TECHNICAL UNIVERSITY OF BUDAPEST
HUNGARIAN ACADEMY OF SCIENCES
STRUCTURAL STABILITY RESEARCH COUNCIL
INTERNATIONAL ASSOCIATION FOR BRIDGE AND
STRUCTURAL ENGINEERING

INTERNATIONAL COLLOQUIUM
EUROPEAN SESSION

STABILITY OF STEEL STRUCTURES

PRELIMINARY REPORT

VOLUME II

HUNGARY, BUDAPEST
SEPTEMBER 21-23, 1995

(1)

FARKAS, József (1)
JÁRMAI, Károly (2)

STABILITY CONSTRAINTS IN THE OPTIMUM DESIGN OF TUBULAR TRUSSES

International Colloquium
STABILITY OF STEEL STRUCTURES
Budapest, Hungary, 1995
PRELIMINARY REPORT

Summary

Several authors have used rough approximations for stability constraints of compressed members in trusses to simplify the optimum design procedure. It is shown that the use of the Euler buckling curve instead of the Eurocode 3 column buckling formula causes 19-35% error in the unsafe side, so it is not suitable for optimum design. Moreover the limiting local slenderness of thin-walled circular hollow sections (CHS) should be taken according to Eurocode 3 (d/t)lim =70*235/f_y (f_y is the yield stress in MPa) instead of 10 also used by several authors, since this low value leads to uneconomic design. The importance of stability constraints is illustrated by a numerical example of a K-type truss with parallel chords and gap joints welded from CHS struts.

1. Introduction

Modern structures should be safe and economic. The safety is achieved by using stability constraints which describe the behaviour of structures realistically. The economy can be realized by using optimum design to minimize the cost or weight of the structure.

Authors dealing with the optimum design of metal structures make in some cases simplifications to solve the problems easier. E.g. in the optimization of trusses they neglect the overall buckling of compressed members or use too simple stability constraints such as the Euler buckling curve.

It is well known that the Euler buckling curve neglects the very important effect of initial crookedness and residual stresses caused by fabrication processes (welding, cold-forming). These effects can be described only by a more complicated mathematical form. It will be shown in the present paper that the use of the Euler buckling curve causes unsafe design which is not permissible.

(1) Professor, Dr. of Techn. Sci.
(2) Associate Professor, PhD. University of Miskolc, H-3515 Miskolc, Hungary

(2)

Furthermore, the suitable optimum design procedure will be described using all stability constraints necessary for safe design. The case of welded thin-walled tubular trusses is selected for this purpose, in which not only the constraints on overall buckling, but also the constraints on local buckling of plate elements should be considered. The consideration of all important constraints will be illustrated by a numerical example of a simple tubular truss welded from CHS rods.

2. Unsafe design using the Euler buckling curve

Authors dealing with the optimum design of tubular trusses have neglected the overall buckling of compression members prescribing constant permissible stresses for tension and compression rods (e.g. *Khot and Berke* 1984), or the overall buckling is considered by the Euler buckling formula (e.g. *Vanderplaats and Moses* 1972, *Saka* 1980, *Amir and Hasegawa* 1994)

$$\sigma_E = \pi^2 E / \lambda^2; \quad \lambda = KL/r; \quad r = \sqrt{I_x / A} \tag{1}$$

where E is the elastic modulus, λ is the slenderness, K is the end restraint factor (for pinned ends $K = 1$), I_x is the moment of inertia, A is the cross-sectional area, r is the radius of gyration.

