# ADVANCED TEMPUS COURSE

**on**

## NUMERICAL METHODS IN COMPUTER AIDED OPTIMAL DESIGN

*Sponsored by TEMPUS JEP 0045*
*"Computer Aided Optimal Design of Structural and Mechanical Systems"*
*Coordinator: M.P.Bendsøe, Coordinator for Poland: Z.Mróz*

Organized by:

**Silesian Technical University of Gliwice**
**Polish Society of Theoretical and Applied Mechanics - Branch in Gliwice**

**Zakopane, May 11 - 15, 1992**

# LECTURE NOTES

**Volume 2**


Editor & Chairman

**Tadeusz BURCZYŃSKI**
**Department of Engineering Mechanics**
**Silesian Technical University of Gliwice, Poland**

# ADVANCED TEMPUS COURSE

**on**

## NUMERICAL METHODS IN COMPUTER AIDED OPTIMAL DESIGN

### LECTURE NOTES: VOLUME 2

**Contents**

# OPTIMUM DESIGN OF METAL STRUCTURES USING SINGLE- AND MULTIOBJECTIVE OPTIMIZATION TECHNIQUES

Dr. Károly JARMAI
associate professor
University of Miskolc, Hungary, H-3515 Miskolc-Egyetemváros

## SUMMARY

The different single- and multiobjective optimization techniques makes the designer able to determine the optimal sizes of structures, to get the best solution among several alternatives. The efficiencies of these mathematical programming techniques (MP) are different. We have shown the efficiency of these techniques at the optimum design of single bay frames, main girders of overhead travelling cranes, spindle-bearing systems of a machine tool, stiffened plates, compressed columns.

## 1. INTRODUCTION

Single- and multiobjective optimization techniques are good tools for finding the best results of the design problem. The developed computer code contains seven various type multiobjective and five single-objective optimization techniques [1].

The efficiency of the computer code is shown at the design of single-bay plane frame, with I-cross section with continuously increasing web height, taking account 3 objective functions and 35 inequality constraints. The second application is the design of a welded, stiffened box girder as a main girder of an overhead travelling crane with 4 objectives and 16 inequality constraints. The third application is the design of a spindle-bearing system with 3 objectives and 10 inequality constraints. The fourth application is the design of cellular plates with 3 objectives and 14 inequality constraints, the fifth application is design of compressed columns with 1 objective and 9 inequality constraints.

In these cases the optimization techniques had different efficiencies, one or two is better to use for that problem than the others, regarding the single-objective optimization techniques. At the multiobjective optimization techniques the main difference is, what kind of Pareto optima can be found and how close is it to the ideal solution. The great number of Pareto optima gives the possibility for the designer to choose the "best" from them [8,9,10,13]. See Table 1.

## 2. SINGLE-CRITERION OPTIMIZATION TECHNIQUES

A large number of algorithm have been proposed for the nonlinear programming solution. Each technique has its own advantages and disadvantages, no one algorithm is suitable for all purposes. The choice of a particular algorithm for any situation depends on the problem formulation and the user.

The general formulation of a single-criterion nonlinear programming problem is the following:

$$\begin{aligned} &\text{minimize} \quad f(x), && x = x_1, x_2, \ldots, x_N, \\ &\text{subject to } g_j(x) \geq 0, && j = 1,2,\ldots\ldots,P, \\ &\qquad\qquad h_k(x) = 0, && k = P+1,\ldots,P+M. \end{aligned} \tag{1}$$

### 2.1 THE FLEXIBLE TOLERANCE (FT) METHOD

The FT [2] algorithm improves the value of the objective function by using information provided by feasible points, as well as certain nonfeasible points termed near-feasible points. The near-feasibility limits are gradually made more restrictive as the search proceeds toward the solution, until in the limit only feasible x vectors are accepted.

With this strategy (1) can be replaced by a simpler problem, having the same solution:

$$\begin{aligned} &\text{minimize} \quad f(x), \\ &\text{subject to } \Phi^k - T(x) \geq 0 \end{aligned} \tag{2}$$

where $\Phi^k$ is the value of the flexible tolerance criterion for feasibility on the kth stage of the search, and $T(x)$ is a positive functional of all the equality and/or inequality constraints of (1), used as a measure of the extent of constraint violation. It is very important to choose a good size of initial polyhedron, which is difficult, when the difference between the values of unknowns is great.

