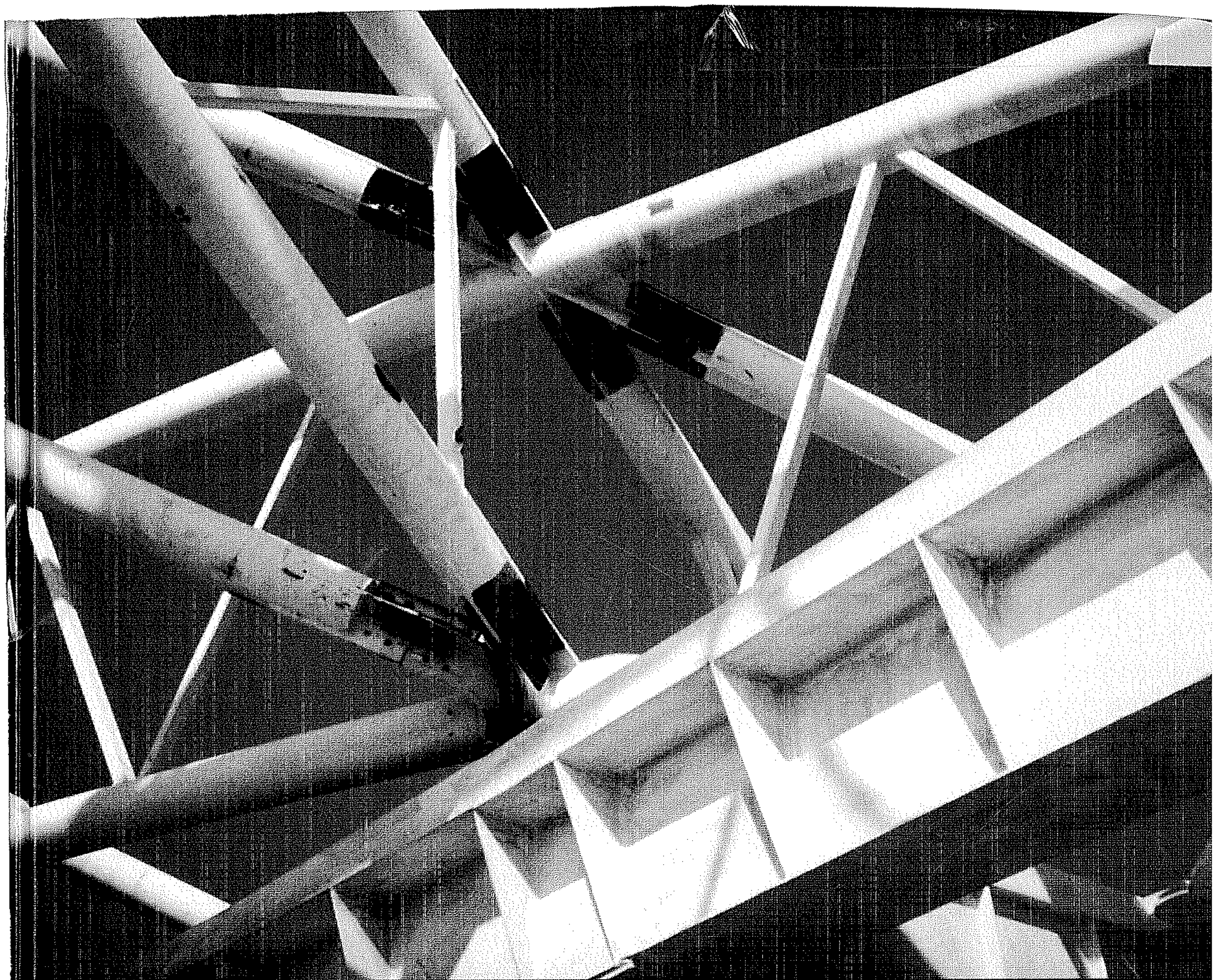

Tubular Structures VII

József Farkas and Károly Jármai, editors

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Cover photo: Tubular roof structure of a new sports hall in Budapest