For CHS, using the notation $\delta = D/t = (d-t)/t$, where D is the mean diameter and d is the outside diameter, t is the thickness, the following formulae are valid

$$I_x = \frac{\pi D^3 t}{8} = \frac{\pi D^4}{8\delta}; \; A = \frac{\pi D^2}{\delta}; \; r = \frac{D}{\sqrt{8}} = a\sqrt{A}; \; a = \sqrt{\frac{\delta}{8\pi}} \tag{2}$$

Thus,

$$\sigma_E = \frac{\pi E A}{8 K^2 L^2} \delta \tag{3}$$

It can be seen that the local slenderness δ plays an important role in the buckling strength, therefore the selection of the limiting value δ_L influences the optimum design significantly. The first author has verified (*Farkas* 1992) that the local buckling constraint is active in the optimum design of a concentrically compressed CHS strut. E.g. *Vanderplaats and Moses* (1972) have selected for steel tubes the value of $\delta_L = 10$, and this value has been used also by *Saka* (1980) and *Amir and Hasegawa* (1994) (note that in *Amir and Hasegawa* (1994) in Eq.(3) the erroneous value of 3 is printed instead of 8). Since in the Eurocode 3 (1992) $\delta_L = 70*235/f_y$ is given for Class 2 sections to be used in tubular trusses, i.e. 70 for a steel of yield stress $f_y = 235$ MPa and 50 for $f_y = 355$ MPa, the value of 10 is incorrect and leads to uneconomic solutions.

In the contrary, the use of the Euler formula leads to unsafe solutions, since it does not take into account the initial crookedness and residual stresses. In (*Saka* 1990) the AISC buckling curve has been used. *Farkas and Jármai* (1994) have applied the Eurocode 3 buckling formulae and have shown that the optimal slope angle of a roof truss depends on the cross-section type of compression members and the use of CHS is much more economic than that of double angle profile.

In the following we compare the cross-sectional areas of a CHS compressed strut calculated from the Euler curve and from the Eurocode 3 buckling formula. In the calculations the values of f_y =355 MPa, $a_L = \sqrt{50/(8\pi)} = 1.4105$ and $K = 1$

(3)

are used. Using Eq. (2) the slenderness can be expressed by A as follows.

$$\lambda^2 = \frac{L^2}{r^2} = \frac{L^2}{a^2 A} = \frac{10^4}{a^2} \cdot \frac{1}{10^4 A / L^2} = \frac{5027}{10^4 A / L^2} \qquad (4)$$

The overall buckling constraint, using the Euler formula, is

$$\frac{N}{A} \le \chi f_y \quad ; \quad \chi = \frac{1}{\bar{\lambda}^2} \quad \text{for} \quad \bar{\lambda} \ge 1 \qquad (5)$$

$$\chi = 1 \quad \text{for} \quad \bar{\lambda} \le 1$$

where $\bar{\lambda} = \lambda / \lambda_E; \quad \lambda_E = \pi \sqrt{E / f_y} = 76.4091$ (6)

From

$$\frac{10^4 N / L^2}{10^4 A / L^2} \le \frac{f_y}{\bar{\lambda}^2} = \frac{f_y \lambda^2{}_E}{\lambda^2} \qquad (7)$$

using Eq. (4) one obtains $\frac{10^4 A}{L^2} = \frac{1}{76.4091} \sqrt{\frac{5027}{355}} \sqrt{\frac{10^4 N}{L^2}} = 0.049247 \sqrt{\frac{10^4 N}{L^2}}$ (8)

valid for $\lambda \ge \lambda_E$. For $\lambda \le \lambda_E$ taking $\chi = 1$ in Eq.(5) we get

$$\frac{10^4 A}{L^2} \ge \frac{10^4 N}{L^2 f_y} \qquad (9)$$

According to the Eurocode 3 the overall buckling constraint is

$$\frac{N}{A} \le \frac{\chi f_y}{\gamma_{M1}}; \; \gamma_{M1} = 1.1; \; \frac{1}{\chi} = \phi + \sqrt{\phi^2 - \bar{\lambda}^2}$$

$$\phi = 0.5\left[1 + 0.34\left(\bar{\lambda} - 0.2\right) + \bar{\lambda}^2\right] \qquad (10)$$

