_m [kg] $k_F/k_m=0$ $k_F/k_m=1.5$ | |
|----------------|----------------|---------------|---------------|-----------|---|--------------|
| 235 | 0,2 | - | - | - | - | - |
| 235 | 0,1 | 38 | 12 | 10 | 8014 | 11758 |
| 235 | 0,05 | 30 | 12 | 6 | 6127 | 8362 |
| 355 | 0,2 | - | - | - | - | - |
| 355 | 0,1 | 28 | 12 | 10 | 6568 | 10137 |
| 355 | 0,05 | 21 | 11 | 8 | 4852 | 7312 |

Table 3. Optimum dimensions with *L*-stiffener for $k_F/k_m=0$, the material minima

| f_y [MPa] | p_0 [MPa] | t_f [mm] | t_s [mm] | φ | K/k_m [kg] $k_F/k_m=0$ $k_F/k_m=1.5$ | |
|----------------|----------------|---------------|---------------|-----------|---|------|
| 235 | 0,2 | 31 | 12 | 5 | 6993 | 8933 |
| 235 | 0,1 | 21 | 12 | 7 | 5686 | 8230 |
| 235 | 0,05 | 20 | 10 | 7 | 4969 | 6952 |
| 355 | 0,2 | 22 | 12 | 7 | 6107 | 8641 |
| 355 | 0,1 | 18 | 9 | 10 | 5036 | 7389 |
| 355 | 0,05 | 17 | 7 | 10 | 4313 | 6302 |

Table 4. Optimum dimensions with *L*-stiffener for $k_F/k_m=1.5$, the cost minima

| f_y [MPa] | p_0 [MPa] | t_f [mm] | t_s [mm] | φ | K/k_m [kg] $k_F/k_m=0$ $k_F/k_m=1.5$ | |
|----------------|----------------|---------------|---------------|-----------|---|-------------|
| 235 | 0,2 | 34 | 11 | 4 | 7132 | 8584 |
| 235 | 0,1 | 27 | 10 | 5 | 5888 | 7422 |
| 235 | 0,05 | 24 | 8 | 6 | 5162 | 6564 |
| 355 | 0,2 | 28 | 9 | 6 | 6528 | 8149 |
| 355 | 0,1 | 22 | 8 | 7 | 5247 | 6801 |
| 355 | 0,05 | 19 | 8 | 7 | 4626 | 6129 |

Table 5. Optimum dimensions with *trapezoidal* stiffener for $k_F/k_m=0$, the material minima

| f_y [MPa] | p_0 [MPa] | t_f [mm] | t_s [mm] | φ | K/k_m [kg] $k_F/k_m=0$ $k_F/k_m=1.5$ | |
|----------------|----------------|---------------|---------------|-----------|---|------|
| 235 | 0,2 | 28 | 12 | 4 | 6974 | 8549 |
| 235 | 0,1 | 24 | 10 | 4 | 5723 | 6975 |
| 235 | 0,05 | 18 | 10 | 5 | 4993 | 6466 |
| 355 | 0,2 | 21 | 11 | 5 | 6108 | 7780 |
| 355 | 0,1 | 15 | 10 | 6 | 4944 | 6635 |
| 355 | 0,05 | 13 | 8 | 7 | 4148 | 5611 |

Table 6. Optimum dimensions with *trapezoidal* stiffener for $k_F/k_m=1.5$, the cost minima

| f_y [MPa] | p_0 [MPa] | t_f [mm] | t_s [mm] | φ | K/k_m [kg] $k_F/k_m=0$ $k_F/k_m=1.5$ | |
|----------------|----------------|---------------|---------------|-----------|---|-------------|
| 235 | 0,2 | 35 | 9 | 3 | 7250 | 8223 |
| 235 | 0,1 | 24 | 10 | 4 | 5723 | 6975 |
| 235 | 0,05 | 23 | 8 | 4 | 5122 | 6132 |
| 355 | 0,2 | 28 | 8 | 4 | 6530 | 7589 |
| 355 | 0,1 | 21 | 7 | 5 | 5111 | 6284 |
| 355 | 0,05 | 16 | 7 | 6 | 4264 | 5560 |

It can be seen from Tables 1 and 2 that there are no solutions for the highest lateral pressure for flat stiffeners due to the size limits.

The results show that the trapezoidal stiffener is the most economic one. The cost saving can be 69 % comparing with various ribs.

Materials with higher yield stress gives the cheaper results. The cost saving can be 40 % comparing with the lower one.

In most cases the material and cost minima are different, the number of stiffeners is smaller at cost minima due to welding cost effects, as it is visible for flat stiffeners on Tables 1 and 2, for L-stiffeners on Tables 3 and 4, and for trapezoidal stiffeners on Tables 5 and 6.

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REFERENCES

- Bonello, M.A., Chryssanthopoulos, M.K., Dowling, P.J. 1993. Ultimate strength design of stiffened plates under axial compression and bending. *Marine Structures* 6, 533-552.
- Davidson, P.C., Chapman, J.C., Smith, C.S., Dowling, P.J. 1992. The design of plate panels subject to biaxial compression and lateral pressure. *Trans Royal Institution of Naval Architects* 134, 149-160.
- Jármai, K., Farkas, J., Simoes, L.C. and Virág, Z. 2002. Minimum cost design of longitudinally stiffened welded steel plates loaded by eccentric compression, *Proceedings of Third European Conference on Steel Structures*, Coimbra, Portugal, 533-540, ISBN 972-98376-3-5.
- Farkas, J. 1984. *Optimum design of metal structures*. Budapest, Akadémiai Kiadó, Chichester, Ellis Horwood.
- Farkas, J., Jármai, K. 1997. Analysis and optimum design of metal structures. Balkema, Rotterdam-Brookfield.
- Farkas, J., Jármai, K. 2000. Minimum cost design and comparison of uniaxially compressed plates with welded flat-, L- and trapezoidal stiffeners. *Welding in the World* 44:3, 47-51.
- Farkas, J., Simoes, L.M.C., Jármai, K. 2001. Minimum cost design of a welded stiffened square plate loaded by biaxial compression. WCSMO-4, *4th World Congress of Structural and Multidisciplinary Optimization*, Dalian China, Extended Abstracts, 136-137.
- Mansour, A.E. 1971. Post-buckling behaviour of stiffened plates with small initial curvature under combined loads. *Internat. Shipbuilding Progress* 18:202, 217-240.
- Mikami, I., Niwa, K. 1996. Ultimate compressive strength of orthogonally stiffened steel plates. *J. Struct. Engng ASCE* 122:6, 674-682.
- Paik, J.K., Thayamballi, A.K., Kim, B.J. 2001. Large deflection orthotropic plate approach to develop ultimate strength formulations for stiffened panels under combined biaxial compression/tension and lateral pressure. *Thin-Walled Structures* 39, 215-246.
- Paik, J.K., Kim, B.J. 2002. Ultimate strength formulations for stiffened panels under combined axial load, in-plane bending and lateral pressure: a benchmark study. *Thin-Walled Structures* 40, 45-83.
- Rosenbrock, H.H. 1960. An automatic method for finding the greatest or least value of a function, *Computer Journal*, 3, 175-184.
- Smith, C.S., Anderson, N., Chapman, J.C., Davidson, P.C., Dowling, P.J. 1992. Strength of stiffened plating under combined compression and lateral pressure. *Trans Royal Institution of Naval Architects* 134, 131-147.