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Optimum design and imperfection-sensitivity of centrally compressed SHS and CHS aluminium struts

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ABSTRACT: A numerical comparison of the optimal solutions obtained without and with the consideration of initial imperfections of aluminium SHS struts shows that the imperfection sensitivity of optimal solution considering the initial imperfections is much smaller than that obtained using buckling formulae which do not consider the initial imperfections. Thus, the buckling formulae given by BS 8118 can be used for optimum design of compressed aluminium SHS and CHS struts. Using a computer method some optimum design results are given for two aluminium alloys and different effective length factors.

KEYWORDS: compressed struts, aluminium struts, hollow section struts, optimum design, imperfection-sensitivity.

1 INTRODUCTION

In the past several authors (e.g. Shanley 1960, Gerard 1962) have solved optimization problems with two active stability constraints using buckling formulae which do not consider the effect of initial imperfections. Some authors (e.g. Thompson & Hunt 1973, Tvergaard 1973, Van der Neut 1973) have shown that this "naive" method leads to imperfection-sensitive solutions. Thompson (1972) has named this optimization "as a generator of structural instability".

The first aim of this paper is to show that the optimization which considers the initial imperfections gives solutions of much smaller imperfection-sensitivity than that without imperfections. Rondal & Maquoi (1981) have already shown that the method considering the imperfections does not lead to severe erosion of compressive strength.

A detailed survey of the coupled instability phenomena has been worked out by Gioncu (1994). The authors have shown (Farkas & Jármai 1995) that the use of Euler buckling formula leads to considerably unsafe solutions in the case of steel CHS compressed struts.

The second aim is to extend the optimization worked out for steel SHS and CHS struts (Farkas & Jármai 1994) to aluminium struts, since it can be shown that the overall buckling formulae given by BS 8118 (1991) are the same as those given by the Eurocode 3 (EC 3) (1992). Some optimum design results are given to enable designers the use of optimization in these cases.

Note that the design of open-section aluminium compressed struts has been treated by Lai and Nethercot (1992).

2 OPTIMUM DESIGN OF COM-PRESSED ALUMINIUM SHS STRUTS WITHOUT THE CONSIDERATION OF INITIAL IMPERFEC-TIONS

In order to compare the optimal solutions obtained without and with initial imperfections we derive first the optimal formulae using buckling formulae which do not consider the initial imperfections. The objective function to be minimized is the cross-sectional area. Neglecting the effect of rounding at the corners, a simple approximate formula can be used as follows (Fig.1)

$$A = 4bt \tag{1}$$

The overall buckling constraint, using the Euler formula for pinned ends, is expressed by

$$\frac{N}{4bt} \le \frac{\pi^2 E}{\lambda^2} = \frac{\pi^2 E b^2}{6L^2} \tag{2}$$

The local buckling constraint, without the effect of initial imperfections, can be written as

$$\frac{N}{4bt} \leq \frac{4\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2 \quad (3)$$

The constraint on yielding is

$$N / A \leq p_0 / \gamma_m \quad (4)$$

where p_0 is the limiting stress for yielding, γ_m is the material factor.

It can be shown (Farkas 1992) that, using a suitable coordinate-system of two unknowns, the optimum point is determined as the intersection point of the limiting curves of constraints (2) and (3) or (4) and (3), respectively. Thus, these constraints are active and can be treated as equalities. Thus, from Eq.(2) one obtains

$$t = 3L^2 N / \left(2\pi^2 E b^3\right) \quad (5)$$

and from Eq.(3)