Introducing the symbols $c_o = 100K/\lambda_E$, $x = 10^4 N/L^2$ and $y = 10^4 A/L^2$, where L [mm] is the strut length, A [mm^2] is the required cross-sectional area, N is the factored compressive force in [N], Eq. (10) can be written as

$$\frac{\gamma_{M1} x}{f_y} \le \frac{y}{\phi + \sqrt{\phi^2 - \frac{c_o^2}{a^2 y}}}$$

$$\phi = 0.5\left[1 + 0.34\left(\frac{c_o}{a\sqrt{y}} - 0.2\right) + \frac{c_o^2}{a^2 y}\right]; \quad \lambda = \frac{100K}{a\sqrt{y}} \qquad (11)$$

Table 1. Required $10^4 A/L^2$-values for some $10^4 N/L^2$-values in the case of a compressed CHS strut, f_y = 355 MPa, K=1

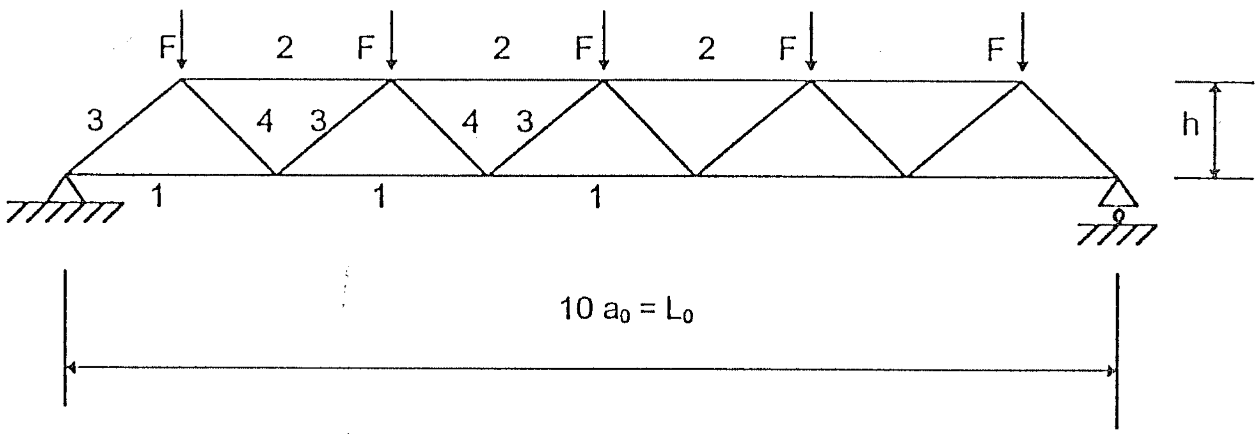
	$10^4 N / L^2 \left[\frac{\text{N}}{\text{mm}^2}\right]$	10	100	305.7	1000	10000
	Euler	0.1557	0.4925	0.8610	2.8169	28.17
$\frac{10^4 A}{L^2}$	Eurocode	0.1766	0.6273	1.3171	3.4975	30.60
	difference %	12	21	35	19	8
λ	Eurocode	168	89	66	38	13

(4)

A computer method is used to calculate y for a given x. Results are summarized in Table 1. It can be seen that the results obtained by the Euler formula are unsafe by 19-35% in the range of λ = 38 - 89, so the Euler formula gives incorrect solutions.

3. Numerical example of a tubular truss

In order to illustrate the role of stability constraints we select a simple planar, statically determinate, K-type truss with parallel chords and gap joints, welded from CHS rods (Fig.1). In the optimum design the optimal distance of chords h is sought which minimizes the total volume of the structure and the dimensions of rods fulfil the design constraints. The structural members are divided to 4 groups of equal cross-section as follows: 1 - lower chord, 2 - upper chord, 3 - compression braces, 4 - tension braces.