### 2.2 THE DIRECT-RANDOM SEARCH (DRS) METHOD

The DRS [3] method combined three techniques: the direct search of Hooke and Jeeves, the random search, and the penalty function concept into one computer code. The penalty function is formed as follows:

$$P(x,r) = f(x) + \delta_i\ r_i\ g_i^2(x), \tag{3}$$

$\delta_i=(1-U_i)$ is zero if the constraints is satisfied and unity otherwise.
The initial value of r is as follows:

$r_i^0 = 0.02/\ (P^* g_i(x^0)\ f(x^0))$, where $P^*$ is the number of constraints.

Minimization of the P function is carried out by the Hooke and Jeeves technique for the successive series of increasing value of $r_i$ from stage to stage. The search terminates when all the constraints are satisfied or when the absolute difference between the value of the constraint at the beginning of the search and at the end is less than some prespecified tolerance.

2.3 THE HILLCLIMB (HI) ALGORITHM

The procedure is based on the "automatic" method proposed by Rosenbrock [4]. The method of rotating coordinates can be considered as a further development of the Hooke and Jeeves method. Before starting the minimization process, define a set of ´initial´ step lengths $S_i$, to be taken along the search directions $M_i$, i= 1,2,...,N. The starting point must satisfies the constraints and does not lie in the boundary zones. The boundary zones are defined as follows:

lower zone: $x_i^L \le x_i \le x_i^L + (x_i^U - x_i^L) * 10^{-4}$

upper zone: $x_i^U \le x_i \le x_i^U - (x_i^U - x_i^L) * 10^{-4}$ i= 1,2,..,M

The variables are stepped a distance $S_i$ parallel to the axis and the function is evaluated.

$$\text{new } x_i^{(k)} = \text{old } x_i^{(k)} + S_j^{(k)} * M_{i,j}^{(k)}$$

If the current point objective function value is worse, than the previous good value, or if the constraints are violated, the trial point is a failure and $S_i$ decreased by a factor $\mu$, $0 < \mu \le 1.0$ and the direction of movement reversed. If the move is termed a success, $S_i$ increased by a factor $\beta$, $\beta \ge 1.0$. The new point is retained and a success is recorded. The values of $\mu$ and $\beta$ are usually taken as 3.0 and 0.5 respectively. If the current point lies within a boundary zone, the objective function is modified by the distance into the boundary zone.

At the algorithm, the coordinate system is rotated in each stage of minimization. The procedure stops if the convergence criterion or the iteration limit is reached. No derivatives are required. The procedure is very quick, but it gives usually local optima, so it is advisable to use more starting points.

## 2.4 THE COMPLEX PROGRAMMING METHOD (BO)

Using random numbers a so-called "complex" is generated from the upper and lower bounds of variables. Computing the coordinates of the centroid some geometrical replacements are used:

$$x_{ij} = x_i^L + r_{ij} (x_i^U - x_i^L) \qquad i=1,\dots,N, \qquad j=2,\dots,K. \quad (4)$$

where $r_{ij}$ are the random numbers with a uniform distribution over the interval $0 \div 1$.
$x_i^U$ and $x_i^L$ are the upper and lower limits of, variables.

- rejection with the coefficient through the centroid. If $\mu = 1$ that is a simple rejection, if $\mu > 1$ ( $\mu = 1.3$) that is an expanded rejection.
- halving the distance between the point and the centroid ( $\Gamma = 0.5$).

The convergence criterion is

$$f_{max} - f_{min} < \beta \qquad (5)$$

when it is fulfilled, the procedure is terminated [5]. The procedure is robust, it gives global optima, but if the number of unknowns (N) and the size of complex (K) are great, it becomes very slow.

## 2.5 THE DAVIDON-FLETCHER-POWELL METHOD (DFP)

The variable metric method of Davidon was extended by Fletcher and Powell [6]. This method is one of the best general-purpose unconstrained optimization techniques making use of the derivatives that are currently available.

The method computes the gradient of the function f(x) at the initial point and sets

$$S_i = -H_i \nabla f(x_i) \qquad (6)$$

Find the optimal length $\Omega_i$ , in the direction $S_i$,

$$x_{i+1} = x_i + \Omega_i S_i \qquad (7)$$

where $H_i$ is taken as the identity matrix.