$$t^3 = 3(1-\nu^2) N b / \left(4\pi^2 E\right) \quad (6)$$

Substituting (5) into (6) we get

$$b = \left[\frac{9L^6 N^2}{2\pi^4 E^2 (1-\nu^2)}\right]^{1/10} \quad (7)$$

and

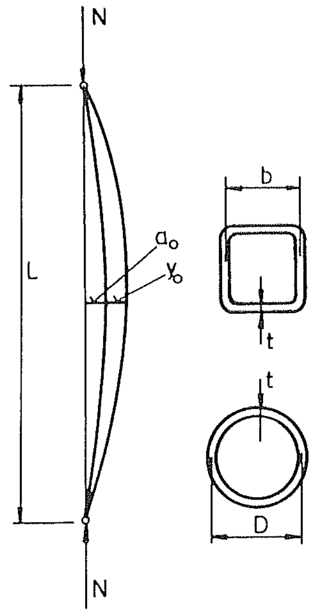

Fig. 1. Compressed SHS and CHS struts

$$t = \frac{3L^2 N}{2\pi^2 E}\left[\frac{2\pi^4 E^2 (1-\nu^2)}{9L^6 N^2}\right]^{3/10} \quad (8)$$

Our aim is to calculate the A/L^2 -values in the function of N/L^2 . Thus, using Eqs.(7) and (8) we get

$$\frac{A}{L^2} = \frac{4bt}{L^2} = 6\left(\frac{2}{9\pi^6}\right)^{1/5}\left[\frac{N^3(1-\nu^2)}{E^3 L^6}\right]^{1/5} \quad (9)$$

Table 1. Limiting stresses for heat-treatable alloys

Alloy	Condition	Product	Thickness over (mm)	Thickness up to and including (mm)	p_0 (MPa)	nearest equivalent to ISO 209-1
6061	T6	Extrusion	--	150	240	AlMg1SiCu
		Drawn tube	--	6	240	
		Drawn tube	6	10	225	
6063	T6	Extrusion	--	150	160	AlMg0.7Si
		Drawn tube	--	10	180	
6082	T6	Extrusion	--	20	225	AlSi1MgMn
		Extrusion	20	150	270	
		Drawn tube	--	6	255	
		Drawn tube	6	10	240	
7020	T6	Extrusion	--	25	280	AlZn4.5Mg1

To scale the dimensions we multiply with 10^4

$$\frac{10^4 A}{L^2}=6\left(\frac{2}{9\pi^6}\right)^{1/5}\left(10^4\right)^{2/5}\left(1-\nu^2\right)^{1/5}E^{-3/5}\left(\frac{10^4 N}{L^2}\right)^{3/5} \tag{10}$$

Note that N/L^2 should have a dimension of N/mm^2. For aluminium alloys it is E=7*10^4 MPa and $\nu = 0.3$, thus

$$\frac{10^4 A}{L^2}=0.05441\left(\frac{10^4 N}{L^2}\right)^{3/5} \tag{11}$$

Using data of p_0 = 240 MPa and $\gamma_m = 1.2$, in the plastic region Eq.(4) can be written as

$$\frac{10^4 A}{L^2}\geq\frac{10^4 N/L^2}{p_0/\gamma_m}=\frac{10^4 N/L^2}{200} \tag{12}$$

Data of some aluminium alloys, according to BS 8118, are given in Table 1.

3 OPTIMUM DESIGN OF COMPRESSED ALUMINIUM SHS AND CHS STRUTS CONSIDERING THE INITIAL IMPERFECTIONS

On the basis of the Ayrton-Perry formulation

$$\frac{N}{A}+\frac{N(a_0+y_0)}{W_x}\leq\frac{p_0}{\gamma_m} \tag{13}$$

where a_0 is the maximal initial imperfection, y_0 is the elastic deformation due to the compressive force (Fig.1), W_x is the elastic section modulus,

the overall buckling constraint can be derived which is used in EC3 and BS 8118 as follows:

$$\frac{10^4 N/L^2}{p_0/\gamma_m}\leq\frac{10^4 A/L^2}{\phi+\sqrt{\phi^2-\bar{\lambda}^2}};\bar{\lambda}^2=\frac{c_0^2}{a^2 y};c_0=\frac{100K}{\lambda_E};\lambda_E=\pi\sqrt{\frac{E}{p_0}};y=\frac{10^4 A}{L^2}; \tag{14}$$

$$\phi=0.5\left[1+\alpha(\bar{\lambda}-0.2)+\bar{\lambda}^2\right];x=10^4 N/L^2 \qquad \text{For } \bar{\lambda}\leq 0.2 \qquad \phi+\sqrt{\phi^2-\bar{\lambda}^2}=1$$