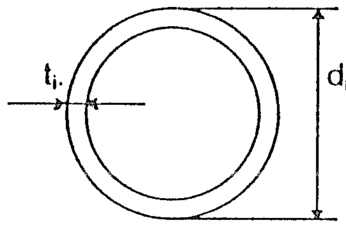


Fig. 1. Planar truss with parallel chords. The numbering relates to groups of members of equal cross-section

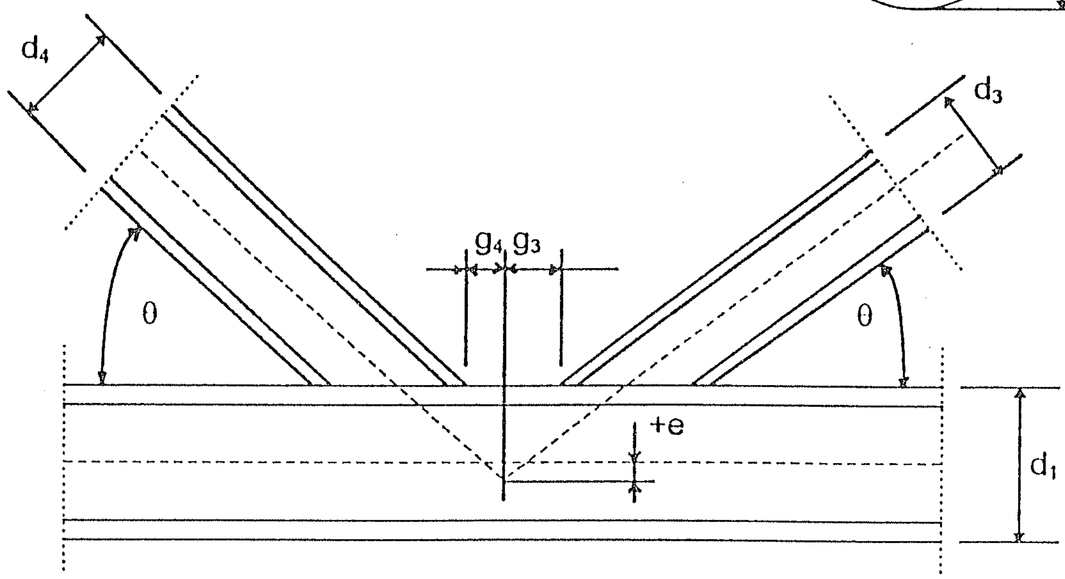


Fig. 2. K-type gap joint with eccentricity e

(5)

According to DIN 2448 and DIN 2458 (*Dutta and Würker* 1988) the available CHS have the following dimensions (discrete values):

d = 133, 139.7, 152.4, 159, 168.3, 177.8, 193.7, 219.1, 244.5, 273, 298.5, 323.9

t = 2.9, 3.2, 3.6, 4, 4.5, 5, 5.6, 6.3, 7.1, 8, 8.8, 10.

All members are made from steel Fe 510 with ultimate strength f_u = 510 MPa and yield stress f_y = 355 MPa.

The load is shown in Fig.1, the factored value of the static forces is F = 200 kN. Calculate the required cross-sections for various values of $\omega = h/a_o$ to select the ω_{opt} which minimizes the total volume V. The variables are as follows: d_i and t_i (i=1,2,3,4). The objective function is expressed as

$$\frac{V}{2\pi a_o} = 5(d_1 - t_1)t_1 + 4(d_2 - t_2)t_2 + 3\sqrt{\omega^2 + 1}(d_3 - t_3)t_3 + 2\sqrt{\omega^2 + 1}(d_4 - t_4)t_4 \qquad (12)$$

The constraints are as follows.