Find the new point $x_{i+1}$ for optimality and if $x_{i+1}$ is optimal, terminate the iterative process, otherwise continue calculation.

The interior penalty function method is used in the algorithm to be able to handle constraints. The $\Phi$ function is defined as follows:

$$\Phi(x,r_k) = f(x) + r_k \Sigma\, 1/\, g_j(x) \quad (8)$$

It can be seen that the value of the function $\Phi$ will always be smaller than f since $g_j(x)$ is negative for all infeasible points x.

The cubic interpolation method is used for finding the minimizing step length $\Omega_i$ in four stages. Sometimes there is an overflow at $\Phi$ function, because $g_j(x)$ is close to zero near the optimum, so the convergence criterion is very important.

The program system can be seen an Table 1.

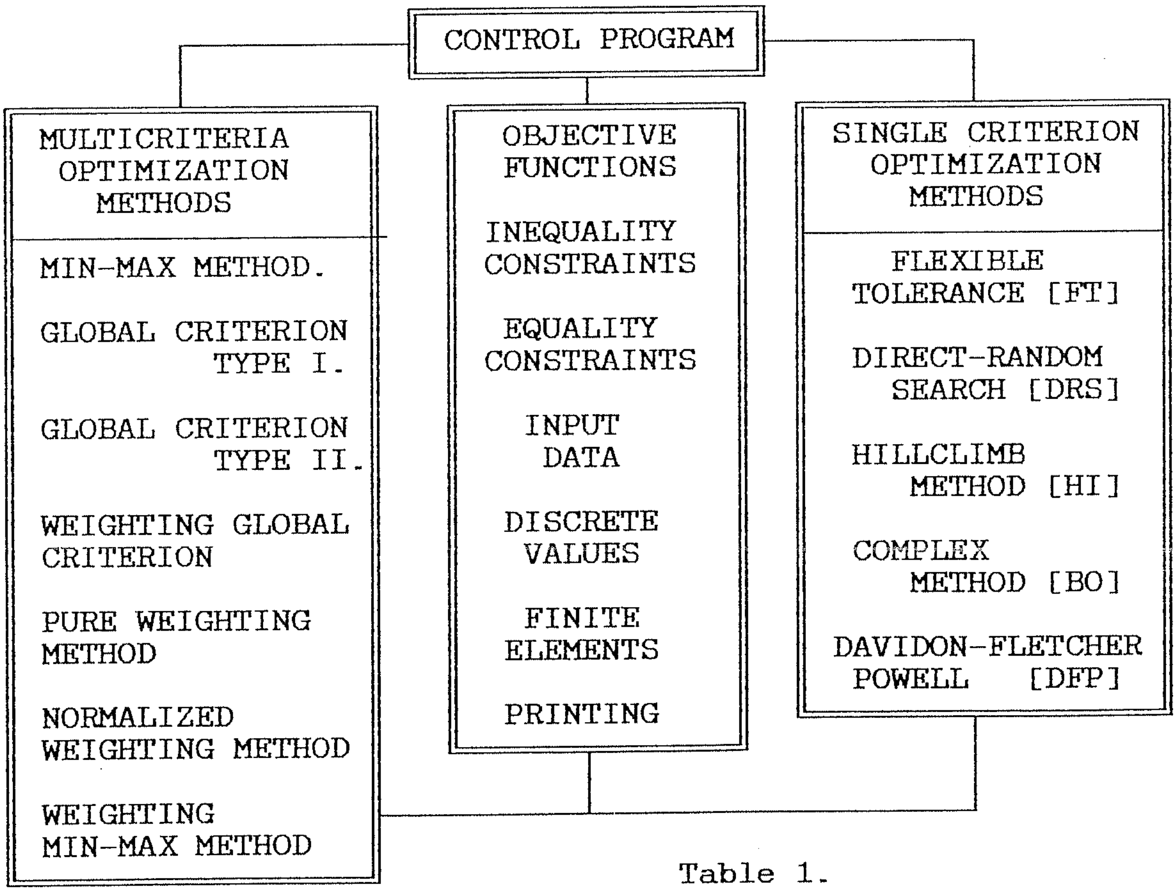


Table 1.