Table 2. Optimal $10^4 A/L^2$-values in the function of $10^4 N/L^2$ for aluminium SHS struts

$10^4 N/L^2$ (N/mm^2)	10	100	390.64	1000	10000
without imperfections elastic Eq.(11)	0.2166	0.8623	1.9532	3.4330	13.6670
without imperfections plastic Eq.(12)	0.0500	0.5000	1.9532	5.0000	50.0000
with imperfections Eq.(14)	0.4463	1.4719	---	6.1390	50.7595

where K is the effective length factor, e.g. for pinned ends K = 1, for struts effectively held in position and restrained in direction at both ends K = 0.7. According to BS 8118 the imperfection factor for unwelded and symmetric profiles is $\alpha = 0.2$.

In the derivation the member proportional to the initial imperfection is expressed by the reduced slenderness

$$\frac{a_0 A}{W_x} = \alpha(\bar{\lambda} - 0.2) \qquad (15)$$

thus, this expression should be changed in the investigation of the imperfection-sensitivity.

The local buckling constraint may be expressed by the limiting local slenderness

$$\delta \le \delta_L; \qquad (16)$$

for SHS $\delta_S = b/t$;

for CHS $\delta_C = D/t$

Neglecting the effect of initial imperfections, for SHS, it can be written that

$$\frac{4\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2 \ge \frac{p_0}{\gamma_m} \qquad (17)$$

From Eq.(17) we get

$$\left(\frac{b}{t}\right)_L = \sqrt{\frac{\pi^2 E \gamma_m}{3(1-\nu^2)p_0}} \qquad (18)$$

With values of $E = 7*10^4$ MPa, p_0 =240 MPa, $\gamma_m = 1.2; \nu = 0.3$ we obtain $(b/t)_L = 35.6$.

According to BS 8118, the limiting plate slenderness is

$$\delta_{SL} = 22\sqrt{250/p_0} = 22.45 \qquad (19)$$

which shows that the value of 35.6 is decreased considering the initial imperfections.

As it has been treated in the authors' previous work (Farkas & Jármai 1994), the radius of gyration is

for SHS $r = a_S\sqrt{A}; a_S = \sqrt{\delta_S/24}$ (20a)

for CHS $r = a_C\sqrt{A}; a_C = \sqrt{\delta_C/(8\pi)}$ (20b)

According to BS 8118, the limiting local slenderness for CHS is

$$\delta_{CL} = \left(\frac{22}{3}\right)^2 \frac{250}{p_0} \qquad (21)$$

4 COMPARISON OF THE OPTIMUM DESIGNS WITHOUT AND WITH INITIAL IMPERFECTIONS

The solutions are summarized in Table 2. The imperfection-sensitivity is investigated for the following numerical data: $10^4 N/L^2$ = 100 N/mm^2 and L = 6 m (N = 360 kN).

The solution without imperfections is (Table 2) $10^4 A/L^2 = 0.8623$, thus A = 3104 mm^2. Using (7) and (8) we get

$$\frac{b}{t} = \sqrt{\frac{A\pi^2 E}{3(1-\nu^2)N}} = 46.71 \quad \text{and}$$

$$b = \sqrt{\frac{Ab}{4t}} = 190.4 \text{ mm}.$$

Based on the value of A = 3104 mm^2 we calculate the buckling strength of this section taking a limiting plate slenderness according to BS 8118 i.e. b/t =22.45 (Eq.19). Then b = 132 mm, with Eq.(14) $\lambda_E = 53.65; \bar{\lambda} = 6000\sqrt{6}/(132*53.65) = 2.0753$.