Local buckling constraints for all sections according to *Wardenier et al.* (1991) are

$$d_i/t_i \le 50 \qquad (13)$$

Stress constraint for tension members are

$$\frac{S_{1max}}{\pi(d_1 - t_1)t_1} \le \frac{f_y}{\gamma_{Mo}}; \quad S_{1max} = \frac{6.5F}{\omega}; \quad \gamma_{Mo} = 1.1 \qquad (14)$$

$$\frac{S_{4max}}{\pi(d_4 - t_4)t_4} \le \frac{f_y}{\gamma_{Mo}}; \quad S_{4max} = \frac{1.5F}{\omega}\sqrt{\omega^2 + 1} \qquad (15)$$

Overall buckling constraints for compression members according to Eurocode 3. are as follows

Upper chord: $$\frac{S_{2max}}{\pi(d_2 - t_2)t_2} \le \frac{\chi_2 f_y}{\gamma_{M1}}; \quad S_{2max} = \frac{6F}{\omega}; \quad \gamma_{M1} = 1.1 \qquad (16)$$

$$\chi_2 = \frac{1}{\phi_2 + \sqrt{\phi_2^2 - \bar{\lambda}_2^2}}; \quad \phi_2 = 0.5\left[1 + 0.34(\bar{\lambda}_2 - 0.2) + \bar{\lambda}_2^2\right]$$

$$\bar{\lambda}_2 = \frac{\lambda_2}{\lambda_E} = \frac{K_2 L_2}{\lambda_E r_2} = \frac{0.9 * 2a_o\sqrt{8}}{\lambda_E(d_2 - t_2)}.$$

With $E = 2.1 \cdot 10^5$ MPa and $f_y = 355$ MPa $\lambda_E = \pi\sqrt{E/f_y} = 76.4091$.

$K_2 = 0.9$ is the end restraint factor according to *Rondal et al.* (1992), $r_2 = (d_2 - t_2)/\sqrt{8}$ is the radius of gyration.

Compression braces:

$$\frac{S_{3max}}{\pi(d_3 - t_3)t_3} \le \frac{\chi_3 f_y}{\gamma_{M1}}; \quad S_{3max} = \frac{2.5F}{\omega}\sqrt{\omega^2 + 1} \qquad (17)$$

$$\chi_3 = \frac{1}{\phi_3 + \sqrt{\phi_3^2 - \bar{\lambda}_3^2}}; \quad \phi_3 = 0.5\left[1 + 0.34(\bar{\lambda}_3 - 0.2) + \bar{\lambda}_3^2\right]$$

$$\bar{\lambda}_3 = \frac{\lambda_3}{\lambda_E} = \frac{K_3 L_3}{\lambda_E r_3} = \frac{0.75a_o\sqrt{\omega^2 + 1}\sqrt{8}}{\lambda_E(d_3 - t_3)}$$

(6)

In order to ease the fabrication the diameter of braces should be smaller than those of chords:

$$d_3 = 0{,}92d_1;\quad d_3 \le 0.92d_2;\quad d_4 \le 0.92d_1;\quad d_4 \le 0.92d_2 \tag{18}$$

Prescription for the joint eccentricity to avoid too large additional bending moment in the vicinity of nodes is as follows (Fig. 2.):

$$e \le 0.25d_1;\quad e \le 0.25d_2 \tag{19}$$

The eccentricity can be expressed by d_i, angle θ and gap parts g_3 and g_4 as follows:

$$\mathrm{tg}\theta = \frac{e + d_1/2}{g_3 + d_3/(2\sin\theta)} \quad \text{or} \quad \mathrm{tg}\theta = \frac{e + d_1/2}{g_4 + d_4/(2\sin\theta)} \tag{20}$$

Assuming that

$$g_3 = g_4 = 0{,}05\,d_1 \quad \text{or} \quad 0.05\,d_2 \tag{21}$$

the geometry constraints can be given by:

$$\frac{d_3}{2}\sqrt{\omega^2 + 1} + d_1(0.05\omega - 0.75) \le 0 \tag{22}$$

and

$$\frac{d_3}{2}\sqrt{\omega^2 + 1} + d_2(0.05\omega - 0.75) \le 0 \tag{23}$$

Constraint on static strength of welded joints between chords and braces according to Eurocode 3 is

$$\sqrt{\sigma_\perp^2 + 3\left(\tau_\perp^2 + \tau_{II}^2\right)} \le f_u/(\beta_w \gamma_{MW}) \tag{24}$$