## 3. MULTICRITERIA OPTIMIZATION TECHNIQUES

A multicriteria optimization problem can be formulated as follows :

$$\text{Find } f(x^*) = \text{opt } f(x), \quad (9)$$

$$\text{such that } \quad g_j(x) \geq 0, \qquad j = 1,\ldots,M,$$

$$h_i(x) = 0, \qquad i = 1,\ldots,P.$$

where x is the vector of decision variables defined in N-dimensional Euclidean space and $f_k(x)$ is a vector function defined in K-dimensional Euclidean space. $g_j(x)$ and $h_l(x)$ are inequality and equality constraints.

The solutions of this problem are the Pareto optima. The definition of this optimum is based upon the intuitive conviction that the point x* is chosen as the optimal, if no criterion can be improved without worsening at least one other criterion.

### 3.1 THE MIN-MAX METHOD

The min-max optimum compares relative deviations from the separately reached minima. The relative deviation can be calculated from

$z_i'(x) = |(f_i(x)-f_i^0)| / |f_i^0|$ and $z_i''(x) = |(f_i(x)-f_i^0)| / |f_i(x)|$

If we know the extremes of the objective functions which can be obtained by solving the optimization problems for each criterion separately, the desirable solution is the one which gives the smallest values of the increments of all the objective functions. The point x* may be called the best compromise solution considering all the objective functions simultaneously and on equal terms of importance.

$$z_i(x) = \max \{ z_i'(x), z_i''(x) \} \qquad i \in I \tag{10}$$

$$\mu(x^*) = \min \max \{ z_i(x) \} \qquad x \in X, \quad i \in I \tag{11}$$

where X is the feasible region.

### 3.2 THE WEIGHTING MIN-MAX METHOD

Combining the min-max approach with the weighting method, a desired representation of Pareto optimal solutions can be obtained [9]

$$z_i(x) = \max \{ w_i\, z_i'(x), w_i\, z_i''(x) \} \qquad i \in I \tag{12}$$

The weighting coefficients $w_i$ reflect exactly the priority of the criteria, the relative importance of it. We can get a distributed subset of Pareto optimal solutions.

### 3.3 GLOBAL CRITERION METHOD

A function which describes a global criterion is a measure of closeness the solution to the ideal vector of $f^0$. The common form of this function is (type I) :

$$f(x) = \sum_i ( ( f_i^0 - f_i(x) ) / f_i^0 )^p \tag{13}$$

It is suggested to use P=2, but other values of P such as 1,3,4, etc. can be used. Naturally the solution obtained will differ greatly according to the value of P chosen.

It is recommended to use relative deviations (type II) :

$$L_P(f) = \left[ \sum_i \left| f_i^0 - f_i(x) \right|^P \right]^{1/P} \qquad 1 \le P \le \infty \qquad (14)$$

3.4 WEIGHTING GLOBAL CRITERION METHOD

Using weighting parameters we could get a great number of Pareto optima with (13). If we choose P=2, which means the Euclidean distance between Pareto optimum and ideal solution [1]. The coordinates of this distance are weighted by the parameters as follows:

$$L_r(f) = \left[ \sum_i w_i \left| f_i^0 - f_i(x) \right|^P \right]^{1/P} \qquad 1 \le P \le \infty \qquad (15)$$

3.5 PURE WEIGHTING METHOD

The basis of this method consists in adding all the objective functions together using different weighting coefficients for each. It means, that we transform our multicriteria optimization problem to a scalar one by creating one function of the form:

$$f(x) = \sum_i w_i f_i(x) \qquad \text{where} \quad w_i \ge 0 \quad \text{and} \quad \sum_i w_i = 1 \qquad (16)$$

If we change the weighting coefficients result of solving this model can vary significantly, and depends greatly from the nominal value of the different objective functions.

3.7 NORMALIZED WEIGHTING METHOD

At the pure weighting method, the weighting coefficients do not reflect proportionally the relative importance of the objective, because of the great difference on the nominal value of the objective functions. At the normalized weighting method $w_i$ reflect closely the importance of objectives, all functions are expressed in units.

$$f(x) = \sum_i w_i f_i(x) / f_i^0 \qquad (17)$$

The condition $f_i^0 \neq 0$ is assumed.

4.1 OPTIMUM DESIGN OF PLANE FRAMES USING FEM

Objective functions:

- mass of the frame,
- cost of the frame containing material, welded and surface preparation costs,
- mass/floor space ratio to be minimized.