The maximal initial imperfection using Eq.(15) with $W_x/A = b/3$ is a_0 = 16.5 mm. The overall buckling force according to Eq.(14) is N_1 = 130.0 kN. Changing the previous value of b/t = 22.45 to 15 and taking $2a_0$ instead of a_0 we get N_2 = 82.5 kN.

The solution with imperfections is (Table 2) $10^4 A/L^2$=.1.4719. A = 5299 mm^2, with b/t = 22.45 we obtain b = 172.45 and t = 7.68 mm. Investigating the imperfection-sensitivity of this solution we take b/t = 15 and $0.4(\bar{\lambda} - 0.2)$ instead of $0.2(\bar{\lambda} - 0.2)$ and get N_3 = 227.2 kN.

Summarizing the results in a diagram (Fig.2) it can be seen that the sensitivity of the solution with

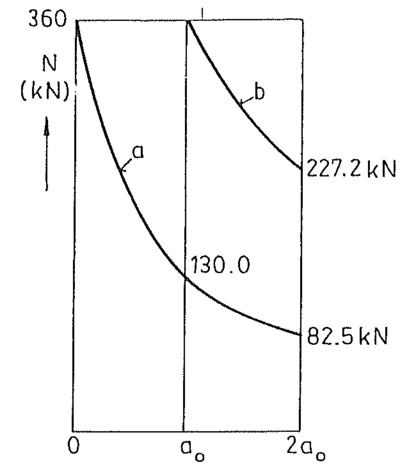

Fig. 2. The imperfection-sensitivity of compressed struts a) without b) with initial imperfections

initial imperfections is much smaller than that without imperfections. Thus, the concept of two simultaneously active buckling constraints can be applied using buckling formulae which consider the initial imperfections.

5 TABLE FOR THE OPTIMUM DESIGN OF COMPRESSED ALUMINIUM SHS AND CHS STRUTS

Since a closed solution of Eq.(14) cannot be given, it is solved numerically using a computer program. In the design practice the following data can be varied: N, L, p_0, K, SHS or CHS, α (for unwelded symmetric profiles 0.2, for welded symmetric profiles 0.45).

In Table 3 results are given for $p_0 = 240$ and 160 MPa (see Table 1), for $K = 1$ and 0.7 and for SHS and CHS. If the $10^4\ A/L^2$ -values are plotted in the function of $10^4\ N/L^2$ in a doubly logarithmic coordinate system, it can be seen that a linear interpolation can be used between the values given in Table 3.

Similar diagrams have been given by the authors for steel CHS and SHS struts (Farkas & Jármai 1994).

Table 3. Optimal $10^4\ A/L^2$ -values in the function of $10^4\ N/L^2$

			$10^4\ N/L^2$ (N/mm^2)			
p_0 (MPa)	K	section	10	100	1000	10000
240	1	CHS	0.2921	0.9948	5.4534	50.0000
240	1	SHS	0.4463	1.4719	6.1390	50.7595
240	0.7	CHS	0.2070	0.7507	5.2127	50.0000
240	0.7	SHS	0.3150	1.0640	5.5316	50.0000
160	1	CHS	0.2965	1.0894	7.7846	75.0000
160	1	SHS	0.4973	1.6719	8.3891	75.0163
160	0.7	CHS	0.2117	0.9047	7.5947	75.0000
160	0.7	SHS	0.3520	1.2392	7.9248	75.0000

6 CONCLUSIONS

It has been shown that the structural optimization with two simultaneously active overall and local buckling constraints may lead to imperfection-sensitive solutions when it is based on buckling formulae which do not consider the initial imperfections. On the contrary, the optimum design results in practically safe and non-imperfection-sensitive solutions when it is based on formulae considering the initial imperfections.

The sensitivity of rods designed with initial imperfections is less since they are sensitive only against additional imperfections.

Optimal solutions are given in Table 3. for two values of p_δ and K to ease the optimum design of compressed aluminium SHS and CHS struts.

7 ACKNOWLEDGEMENTS

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