$$f_u = 510\ \text{MPa},\quad \beta_w = 0.9,\quad \gamma_{Mw} = 1.25.$$

From the force S in a brace the following stress components arise in welds:

$$\sigma_\perp = \tau_\perp = \frac{S\sin\theta}{\pi d a_w}\cdot\frac{\sqrt{2}}{2};\quad \tau_{II} = \frac{S\cos\theta}{\pi d a_w} \tag{25}$$

where a_w is the fillet weld dimension. Substituting Eq. (25) into Eq. (24) we get

$$\frac{S}{\pi d a_w}\sqrt{\frac{2\omega^2 + 3}{\omega^2 + 1}} \le 453\text{MPa} \tag{26}$$

For the maximal value of a_w the corresponding brace thickness can be taken. This constraint should be fulfilled for S_3 and S_4.

For the node strength the following constraints should be fulfilled (*Wardenier et al.* 1991).

Constraints on chord plastification.

In the joint of rods 1 and 3:

$$S_{3\max} \le S_{31}^{*} = \frac{f_y t_1^2}{\sin\theta}\left(1.8 + 10.2\frac{d_3}{d_1}\right) f_1(\gamma_1, g_1') \tag{27}$$

$$f_1(\gamma_1, g_1') = \gamma_1^{0.2}\left[1 + \frac{0.024\gamma_1^{1.2}}{\exp(0.5g_1' - 1.33) + 1}\right];\quad \gamma_1 = \frac{d_1}{2t_1}$$

$$g_1' = g_1/t_1;\quad \text{we assume that}\quad g_1 = g_3 + g_4 = 0.1d_1$$

Constraints on chord plastification for joints of rods 1 - 4, 2 - 3 and 2 - 4 can be formulated similarly to Eq. (27), therefore these constraints are not detailed here.

Constraints on punching shear.

(7)

In the joint of rods 2 and 3:

$$S_{3\max} \le \frac{f_y}{\sqrt{3}} t_2 \pi d_3 \frac{1+\sin\theta}{2\sin^2\theta} \tag{28}$$

Note that the constraint on punching shear was in our calculations always passive, so it is not necessary to investigate it for other joints.

For the computations the Rosenbrock's hillclimb mathematical programming method has been used treating the unknowns as continuous variables. After the determination of the optimal dimensions the discrete optima have been found by using an additional search. The results are summarized in Table 2.

Table 2. Optimal discrete dimensions [mm] and $V/(2\pi a_0)$ - values [mm^2] for various $\omega = h/a_0$ - values.

$\omega = h/a_0$	0.8	0.9	1.0	1.1	1.2	1.3	1.4
d_1/t_1	244.5/8	244.5/8	244.5/8	219.1/8	273/8	273/8	298.5/8.8
d_2/t_2	273/8	244.5/8	244.5/8	219.1/8.8	273/8	273/8	298.5/8.8
d_3/t_3	219.1/4.5	219.1/4.5	219.1/4.5	193.7/4.5	219.1/4.5	219.1/4.5	293.7/4.5
d_4/t_4	159/3.6	152.4/3.6	152.4/3.2	152.4/3.2	139.7/3.2	139.7/3.2	139.7/2.9
$V/(2\pi a_0)$	23083	22367	22475	21063	24970	25264	28704

The optimal value is ω = 1.1, the difference between the best and worst solution in the range of ω =0.8 - 1.4 is 100/(28704 - 21063)/21063 = 36%. The checks of constraints are summarized in Table 3.