Unknowns: height of webs at columns and rafters at pin, apex and eaves points, thickness of webs at columns and rafters, width and thickness of flanges both at columns and rafters. The 10th variable is the span length of the single bay frame.

Constraints: static stress constraints at different stress-maximum points at columns and rafters, local web and flange buckling, lateral buckling for the compressed flange, elastic lateral buckling at the eaves points both in columns and rafters. Vertical and horizontal displacement of the frame, size constraints.

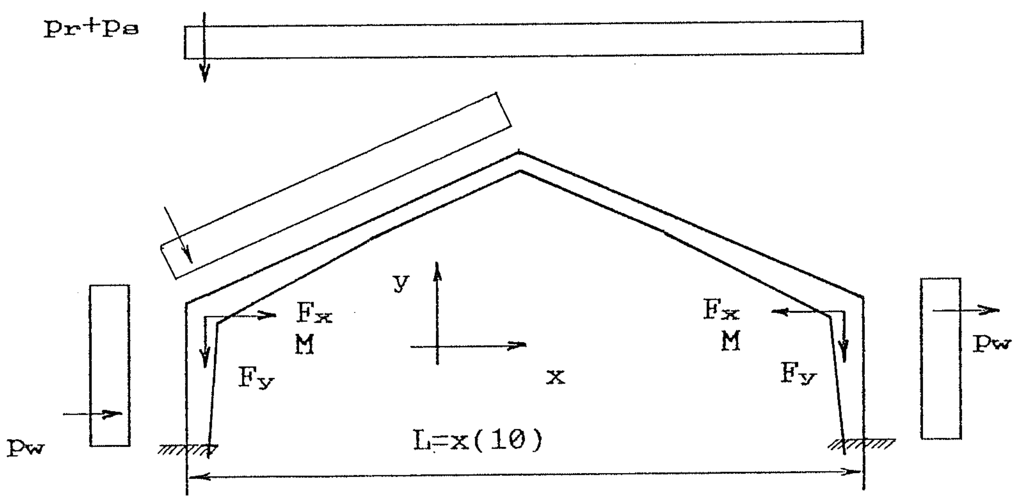


Fig. 1. The single bay welded plane frame structure

The frame can be a gantry one, with a crane runway on it. In this case the Hillclimb method was the most efficient, it could find more quickly the optima, using FEM subprograms for stress and displacement calculations at different topology. It is possible to use higher strength steel.

## 4.2 OPTIMUM DESIGN OF A MAIN GIRDER OF AN OVERHEAD TRAVELLING CRANES

Objective functions:
- mass of the main girder,
- welded cost,
- surface preparation costs,
- total cost to be minimized.

Unknowns: height and thicknesses of webs, h, $t_{w1}$, $t_{w2}$, width b, and thickness of flanges $t_f$.

Constraints: stresses, web bucklings, flange bucklings due to main loading, stresses, web bucklings, flange buckling due to total loading, fatigue constraints on weldments, deflection of the girder, size constraints. There is a possibility of using higher strength steels [11].

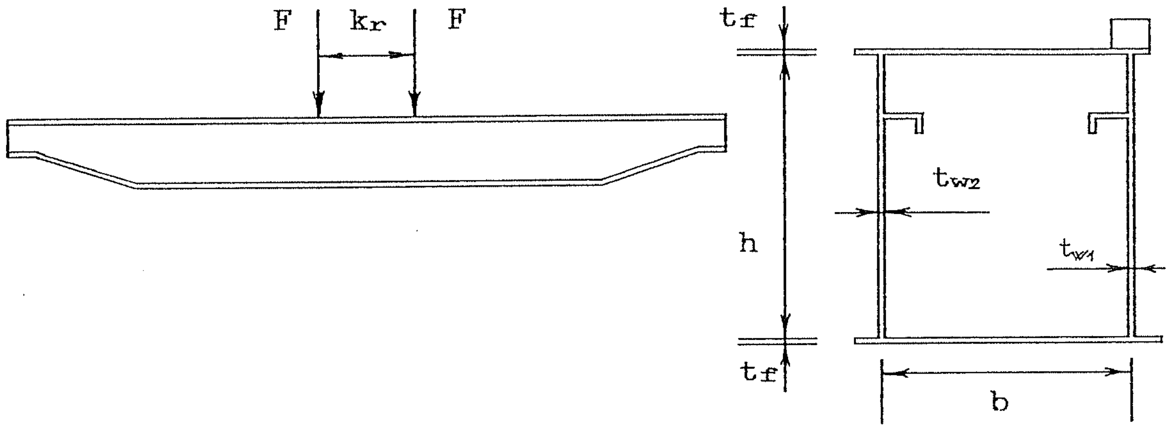


Fig. 2. Cross section of the welded main girder of overhead travelling crane

The Complex method could find quickly the global optima of the multiobjective optimization problem in this case.

## 4.3 OPTIMUM DESIGN OF MACHINE-TOOLS SPINDLE-BEARING SYSTEMS

Objective functions:
- mass of the spindle to be minimized,
- rigidity of the spindle-bearing system to be maximized,
- eigenfrequency of the system to be maximized.

Unknowns: length between the bearings, diameter of the spindle at bearing.

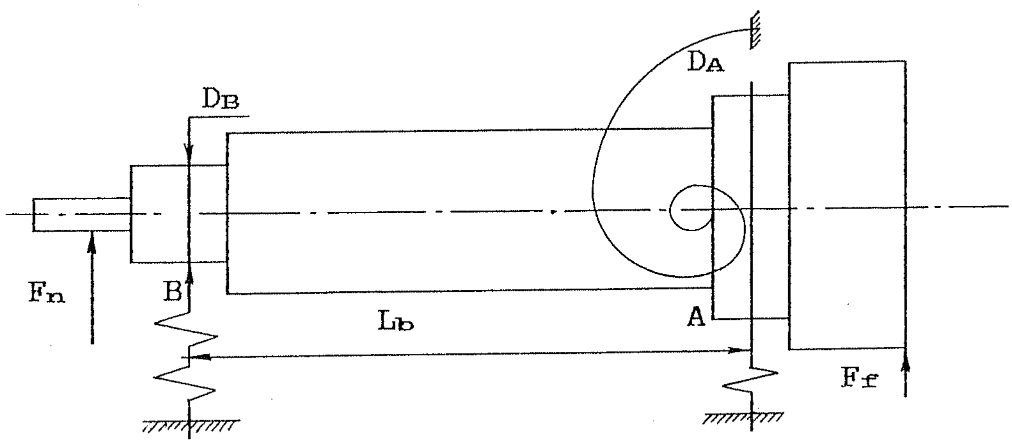


Fig. 3. The spindle-bearing system

Constraints: radial displacement, radial rigidity, eigenfrequency, size constraints.

The Flexible Tolerance method was very efficient in finding the optimum using the finite strip method, but it needed more computation time [12]. The values of the three objectives at single-objective optimization and using the min-max multiobjective method, where the relative importances are the same, can be seen on Fig. 4.

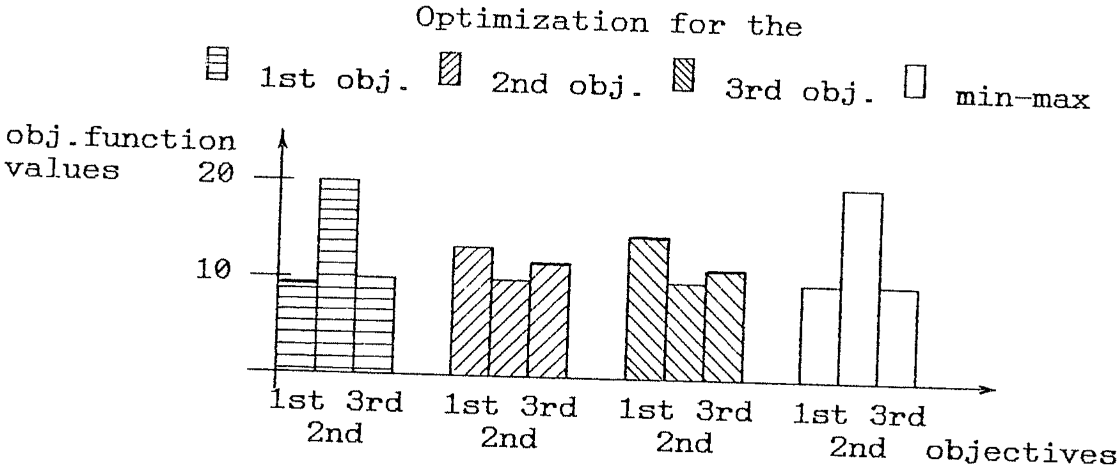


Fig. 4. Solution of the single- and multiobjective optimization

## 4.4 OPTIMUM DESIGN OF CELLULAR PLATES

Objective functions:

- mass of the plate,
- cost of the plate including material and welding costs,
- deflection of the cellular plate to be minimized.