Table 3. Check of the constraints for the optimal solution ω = 1.1

Constraint	Dimension	Eq.	Rod 1	2	3	4	Remarks
Local buckling	-	(13)	27<50	25<50	43<50	48<50	active for rods 3, 4
Tensile stress	MPa	(14) (15)	223<323	-	-	270<323	near active for rod 4
Overall buckling	MPa	(16) (17)	-	188<204	240<261	-	active for rods 2, 3
Fabrication	mm	(18)	-	-	194<202	152<202	active for rod 3
Eccentricity	mm	(22) (23)	-	-	-8.32	-	near active for rods 1,2,3
Weld strength	MPa	(26)	-	-	368<453	414<453	near active for rod 4
Chord plastification	kN	(27)	-	-	642<713	405<586	active for rods 3-1
Punching shear	kN	(28)	-	-	642<1744	-	passive

It can be seen that the overall buckling constraint is always active, the local buckling constraint is passive only for chord 2, since for thickness t_2 the chord plastification is governing. Thus, it can be stated that the effect of stability constraints in the optimum design of tubular trusses is significant.

(8)

4. Conclusions

It is shown that the use of the Euler buckling curve instead of the Eurocode 3 overall buckling formula causes 19 - 35% error in the unsafe side in the most important slenderness range of 38 - 89, so it should not be used in the optimization of tubular trusses. The application of limiting tube local slenderness d/t = 10 instead of 50 leads to uneconomic solutions.

The significant role of the stability constraints in the optimum design of tubular trusses is illustrated by a numerical example. In this optimum design procedure the dimensions of CHS truss members and the optimal distance of chords are determined which give the minimum volume (weight) of the structure and fulfil the design constraints. The constraints relate to the overall buckling of compression members, to the joint eccentricity and static strength of joints. For the final optimal version realistic available discrete tube dimensions are determined.

Acknowledgements

This work received support from the Hungarian Fund for Scientific Research grants OTKA T-4479 and T-4407 as well as from the Hungarian Ministry of Culture and Education grant MKM 474/94.

References

Amir,H.M., Hasegawa,T. 1994: Shape optimization of skeleton structures using mixed-discrete variables. Structural Optimization 8, 125-130.

Dutta,D., Würker,K-G. 1988: Handbuch Hohlprofile in Stahlkonstruktionen. Köln, TŰV Rheinland GmbH.

Eurocode 3. 1992: Design of steel structures. Part 1.1. Brussels, CEN European Committee for Standardization.

Farkas,J. 1990: Minimum cost design of tubular trusses considering buckling and fatigue constraints. In "Tubular Structures. Eds. Niemi,E., Mäkeläinen,P. Elsevier, London-New York."pp.45l-459.

Farkas,J. 1992: Optimum design of circular hollow section beam-columns. In "Proceedings of the Second International Offshore and Polar Engineering Conference, San Francisco, 1992. ISOPE, Golden, Colorado, USA."pp.494-499.

Farkas,J., Jármai,K. 1994: Savings in weight by using CHS or SHS instead of angles in compressed struts and trusses. In "Tubular Structures VI. Proceedings of the 6th International Symposium, Melbourne, 1994. Eds. Grundy,P.,Holgate,A.,Wong,B. Balkema, Rotterdam - Brookfield."pp.4l7-422.

Khot,N.S.,Berke,L. 1984: Structural optimization using optimality criteria methods. In "New directions in optimum structural design. Eds. Atrek,E., Gallagher,R.H. et al. Wiley &Sons, Chichester, New York, etc."pp.47-74.

Rondal,J., Würker,K-G. et al. 1992: Structural stability of hollow sections. Köln,TŰV Rheinland.

Saka,M.P. 1980: Shape optimization of trusses. Journal of Structural Division Proc. ASCE 106, No.ST5, 1155-1174.

Saka,M.P. 1990: Optimum design of pin-jointed steel structures with practical applications. Journal of Structural Division Proc. ASCE 116, No.10. 2599-2620

Vanderplaats,G.N., Moses,F. 1972: Automated design of trusses for optimum geometry. Journal of Structural Division Proc. ASCE 98, No.ST3, 671-690.

Wardenier,J., Kurobane,Y. et al. 1991: Design guide for circular hollow section joints under predominantly static loading. Köln, TŰV Rheinland.