Unknowns: thicknesses of cover plates and stiffeners, height of stiffeners, number of stiffeners.

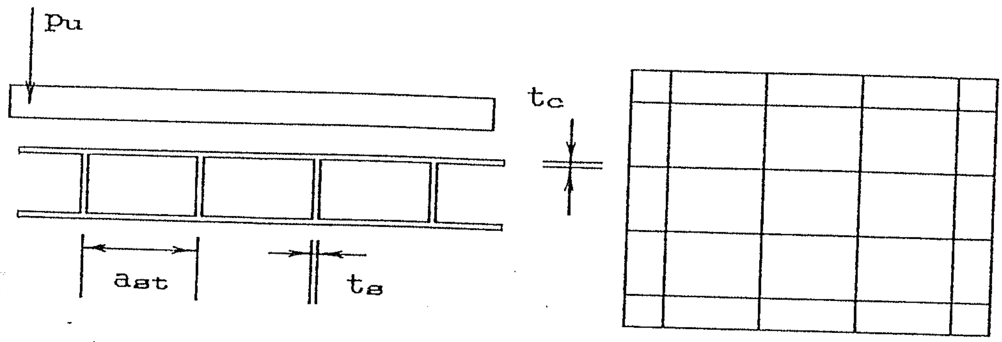


Fig. 5. The stiffened, welded plate structure

Constraints: normal stress constraints at cover plates, shear constraints at stiffeners, deflection constraints, local buckling constraints, size constraints. There is a possibility of using higher strength steels.

The Direct-Random Search technique was very useful in this case. It usually gives global optimum for the structure.

## 4.5 OPTIMUM DESIGN OF COMPRESSED TRUSS MEMBERS

A mathematical programming technique is used to determine the optimal sizes of a K-type truss of parallel chords with gap joints, welded from tubular hollow sections.

Objective function is the volume (weight) of the whole truss.

Unknown variables are the diameter and the thicknesses of chord and brace members.

Constraints: there are 9 inequality constraint considered; overall buckling of compression members, static strength of welded joints and geometric limitations on sizes are considered. The regulations of the Eurocode 3. is used.

The Hillclimb method of Rosenbrock is used in an illustrative numerical example.
The truss can be seen on Fig. 6. The optimum is at $\Omega=h/a=1.23$

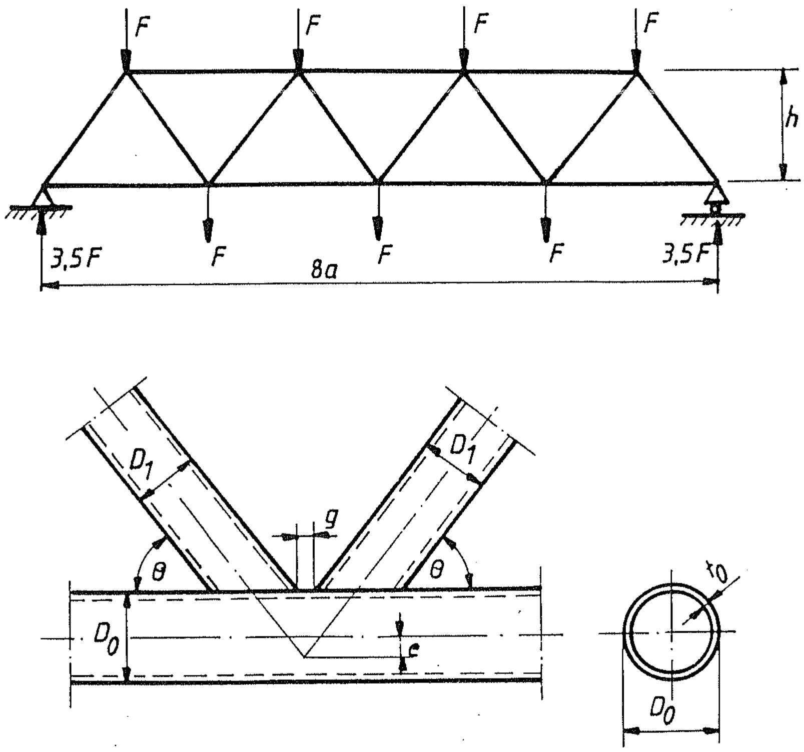


Fig. 6.

The program system was made in MS FORTRAN 5.0 on PC/AT/386 compatible computer. If we write the programs for example in Turbo C language, at that case the Complex method was quicker than the Hillclimb, but in FORTRAN the Hillclimb method was the quickest one, but usually gave local optimum. We have made some of the optimization programs in Quick Basic and the developing time was much smaller, but the runtime was longer. All the single-objective methods can find a feasible starting point, and give an optimum with unbounded and with discrete values.

There is a range of discrete values is given for every variables. We would like to install to the system some other techniques such as Sequential Unconstrained Minimization Technique (SUMT) of Fiacco, Mc Cormic [14], the Sequential Quadratic Programming (SQP) of Zhou and Tits [15], the Method of Moving Asymptotes (MMA) of Svanberg [16]. Similar in these techniques that all of them need the computation of derivatives.

## 5. REFERENCES

[1] Jármai,K.: Single- and multicriteria optimization as a tool of decision support system. Computers in Industry, 1989, Vol.11. No.3. p.249-266.

[2] Himmelblau,D.,M.: Applied nonlinear programming. McGraw-Hill , New York, 1972.

[3] Weisman,J.: MINIMAL, a combined optimization technique. Phd Dissertation, University of Pittsburgh, PA, 1968.

[4] Rosenbrock,H.,H.: An automatic method for finding the greatest or least value of a function. Computer Journal, 1960, Vol.3. No.3. p.175-184.

[5] Box,M.,J.: A new method of costrained optimization and a comparison with other methods. Computer Journal, 1965, Vol.8. p.42-52.

[6] Fletcher,R.,Powell,M.,J.,D.: A rapidly convergent descent method for minimization. Computer Journal, 1963, Vol.6. No.2. p.163-168.

[7] Osyczka,A.: Multicriterion optimization in engineering. Ellis Horwood, Chichester, United Kingdom, 1984.

[8] Jármai,K.: Application of decision support system on sandwich beams, verified by experiments. Computer in Industry. 1989, Vol.11. No.3. p.267-274.

[9] Farkas,J.,Jármai,K.: Minimum cross-sectional area of centrally compressed struts of square section with longitudinal stiffeners. Stability of Steel Structures Colloquium, Budapest, Apr. 25-27. 1990, Preliminary report, Vol.III. p.261-268.

[10] Jármai,K.: Design of economic stiffened box girders, experiments for the local buckling interaction. Publications of the Techn. Univ. Miskolc, Series C. Mechanical Engineering, 1990, Vol.43. p.107-126.

[11] Jármai,K.: Decision support system on IBM PC for design of economic steel structures, applied to crane girders. Thin-walled structures, Elsevier Applied Science Publishers, 1990, Vol. 10. p.143-159.

[12]Jármai,K.,Tompa,S.: Multiobjective optimization of spindle-bearing systems.(in Hungarian) MicroCAD'91, University of Miskolc, 1991. Febr.26 - March 2. Proceedings Vol. 1. p.VI/10-18.

[13] Farkas,J.: Optimum design of steel structures. Ellis Hoorwood Ltd. Chichester, 1984.

[14] Fiacco,A.,V.,Mc Cormic,G.,P.: Nonlinear programming: sequential unconstrained minimization techniques, Wiley, New York, 1968.

[15] Zhou,J.L., Tits,A.L: FSQP Version 2.3b Sept. 1991. Electrical Engineering Department and Systems Research Centre, University of Maryland. Md 20742.

[16] Svanberg,K.: Method of Moving Asymptotes (MMA) with some extensions. NATO/DFG ASI seminar on Optimization of Large Structural Systems, Berchtesgaden, Sept. 23- Oct. 4, 1991. Lecture notes, Vol.1. p. 55-66.